

Slow-roll inflation and the BB-mode correlation spectrum of the Cosmic Microwave Background

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Abstract. The BB-mode correlation angular spectrum of cosmic microwave background is studied for several inflation models. A comparative study of the resulting angular power spectrum with the recent joint analysis data of the BICEP2/Keck Array and Planck shows constraint to several slow-roll inflation models of cosmology.

1. Introduction

The theory of cosmic inflation is the most widely known scenario proposed to resolve several issues associated with the standard model of cosmology, like large scale isotropy and homogeneity, horizon and flatness problems, monopole problem, etc.,. A large number of inflation models have been studied over several years [1, 2, 3, 4, 5]. It is believed that quantum fluctuations during inflation gave rise to primordial fluctuations like scalar and tensor perturbations. The scalar perturbations seeded the formation of large scale structures in the universe and the tensor perturbations represent the primordial gravitational waves (GWs).

The primordial GWs were generated during inflation by the strong and variable gravitational field of the early universe through the mechanism of parametric amplification of the zero-point quantum fluctuations which transforms the initial vacuum state with no particle into a multi-particle quantum state [6, 7] called squeezed vacuum state [8, 9]. All primordial GW modes start in the same vacuum state and this vacuum state gets transformed into a strongly squeezed state. Due to the squeezing effect, the variance of the GW mode's phase is strongly squeezed while the variance of its amplitude is being enhanced so that the uncertainty product remains intact. The parameter of squeezing grows all the way up in the amplifying regime and can vary from zero in the vacuum state up to a very large value by the end of the amplifying regime. From the vacuum state to the inflationary period, it has grown from zero up to 1 ($0 \leq r_s \leq 1$) which we use for the present study. The primordial GWs are believed to exist in a strongly squeezed vacuum state.

The primordial GWs are believed to have left an imprint on the cosmic microwave background (CMB) anisotropy in the form of B-mode polarization. Hence, if the primordial GWs do exist in the squeezed vacuum state, then the effect of squeezing should also be observed in the B-mode angular power spectrum of CMB [10]. This phenomenon is employed as a platform to provide further constraint on the inflation models. The results are compared with the recent BICEP2/Keck Array and Planck collaboration data [11] which sets the upper and lower bounds for the tensor-to-scalar ratio respectively as $r < 0.07$ and $r \simeq \mathcal{O}(10^{-3})$ [12].



2. Primordial GWs in the Squeezed Vacuum State

The perturbed metric for a flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe can be written as,

$$dS^2 = a^2(\eta)[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j], \quad (1)$$

where a is the scale factor and η is the conformal time defined by $d\eta = \frac{dt}{a}$, h_{ij} is a transverse-traceless perturbation of space-time, $|h_{ij}| \ll \delta_{ij}$,

$$\partial_i h^{ij} = 0, \quad \delta^{ij} h_{ij} = 0.$$

where δ_{ij} is the flat space metric.

The GW field $h_{ij}(\mathbf{x}, \eta)$ can be decomposed into a collection of Fourier modes as,

$$h_{ij}(\mathbf{x}, \eta) = \frac{C}{(2\pi)^{\frac{3}{2}}} \int_{-\infty}^{+\infty} \frac{d^3\mathbf{k}}{\sqrt{2k}} \sum_{p=1}^2 [h_k^{(p)}(\eta) c_k^{(p)} \varepsilon_{ij}^{(p)}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} + h_k^{(p)*}(\eta) c_k^{(p)\dagger} \varepsilon_{ij}^{(p)*}(\mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{x}}], \quad (2)$$

where $C = \sqrt{16\pi} l_{pl}$ is the quantum normalization of the GW field and $l_{pl} = \sqrt{G}$ is Planck's length.

The creation operator $c_k^\dagger$ and the annihilation operator c_k satisfy the relationships $[c_k^{(p)}, c_{k'}^{(p')\dagger}] = \delta_{pp'} \delta^3(k - k')$, $[c_k^{(p)}, c_{k'}^{(p')}] = [c_k^{(p)\dagger}, c_{k'}^{(p')\dagger}] = 0$. The Heisenberg equations of motion govern the evolution of these operators.

The two polarization states $\varepsilon_{ij}^{(p)}$, $p = 1, 2$ are symmetric and transverse-traceless and satisfy the conditions $\varepsilon_{ij}^{(p)} \delta^{ij} = 0$, $\varepsilon_{ij}^{(p)} k^i = 0$, $\varepsilon_{ij}^{(p)} \varepsilon^{(p')ij} = 2\delta_{pp'}$, $\varepsilon_{ij}^{(p)}(-\mathbf{k}) = \varepsilon_{ij}^{(p)}(\mathbf{k})$. These polarizations are linear and are called the plus (+) polarization and cross (×) polarization. The contribution from each polarization is same. Hence, from here onward, the superscript (p) is dropped for convenience.

