

# Inflationary Magnetogenesis with On-shell Local $U(1)$ Symmetry

Guillem Domènech, Chunshan Lin and Misao Sasaki

Center for Gravitational Physics, Yukawa Institute for Theoretical Physics, Kyoto University

**Abstract.** We propose a new mechanism for inflationary magnetogenesis in which the local  $U(1)$  symmetry is broken during inflation. Nevertheless it is shown that the  $U(1)$  symmetry is recovered on-shell. We find that it is free from both the strong coupling and back reaction problems, and can explain the origin of cosmic magnetic fields on intergalactic scales, whose existence has been strongly suggested by recent observations.

## 1. Introduction

Magnetic fields are present throughout the universe and play an important role in many astrophysical process. However, their origin has not been well understood yet. Recently it was reported that large scale magnetic fields of  $\gtrsim 10^{-15}\text{G}$  with coherence length of  $\gtrsim \text{Mpc}$  may be present in inter-galactic space [1, 2, 3, 4]. If such a magnetic field exists, no causal process, not even dynamo, after the recombination time ( $z \sim 1000$ ) can explain its origin. And it becomes even more difficult to explain with processes at  $z > 1000$  because of the horizon problem.

It is then natural to attribute the origin of such a large scale magnetic field to inflation. However, because the Friedmann universe is conformally flat, the conformal invariance of the electromagnetic field makes it impossible to allow any process that amplifies the magnetic field. The electromagnetic field remains in vacuum if it is in vacuum in the beginning, while any excitation from vacuum would have been diluted exponentially during inflation.

Many models with broken conformal invariance have been proposed [5, 6] One of the simplest,  $U(1)$  invariant models is [7, 8]

$$\mathcal{L}_{EM} = -\frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}, \quad (1)$$

where  $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ . This model is equivalent to Bekenstein's variable charge theory [9].

Unfortunately, it was shown that the model suffers from either back-reaction or strong coupling problem [10]. If  $f$  grows in time, the effective coupling becomes too large in the past, while if  $f$  decays in time, the energy density of the electromagnetic field becomes too large to be regarded as a small perturbation to inflation.



## 2. On-shell $U(1)$ symmetric model

To circumvent the above mentioned difficulty, we propose a model in which  $f$  grows in time, but it is free from both strong coupling and back-reaction problems. We consider the Lagrangian,

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu} - e_0f(\phi)\gamma^\mu A_\mu\bar{\psi}\psi \\ & +i\bar{\psi}\gamma^\mu(\partial_\mu + \Gamma_\mu)\psi - m\bar{\psi}\psi, \end{aligned} \quad (2)$$

where  $e_0$  is the value of the electric charge today,  $\psi$  is a charged fermion and  $\Gamma_\mu$  is the spin connection. For slowly varying  $f$ , e.g.,  $\dot{f}/f = O(H)$  where  $H$  is the Hubble parameter, we may absorb  $f$  into  $A_\mu$  by scaling  $A_\mu \rightarrow f^{-1}A_\mu$ . This gives

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi \\ & -F_{\mu\nu}Q^\mu A^\nu - \frac{1}{2}Q_\mu Q^\mu A_\nu A^\nu + \frac{1}{2}(Q_\mu A^\mu)^2, \end{aligned} \quad (3)$$

where  $Q_\mu \equiv -\partial_\mu(\ln f)$  and  $D_\mu \equiv \partial_\mu + ie_0A_\mu + \Gamma_\mu$ . From this it is obvious that the model is free from the strong coupling problem even if  $f$  grows in time.

Of course the price to pay is the broken local  $U(1)$  gauge symmetry. Nevertheless, if we use the equations of motion,

$$\nabla_\mu(f^2F^{\mu\nu}) - e_0f\bar{\psi}\gamma^\nu\psi = 0, \quad (4)$$

the divergence of it gives

$$\nabla_\mu(e_0f\bar{\psi}\gamma^\mu\psi) = 0, \quad (5)$$

which implies

$$\begin{aligned} \delta_g S = & -\int \sqrt{-g} e_0 f \partial_\mu \sigma \bar{\psi} \gamma^\mu \psi \\ = & -\int \sqrt{-g} [\nabla_\mu (e_0 f \sigma \bar{\psi} \gamma^\mu \psi) - \sigma \nabla_\mu (e_0 f \bar{\psi} \gamma^\mu \psi)], \end{aligned} \quad (6)$$

where  $\delta_g A_\mu = \partial_\mu \sigma$ . Thus the  $U(1)$  gauge symmetry is recovered on-shell. This suggests that this  $U(1)$  symmetry breaking may not be so harmful after all.

