

The theory of Minimal Massive Gravity

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Abstract. Here we give a short summary of the theory of minimal massive gravity. This theory, by construction, possesses only two tensor modes, exactly as in General Relativity. This theory is able to reconcile the original background of dRGT – which was proven to possess a ghost – with a stable homogeneous and isotropic cosmological evolution. The theory is constructed by imposing two constraints in a non-linear, background-independent way. The phenomenology of the theory is also briefly discussed.

1. Introduction

Recently the construction of a Lagrangian which was able to give a non-zero mass to the graviton, has attracted many people to try to connect its value to the tiny energy scale which is supposed to be the source of the measured acceleration of our universe.

Unfortunately, this same model proved to lead, in general, to unstable homogeneous and isotropic backgrounds, so that in order to do cosmology with massive gravity one had to renounce either to isotropy or to homogeneity.

In order to try to reconcile a massive graviton to the standard Friedmann-Lemaître-Robertson-Walker (FLRW) background, several authors tried to introduce more degrees of freedom, e.g. by coupling the gravity sector to a scalar field.

Here we discuss a recently proposed theory, which acts in the opposite way, by reducing degrees of freedom. In fact, the theory is built in order to remove all the degrees of freedom of massive theory but the tensor modes. In this way, both scalar and vector modes, in vacuum, are not dynamical leading to the same number of degrees of freedom as in General Relativity (GR). However, the tensor modes acquire a non-zero mass, leading, in general, to a different phenomenology than GR.

2. The model

In order to build up the Lagrangian of the theory we first need to give a three-dimensional fiducial metric as

$$\tilde{\gamma}_{ij} = \delta_{IJ} E^I_i E^J_j, \quad (1)$$

where E^I_i is the fiducial 3-vielbein, together with a second three-tensor as in

$$\tilde{\zeta}^i_j = \frac{1}{M} E_L^i \dot{E}^L_j, \quad (2)$$



Both these fields are considered to be given external fields (as long as we make use of the unitary gauge for the Stückelberg fields). In addition to these fields we need the three-dimensional physical metric

$$\gamma_{ij} = \delta_{IJ} e^I_i e^J_j, \quad (3)$$

where e^I_i is the physical 3-vielbein. After introducing $\tilde{\gamma}_{ij}$, $\tilde{\zeta}^i_j$, and γ_{ij} , we can define

$$\kappa^m_l \kappa^l_n = \tilde{\gamma}^{ms} \gamma_{sn}, \quad k^m_j \kappa^j_n = \delta^m_n, \quad (4)$$

so that k^m_j is the inverse tensor of κ^j_n .

Out of these quantities, we introduce the following 3-tensor

$$\Theta^{ij} = \frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}} \{c_1 (\gamma^{il} \kappa^j_l + \gamma^{jl} \kappa^i_l) + c_2 [\kappa (\gamma^{il} \kappa^j_l + \gamma^{jl} \kappa^i_l) - 2\tilde{\gamma}^{ij}] \} + 2c_3 \gamma^{ij}, \quad (5)$$

which is proportional to the difference between the values of the canonical momenta π^{ij} between this theory and GR.

Having defined all the building blocks of the theory we can write down the Lagrangian in the metric formalism as

$$\begin{aligned} S &= \frac{M_P^2}{2} \int d^4x N \sqrt{\gamma} [(^{(3)}R + K^{ij} K_{ij} - K^2)] + \frac{M_P^2}{2} \sum_{i=1}^4 \int d^4x \mathcal{L}_i \\ &+ \frac{M_P^2}{2} \int d^4x N \sqrt{\gamma} \left(\frac{m^2 N \lambda}{4N} \right)^2 \left(\Theta_{ij} \Theta^{ij} - \frac{1}{2} \Theta^2 \right) \\ &- \frac{M_P^2}{2} \int d^4x \sqrt{\gamma} [\lambda \mathcal{C}_0 - (D_n \lambda^i) \mathcal{C}^n_i] + S_{\text{matter}}, \end{aligned} \quad (6)$$

$$\mathcal{L}_1 = -m^2 c_1 \tilde{a}^3 (N + M \kappa), \quad (7)$$

$$\mathcal{L}_2 = -\frac{1}{2} m^2 c_2 \tilde{a}^3 (2N \kappa + M \kappa^2 - M \kappa^i_j \kappa^j_i), \quad (8)$$