The GW mode $h_k(\eta)$ can be rescaled in terms of the mode function μ_k as,

$$h_k = \frac{\mu_k}{a}. \quad (3)$$

The mode function can have the following form,

$$\mu_k(\eta) = u_k(\eta) + v_k^*(\eta), \quad (4)$$

where $u_k(\eta)$ and $v_k(\eta)$ are complex functions.

The mode function satisfies the equation of motion,

$$\mu_k'' + \left(k^2 - \frac{a''}{a}\right) \mu_k = 0, \quad (5)$$

where prime indicates the derivative with respect to the conformal time η .

The two-point correlation function of the modes leads to the power spectrum of tensor perturbations as,

$$\langle h_k h_{k'}^* \rangle = \frac{2\pi^2}{k^3} P_T(k) \delta^3(\mathbf{k} - \mathbf{k}'), \quad (6)$$

where P_T is the gravitational wave power spectrum.

The squeezed vacuum state is defined as [13],

$$|\xi\rangle = S(\xi)|0\rangle, \quad (7)$$

where $S(\xi)$ is the squeezing operator which can be written as,

$$S(\xi) = \exp \left[\frac{1}{2} \xi^* b^2 - \frac{1}{2} \xi b^{\dagger 2} \right], \quad (8)$$

where $\xi = r_s e^{i\gamma}$ is a complex number, r_s is the squeezing parameter and γ is the squeezing angle. The unitary transformations of the squeezing operator S on the annihilation and creation operators lead to

$$\begin{aligned} S^\dagger(\xi) b S(\xi) &= b \cosh r_s - b^\dagger e^{i\gamma} \sinh r_s, \\ S^\dagger(\xi) b^\dagger S(\xi) &= b^\dagger \cosh r_s - b e^{-i\gamma} \sinh r_s. \end{aligned} \quad (9)$$

Using Eqs.(2) and (9), the two point correlation function in the squeezed vacuum state can be written as,

$$\langle h_k h_{k'}^* \rangle = \frac{C^2}{a^2} \left[(1 + 2 \sinh^2 r_s) |\mu_k|^2 + \frac{1}{2} \sinh 2r_s (\mu_k^2 e^{i\gamma} + \mu_k^{*2} e^{-i\gamma}) \right] \delta^3(\mathbf{k} - \mathbf{k}'). \quad (10)$$

From Eq.(6) and Eq.(10), we get the power spectrum in the squeezed vacuum state as,

$$P_T(k) = \frac{k^3}{2\pi^2} \frac{C^2}{a^2} \left[(1 + 2 \sinh^2 r_s) |\mu_k|^2 + \frac{1}{2} \sinh 2r_s (\mu_k^2 e^{i\gamma} + \mu_k^{*2} e^{-i\gamma}) \right]. \quad (11)$$

In the quasi de Sitter universe during inflation, one can define the relation between the conformal time and the scale factor as $a(\eta) = \frac{-1}{H\eta(1-\epsilon)}$, where H is the Hubble parameter, the slow roll parameter is $\epsilon = \frac{m_{pl}^2}{2} \left(\frac{V'}{V} \right)^2$ and V is the potential of the scalar field. For small ϵ , $\vartheta = \frac{3}{2} + \epsilon$, and $n_T = -2\epsilon = 3 - 2\vartheta$.

For constant ϵ , the equation of motion can be written as [14],

$$\mu_k'' + \left[k^2 - \frac{1}{\eta^2} \left(\vartheta^2 - \frac{1}{4} \right) \right] \mu_k = 0. \quad (12)$$

The general solution for the above equation (Eq.(12)) is,

$$\mu_k(\eta) = \sqrt{-\eta} [C_1(k) \mathcal{H}_\vartheta^{(1)}(-k\eta) + C_2(k) \mathcal{H}_\vartheta^{(2)}(-k\eta)], \quad (13)$$

where $\mathcal{H}_\vartheta^{(1)}$ and $\mathcal{H}_\vartheta^{(2)}$ are the Hankel functions of the first and second kind, and C_1 and C_2 are the constants of integration. Using the flat spacetime solutions $\mu_k^0(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}$, the constants of integration become $C_1(k) = \frac{\sqrt{\pi}}{2} \exp \left[i \left(\vartheta + \frac{1}{2} \right) \left(\frac{\pi}{2} \right) \right]$ and $C_2(k) = 0$.

Then Eq.(13) implies that for long wavelength limit ($k \ll aH$),

$$\mu_k(\eta) = e^{i(\vartheta - \frac{1}{2})(\frac{\pi}{2})} 2^{\vartheta - \frac{3}{2}} \frac{\Gamma(\vartheta)}{\Gamma(\frac{3}{2})} \frac{1}{\sqrt{2k}} (-k\eta)^{\frac{1}{2} - \vartheta}. \quad (14)$$

Using Eq.(14) in Eq.(11), the power spectrum in the superhorizon limit ($k \ll aH$) is,

$$P_T(k) = C^2 \left(\frac{H}{2\pi} \right)^2 \left(\frac{k}{aH} \right)^{3-2\vartheta} \left[1 + 2 \sinh^2 r_s + \sinh 2r_s \cos \left(\gamma + \left(\vartheta - \frac{1}{2} \right) \pi \right) \right]. \quad (15)$$