## 3. Magnetogenesis

It is known that if the function  $f$  behaves as  $f^2 = a^4$ , one would obtain a scale-invariant spectrum for the  $B$  field [11, 10]. But this implies the logarithmic IR divergence in the energy density. To avoid this, we assume

$$f^2(N) = \exp[-4N + (4\epsilon + \nu)N]; \quad N > 0, \quad (7)$$

where  $\epsilon \equiv -\dot{H}/H^2$  is the slow-roll parameter during inflation,  $\nu$  is a new parameter which is required to satisfy  $0 < \nu \ll 1$ , and  $N$  is the number of  $e$ -folds counted backward from the end of inflation. The requirement  $0 < \nu \ll 1$  guarantees the finiteness of the total energy density and an almost scale-invariant  $B$  spectrum. Note that the above behavior of  $f$  may be fairly easily realized, e.g. in power-law inflation, by

$$f = f_0 \exp\left[\alpha \frac{\phi}{M_p}\right] \quad (8)$$

with  $\alpha = O(1)$ , and  $M_p = (8\pi G)^{-1/2}$  is the Planck mass.

Because the model is essentially the same as the one discussed, e.g. in [11], apart from the  $U(1)$  breaking interaction with the charged fermion, one can simply repeat the computation given in the literature. The resulting magnetic spectrum is

$$P_B(k; N) = \frac{H(N)^4}{2\pi^2} e^{\nu(N-N_k)}, \quad (9)$$

where  $N_k$  is the number of  $e$ -folds at which the comoving scale  $k$  crossed the horizon. The energy density of the magnetic field is given by integrating over the whole momentum space (with UV cutoff at  $k = aH \equiv k_*(N)$ ),

$$\rho_B(N) = \frac{1}{2} \int_0^{k_*(N)} \frac{dk}{k} P_B = \frac{H^4(N)}{4\pi^2\nu}. \quad (10)$$

Now we require that the model is free from backreaction. Namely  $\rho_B$  must be much smaller than background energy density. This means

$$\frac{H_f^2}{M_p^2} \ll \nu \ll 1. \quad (11)$$

For reasonable choices of  $H_f$  this is easily satisfied.

The magnetic field at the end of inflation is

$$B_f \sim H_f^2 \nu^{-1/2} = 10^{-12} \nu^{-1/2} \left( \frac{H_f}{10^{-6} M_p} \right)^2 M_p^2. \quad (12)$$

Assuming an instantaneous reheating, the current amplitude of the  $B$  field is estimated as

$$B_0 \sim B_f \times \frac{T_{\text{CMB}}^2}{M_p H_f} \sim 10^{-12} \nu^{-1/2} \left( \frac{H_f}{10^{-6} M_p} \right) G, \quad (13)$$

where  $T_{\text{CMB}}$  is the current CMB temperature. This satisfies the most recent observational constraint  $B_{\text{Mpc}} < 10^{-9} G$  by Planck [12], for

$$10^{-6} < \nu \ll 1. \quad (14)$$

#### 4. Conclusion

We proposed a new model for inflationary magnetogenesis in which the local  $U(1)$  symmetry is broken during inflation. Interestingly, in our model the broken  $U(1)$  symmetry was shown to be recovered on-shell. We found that the model is free from both the strong coupling and back reaction problems, and can explain the origin of intergalactic magnetic fields whose existence was strongly suggested by recent observations.

#### Acknowledgments

MS would like to thank the local organizers of the IF-YITP symposium VI at Naresuan University, August 2016, for their warm hospitality during the conference. This work was supported in part by MEXT KAKENHI Nos. 15H05888 and 15K21733, and by JSPS KAKENHI No. 15F15321. This talk is based on [13].

## References

- [1] A. Neronov, I. Vovk, *Science* 328:73-75,2010, [arxiv: 1006.3504].
- [2] F. Tavecchio, G. Ghisellini, G. Bonnoli and L. Foschini, *MNRAS* (2011) 414 (4): 3566-3576.
- [3] F. Tavecchio, G. Ghisellini, L. Foschini, G. Bonnoli, G. Ghirlanda and P. Coppi, *MNRAS* (2010) 406 (1): L70-L74.
- [4] A. M. Taylor, I. Vovk, A. Neronov, *Astronomy & Astrophysics*, Volume 529, 2011.
- [5] A. Kandus, K. E. Kunze and C. G. Tsagas, *Phys. Rept.* **505**, 1 (2011).
- [6] R. Durrer and A. Neronov, *Astron. Astrophys. Rev.* **21**, 62 (2013).
- [7] M. S. Turner and L. M. Widrow, *Phys. Rev. D* 37 (1988) 2743.
- [8] B. Ratra, *Astrophys. J.* 391, L1 (1992).
- [9] J.D. Bekenstein. *Phys. Rev. D*25 (1982) 1527
- [10] V. Demozzi, V. Mukhanov and H. Rubinstein, *JCAP* **0908**, 025 (2009).
- [11] K. Bamba and M. Sasaki, *JCAP* **0702**, 030 (2007)
- [12] P. A. R. Ade *et al.* [Planck Collaboration], arXiv:1502.01594 [astro-ph.CO].
- [13] G. Domenech, C. Lin and M. Sasaki, *Europhys. Lett.* **115**, no. 1, 19001 (2016).