$$\mathcal{L}_3 = -m^2 c_3 \sqrt{\gamma} (M + N k), \quad (9)$$

$$\mathcal{L}_4 = -m^2 c_4 \sqrt{\gamma} N, \quad (10)$$

$$\mathcal{C}_0 = \frac{1}{2} m^2 M K_{ij} \Theta^{ij} - m^2 M \left\{ \frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}} [c_1 \zeta + c_2 (\kappa \zeta - \kappa^m_n \tilde{\zeta}^n_m)] + c_3 k^m_n \tilde{\zeta}^n_m \right\}, \quad (11)$$

$$\mathcal{C}^n_i = -m^2 M \left\{ \frac{\sqrt{\tilde{\gamma}}}{\sqrt{\gamma}} \left[\frac{1}{2} (c_1 + c_2 \kappa) (\kappa^n_i + \gamma^{nm} \kappa^l_m \gamma_{li}) - c_2 \kappa^n_l \kappa^l_i \right] + c_3 \delta^m_i \right\}, \quad (12)$$

$$S_{\text{matter}} = - \int d^4x [\sqrt{-g} \rho(n, s) + J^\mu (\partial_\mu \ell + \theta \partial_\mu s + A_1 \partial_\mu B_1 + A_2 \partial_\mu B_2)]. \quad (13)$$

Here, we have introduced two Lagrange multipliers, one 3-scalar λ , and one 3-vector λ^i in order to implement four constraints. These constraints are the necessary ones needed to suppress the scalar and vector perturbations of dRGT without spoling its self-accelerating FLRW background solution. Notice that we have supposed here $M = M(t)$, and $E^I_j = \tilde{a}(t) \delta^I_j$. Furthermore N is the lapse function, K_{ij} the extrinsic curvature (K being its trace), D_n represents the covariant derivative with respect to the physical 3D metric, γ_{ij} .

Finally, we have also introduced the matter fields in terms of a perfect fluid, whose action is written in the Schutz-Sorkin form, for which n the number density is written as $n = (J^\alpha J^\beta g_{\alpha\beta}/g)^{1/2}$, and s is the entropy per particle. The fields $A_{1,2}$ and $B_{1,2}$ are scalar fields out of which one can study the vector modes for such a fluid.

3. Phenomenology

In terms of the Hubble factor $H = \dot{a}/a$ and $X = \tilde{a}/a$, then the background exactly coincides with the one of dRGT theory, namely

$$3H^2 = \frac{m^2}{2} (c_4 + 3c_3 X + 3c_2 X^2 + c_1 X^3), \quad (14)$$

and two branches still exist as solutions of the following equation

$$(c_3 + 2c_2 X + c_1 X^2) (\dot{X} + NHX - MH) = 0. \quad (15)$$

As for the perturbation, the self-accelerating branch, for which $c_3 + 2c_2 X + c_1 X^2 = 0$, then the dynamics of both scalar and vector modes exactly coincides with the one of GR, whereas the tensor modes acquire a non-zero mass given by

$$\mu^2 = \frac{1}{2} m^2 X [c_2 X + c_3 + r X (c_1 X + c_2)]. \quad (16)$$

However, for the normal branch, the other solution of Eq. (15), we have that the scalar modes for the perfect fluid (modeling dust), gets modified as the dynamics for the $\delta_m = \delta\rho/\rho$ in comoving matter gauge reads as

$$\delta_m'' + \left(2 - \frac{3}{2} \Omega_m\right) \delta_m' + \left(\frac{9\theta Y \Omega_m}{2(\theta Y - 2)^2} + \frac{3}{\theta Y - 2}\right) \Omega_m \delta_m = 0, \quad (17)$$

where $\theta = \mu^2/H_0^2$, and $Y = H_0^2/H^2$.

A non-trivial phenomenology is then predicted to exist for the normal branch, as G_{eff} gets modified. This modification tends to disappear as soon as $Y \ll 1$, i.e. at early times and/or high densities.

4. Conclusions

The theory described here succeeds to build up a theory with a massive graviton with only two tensor degrees of freedom. This theory has non-trivial self-accelerating cosmological solutions which are stable and viable.

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References

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