Finally, taking $A_T(k_0) = C^2 \left(\frac{H_{k_0}}{2\pi} \right)^2$ as the normalization constant for the tensor spectrum, where H_{k_0} is the Hubble parameter at $aH = k_0$, k_0 being the pivot frequency, we get the power spectrum for the gravitational waves in terms of the tensor spectral index n_T as,

$$P_T(k) = A_T(k_0) \left(\frac{k}{k_0} \right)^{n_T} \left[1 + 2 \sinh^2 r_s + \sinh 2r_s \cos \left(\gamma + (2 - n_T) \frac{\pi}{2} \right) \right]. \quad (16)$$

Eq.(16) represents the power spectrum for gravitational waves in the squeezed vacuum state.

3. Inflationary scenario

In the simplest inflationary scenario a canonical scalar field ϕ called the inflaton drives the accelerated expansion of the early universe.

The equation of motion for the inflaton is given by,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (17)$$

where dot and prime indicate derivatives with respect to time (t) and the field (ϕ) respectively. V is the effective potential of the inflaton. The Hubble parameter H gives the strength of friction and is determined by the energy density of the scalar field,

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V,$$

so that the Friedmann equation can be written as,

$$H^2 = \frac{1}{3m_{pl}^2} \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right). \quad (18)$$

In the slow-roll limit, the energy density of the inflaton is dominated by its potential energy, $\frac{\dot{\phi}^2}{2} \ll V$. Hence in this limit, called the slow-roll limit, the Hubble parameter and the inflaton potential are related as,

$$H^2 \simeq \frac{V}{3m_{pl}^2}. \quad (19)$$

This condition is characterized by the slow-roll parameters which can be defined in terms of the inflaton potential (V) and its derivatives as,

$$\begin{aligned} \epsilon &\equiv \frac{m_{pl}^2}{2} \left(\frac{V'}{V} \right)^2, \\ \eta &\equiv m_{pl}^2 \left(\frac{V''}{V} \right), \end{aligned} \quad (20)$$

and so on. Inflation lasts as long as the slow-roll conditions are satisfied, i.e., $\epsilon \ll 1$ and $|\eta| \ll 1$. Once these slow roll conditions are violated, inflation ends followed by reheating and decay of the inflaton which is followed by particle production, which is the case for most inflation models.

In the slow-roll approximation, the power spectra of the scalar perturbation (P_S) and the tensor perturbation (P_T) generated outside the horizon can be given in terms of the potential by,

$$P_S \simeq \frac{1}{12\pi^2 m_{pl}^6} \frac{V^3}{V'^2} \Big|_{k=aH}, \quad (21)$$

$$P_T \simeq \frac{1}{3\pi^2 m_{pl}^4} V \Big|_{k=aH}, \quad (22)$$

where $k = aH$ indicates that both H and V are evaluated at the time when the mode with wave number k crosses the horizon. For all calculations, we take the scalar power spectrum to be $P_S = 2.43 \times 10^{-9}$.

The tensor-to-scalar ratio can be written in terms of the parameter ϵ as,

$$r \equiv \frac{P_T(k)}{P_S(k)} \simeq 16\epsilon. \quad (23)$$

This parameter measures the strength of the tensor spectrum relative to that of the scalar spectrum and can be very useful to distinguish between the many inflation models.

4. Inflation Models

In this section, we briefly discuss several slow-roll inflation models for which the upper bound for the tensor-to-scalar ratio is set to be $r < 0.07$ and lower limit is $r \simeq \mathcal{O}(10^{-3})$ and are constrained with various CMB observations [12]. We compute the slow roll parameters, tensor spectral index, tensor power spectrum and tensor-to-scalar ratio for the following slow-roll inflation models.

4.1. Natural Inflation model

In this model the inflaton is considered to be the Nambu Goldstone boson which arises whenever global symmetry is broken [15, 16].

The effective potential for the model is given by,

$$V(\phi) = M^4 \left[1 + \cos \left(\frac{\phi}{f} \right) \right], \quad (24)$$

where $f/m_{pl} = 10^2$ is the energy scale at which symmetry is broken, $M/m_{pl} = 10^{-2}$.

Using Eq.(20), the slow-roll parameters are calculated as,

$$\epsilon = 1.29 \times 10^{-3}, \quad (25)$$

$$\eta = 1.24 \times 10^{-3}. \quad (26)$$

The equation of state parameters are calculated as $r = 2.06 \times 10^{-2}$ and $n_T = -2.58 \times 10^{-3}$. The tensor power spectrum is calculated to be $P_T = 5.027 \times 10^{-11}$.

4.2. Arctan Inflation model

This model is often studied as a toy model [17, 18]. In the present scenario, we consider a large field model which starts at a large ϕ which later evolves to a minimum at the origin.

The effective potential for the model is,

$$V(\phi) = M^4 \left[1 - \arctan \left(\frac{\phi}{\mu} \right) \right], \quad (27)$$

where $\mu/m_{pl} = 10^{-2}$ is a free parameter which characterizes the typical vacuum expectation value at which inflation takes place, $M/m_{pl} = 10^{-3}$.

Using Eq.(20), the slow-roll parameters are found to be,

$$\epsilon = 8.62 \times 10^{-4}, \quad (28)$$

$$\eta = 3.0 \times 10^{-2}. \quad (29)$$

The equation of state parameters are calculated as $r = 1.38 \times 10^{-2}$ and $n_T = -1.72 \times 10^{-3}$. The tensor power spectrum is obtained as, $P_T = 3.35 \times 10^{-11}$.

4.3. Inverse Monomial Inflation Model

This model is considered here in the context of quintessential inflation [19, 20, 21]; the inflaton need not necessarily decay and reheating arises naturally even when the potential does not have a global minimum, radiation is created via gravitational particle production.

The effective potential for the model is,

$$V(\phi) = M^4 \left(\frac{\phi}{m_{pl}} \right)^{-p}, \quad (30)$$

here $p = 3$ is a positive parameter, $M/m_{pl} = 10^{-1}$.

Using Eq.(20), the slow-roll parameters are calculated as,

$$\epsilon = 1.25 \times 10^{-4}, \quad (31)$$

$$\eta = 3.33 \times 10^{-4}. \quad (32)$$

The corresponding equation of state parameters are calculated as $r = 2.0 \times 10^{-3}$ and $n_T = -2.50 \times 10^{-4}$. The tensor power spectrum for this model is calculated as $P_T = 4.86 \times 10^{-12}$.

4.4. Hybrid Inflation model

This is a multi-scalar field [22, 23] model where ϕ field drives the inflation and the inflation ends abruptly by the symmetry breaking of σ field.

The effective potential of hybrid inflation model is,

$$V = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\sigma^2, \quad (33)$$

where $M = 1.21 \times 10^{16}$ GeV, $m = 3.65 \times 10^{11}$ GeV, $\lambda = 1$, $g = 8 \times 10^{-4}$.

Using Eq.(20), the slow-roll parameters are found to be,

$$\epsilon = 2.65 \times 10^{-4}, \quad (34)$$

$$\eta = 1.47 \times 10^{-4}.$$

The equation of state parameters for this model are calculated as $r = 4.24 \times 10^{-3}$ and $n_T = -5.3 \times 10^{-4}$. The tensor power spectrum for this model is obtained as $P_T = 1.03 \times 10^{-11}$.

4.5. Loop Inflation Model

In this model, the flatness of the inflaton potential is altered by symmetry breaking along the flatness of the potential which produces quantum radiative corrections in which one loop order correction takes the form of a logarithmic function [24, 25, 26].

The effective potential for this model can be written as,

$$V(\phi) = M^4 \left[1 + \alpha \ln \left(\frac{\phi}{m_{pl}} \right) \right], \quad (35)$$

where $\alpha = g^2/16\pi^2$ tunes the strength of radiative effects, $M = 10^{16}$ GeV.

Using Eq.(20), the slow-roll parameters are found to be,

$$\epsilon = 3.09 \times 10^{-3}, \quad (36)$$

$$\eta = -2.06 \times 10^{-2}. \quad (37)$$

The corresponding equation of state parameters are $r = 4.34 \times 10^{-2}$ and $n_T = -6.18 \times 10^{-3}$. The tensor power spectrum is calculated as, $P_T = 1.2 \times 10^{-10}$.

4.6. Coleman-Weinberg Inflation model

In this scenario, the potential is introduced in the context of spontaneous symmetry breaking generated by radiative corrections [27, 28, 29].

The effective potential for the model is,

$$V(\phi) = M^4 \left[1 + \alpha \left(\frac{\phi}{\sigma} \right)^4 \ln \left(\frac{\phi}{\sigma} \right) \right], \quad (38)$$

where $\alpha = 4e$, $M = 10^{16}$ GeV, $\sigma = 10m_{pl}$ sets the typical vacuum expectation value at which inflation takes place.

Using Eq.(20), the slow-roll parameters are found to be,

$$\epsilon = 4.86 \times 10^{-4}, \quad (39)$$

$$\eta = -4.42 \times 10^{-2}. \quad (40)$$

The equation of state parameters are obtained as $r = 7.77 \times 10^{-3}$ and $n_T = -9.72 \times 10^{-4}$. The tensor power spectrum is calculated as, $P_T = 1.89 \times 10^{-11}$.

4.7. Quadratic Chaotic Inflation model with radiative corrections

This model is a simple quadratic chaotic inflation model [30] studied under the assumption that the scalar field interacts with the fermion field thus leading to the quantum radiative correction which takes the form of a logarithmic function [31]. Introduction of radiative corrections lowers the tensor-to-scalar ratio and the power level of the model.

The effective potential with radiative correction is,

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{g^4}{16\pi^2}\phi^4 \ln\left(\frac{\phi}{m_{pl}}\right), \quad (41)$$

where g represents the Yukawa coupling, $m = 3.44 \times 10^{12}$ GeV.

Using Eq.(20), the slow-roll parameters are found to be,

$$\epsilon = 1.86 \times 10^{-3}, \quad (42)$$

$$\eta = 1.86 \times 10^{-3}.$$

The equation of state parameters are obtained as $r = 2.98 \times 10^{-2}$ and $n_T = -3.72 \times 10^{-3}$. The tensor power spectrum for this model is calculated as, $P_T = 7.25 \times 10^{-11}$.

5. BB-mode correlation spectrum

It is believed that the scalar and tensor fluctuations have left their own signatures on the CMB and can be realized in the form of E-mode and B-mode polarizations respectively [32, 33, 34]. The BB -mode correlation angular power spectrum of CMB is given by [35, 36],

$$C_l^{BB} = (4\pi)^2 \int dk k^2 P_T(k) \left| \int_0^{\eta_0} d\eta g(\eta) h_k(\eta) \times \left\{ (8x + 2x^2 \partial_x) \frac{j_l(x)}{x^2} \right\}_{x=k(\eta_0-\eta)} \right|^2, \quad (43)$$

where $g(\eta) = \kappa e^{-\kappa}$ is the probability distribution of the last scattering, κ is the differential optical depth, $x = k(\eta_0 - \eta)$ and $j_l(x)$ is the spherical Bessel function.

We obtain the BB -mode correlation spectrum of CMB for primordial gravitational waves for the aforementioned slow-roll inflationary models. The results are compared with the data from the joint analysis of BICEP2/Keck Array and Planck at 353 GHz [11]. The limit (BK x BK - α BK x P)/(1 - α) at $\alpha = \alpha_{fid} = 0.04$ is evaluated from the auto-spectra and cross-spectra of the joint BICEP2/Keck 150 GHz maps and Planck 353 GHz maps to clean out the dust contribution which is 0.04 times in the BICEP2 band when compared to that in the Planck 353 GHz band; BK x BK indicates the BICEP2/Keck auto-spectra at 150 GHz and BK x P indicates the cross-spectra of BICEP2/Keck maps at 150 GHz and Planck maps at 353 GHz. This combination for the limit is taken after the subtraction of the dust contribution.

The correlation spectrum for the different inflation models is generated using the CAMB code with the tensor spectral index n_T , the tensor-to-scalar ratio r and tensor power spectrum

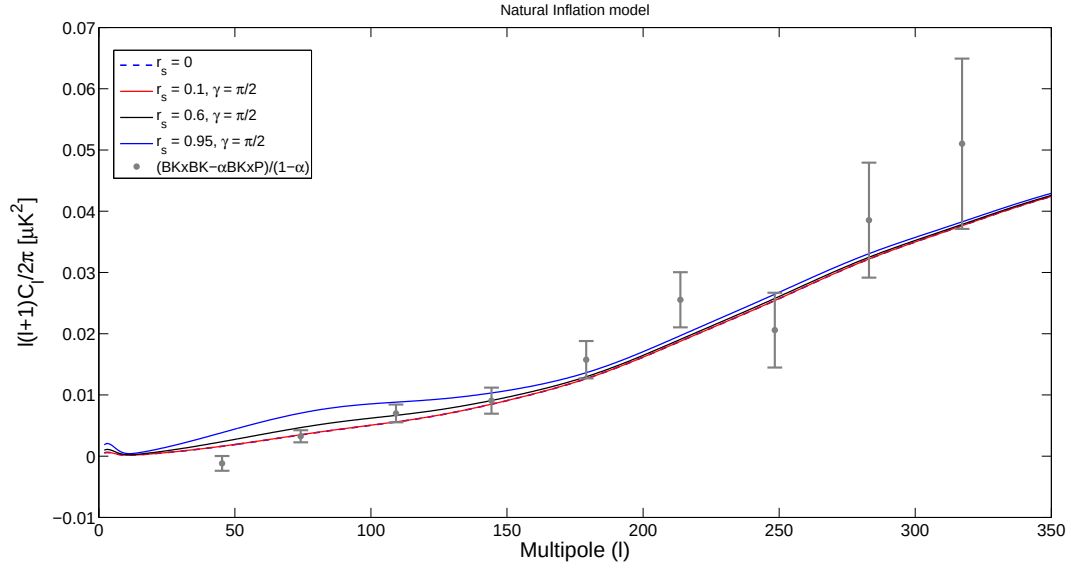


Figure 1. BB-mode angular power spectrum for the Natural inflation model for various values of squeezing parameter.

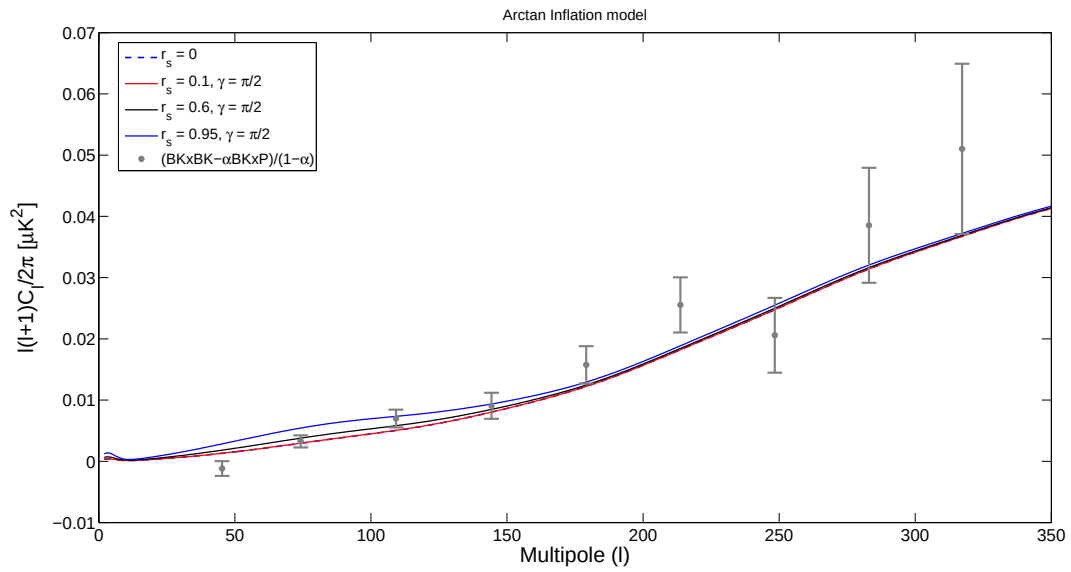


Figure 2. BB-mode angular power spectrum for the Arctan inflation model for various values of squeezing parameter.

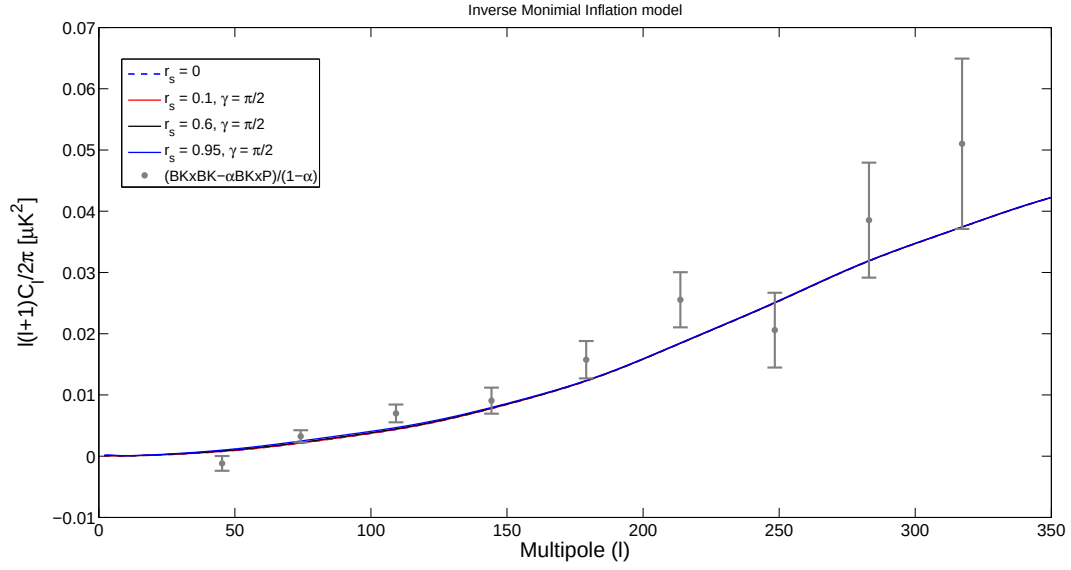


Figure 3. BB-mode angular power spectrum for the Inverse Monomial inflation model for various values of squeezing parameter.

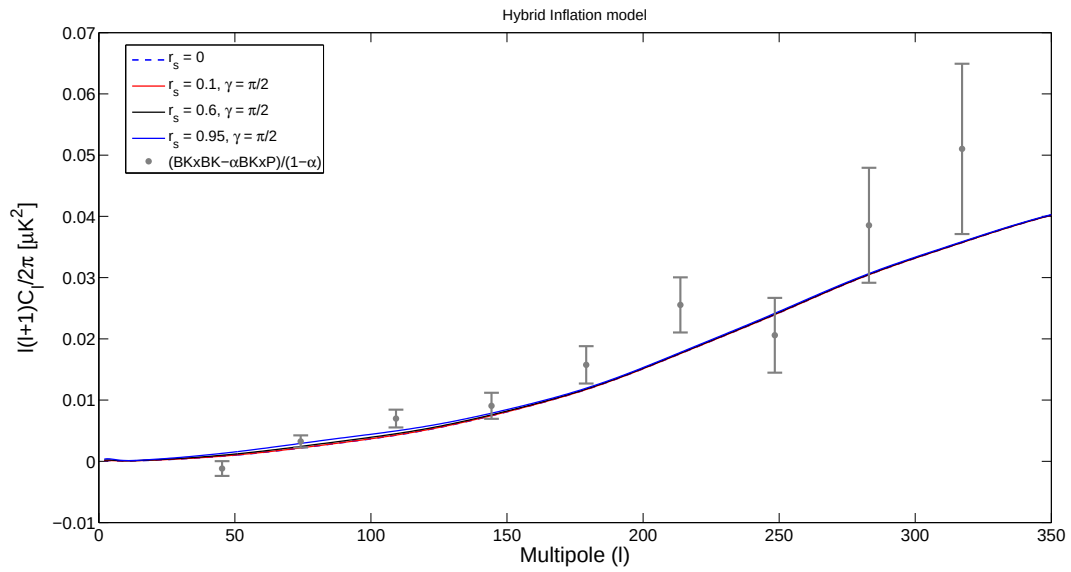


Figure 4. BB-mode angular power spectrum for the Hybrid inflation model for various values of squeezing parameter.

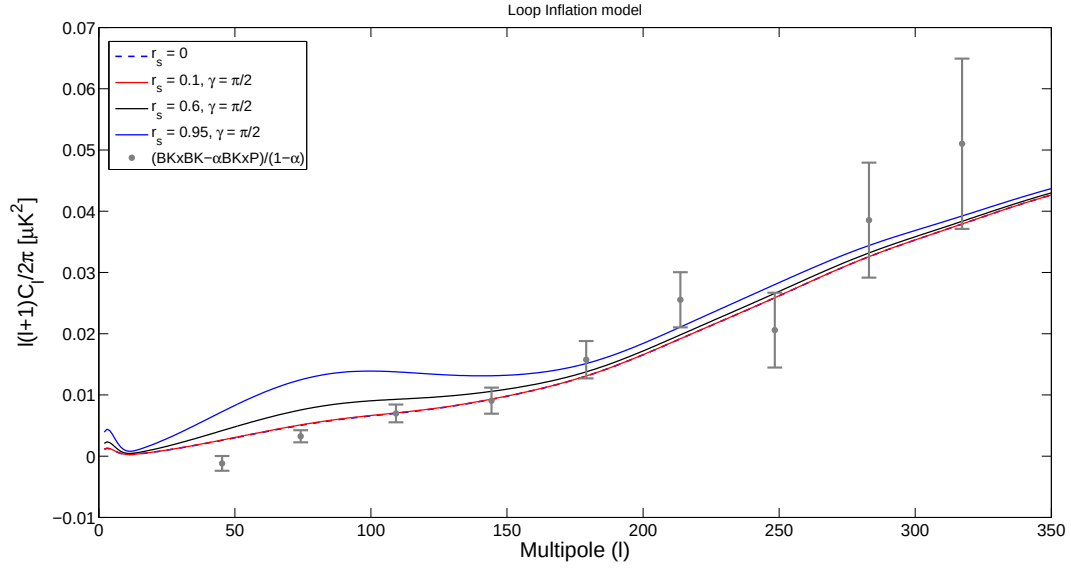


Figure 5. BB-mode angular power spectrum for the Loop inflation model for various values of squeezing parameter.

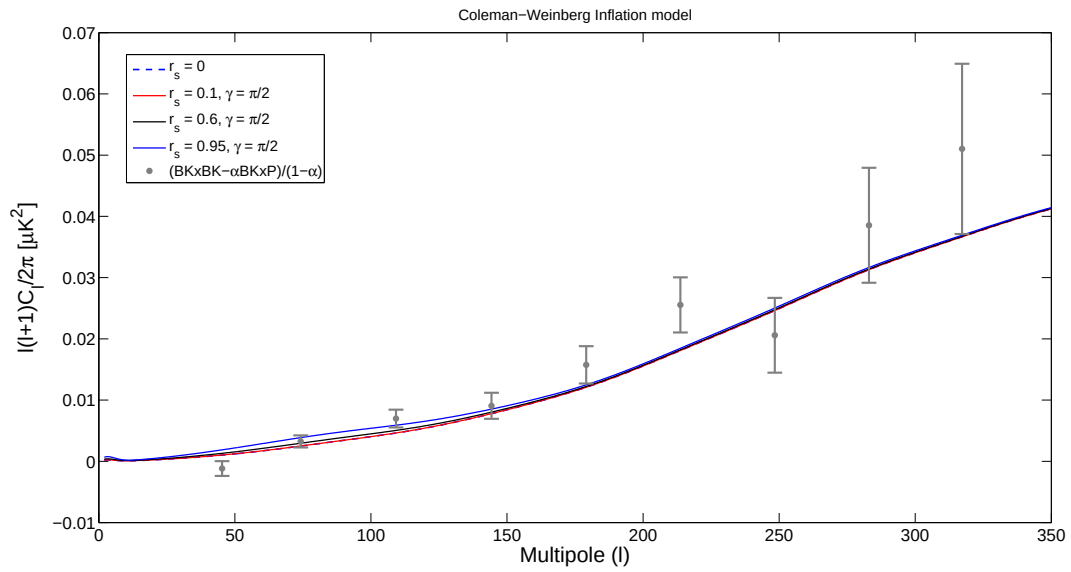


Figure 6. BB-mode angular power spectrum for the Coleman-Weinberg inflation model for various values of squeezing parameter.

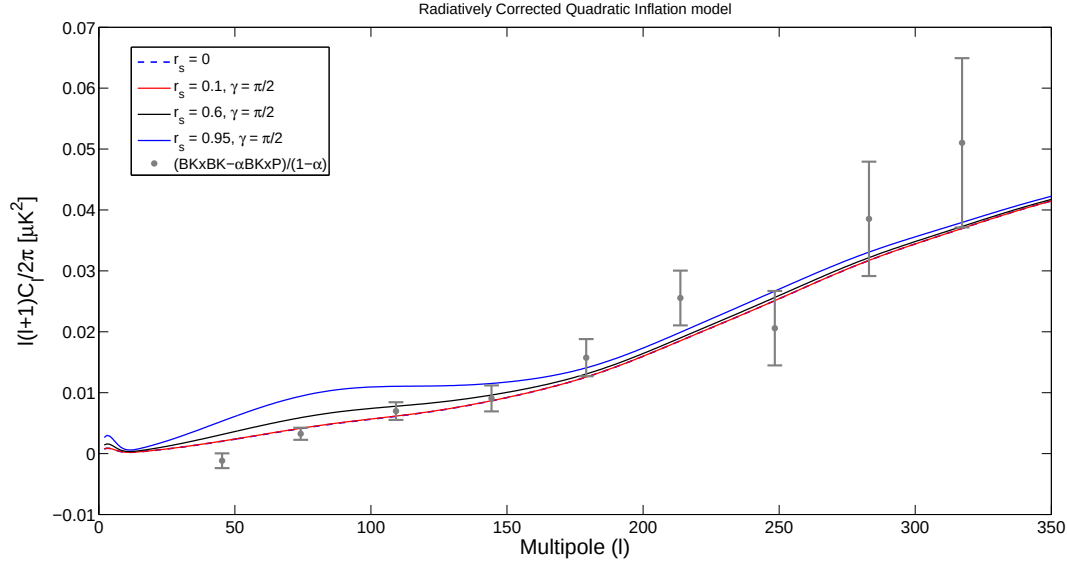


Figure 7. BB-mode angular power spectrum for the Quadratic Chaotic inflation model with radiative corrections for various values of squeezing parameter.

P_T corresponding to each model. For all models, the optical depth is taken to be $\kappa = 0.08$, the pivot wave number for tensor modes is taken as $k_0 = 0.002 \text{ Mpc}^{-1}$ and that for scalar modes is $k_* = 0.05 \text{ Mpc}^{-1}$.

The BB-mode correlation spectrum for the different inflation models with squeezing effect are obtained and are given in Figs.1, 2, 3, 4, 5, 6 and 7. It can be observed that for inflation models like Natural Inflation model (Fig.1), Arctan Inflation model (Fig.2), Loop Inflation model (Fig.5), Quadratic Chaotic Inflation model with radiative corrections (Fig.7), the angular spectra with all squeezing parameters ($0 \leq r_s \leq 0.95$) do not all lie within the BKP limit, which implies that these models are not ruled out, but are marginally favorable according to the BKP collaboration data, while spectra for the rest of the models (Fig.3, Fig.4, Fig.6), with and without squeezing effect lie completely within the BKP limit and are highly favorable.

6. Conclusion

We studied the BB mode correlation spectrum with squeezing effect for several slow-roll inflation models which are found in agreement with the constraint of the recent collaboration data of BICEP2/ Keck Array at 150 GHz and Planck 353 GHz; we use squeezing effect to help in further constraining these models. Note that the angular power spectrum at higher multipoles (smaller angles) are ignored due to contamination from lensing (B-mode produced due to lensing of E-mode). It is observed that with increase in squeezing parameter for each model, the corresponding GW spectrum also increases. Further, larger the deviation from scale invariance, stronger is the squeezing effect, i.e, for models with larger values of tensor-to-scalar ratio and smaller values of tensor spectral index (n_T), squeezing effect is more prominent, especially at larger values of r_s . All the inflationary models that are studied in the present work are large single field models except the Hybrid Inflation model which is a multi-field inflation model.

The phenomenon of squeezing allows us to treat the resulting state as a stochastic collection of standing waves [37] and thus leads to oscillatory features in the angular spectrum of CMB. Note that for our study, we include anisotropy due to lensing (which acts to smooth out the acoustic peaks in the spectrum on intermediate to small angular scales) in correspondence with

the data given by BICEP2/Keck Array and Planck. The primordial GWs, due to squeezing, form a non-stationary background. Hence, the primordial GWs' Gaussian, but non-stationary property can provide a unique signature that could be essential to distinguish a background created from stationary stochastic background by other types of processes which do not produce oscillatory features in the C_l multipole.

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