

Mass-varying massive gravity with k-essence

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Abstract. In this work I review the specific class of mass-varying massive gravity in which a corresponding graviton is given nonzero varying mass. Particularly in this theory, the graviton mass varies which is driven by a rolling k-essence field. Firstly, the model action is expressed along with its equations of motion. It is found from those equations of motion that not only the cosmic accelerating expansion but also the presence of dark matter can be realized in this context. Due to the complexity of its dynamics, the method of dynamical analysis is used to investigate evolutions of the cosmological system. It is found that the corresponding cosmological evolutions can explain the cosmic coincidence problem, where the effective dark matter is presented while the system is in the phase of the cosmic accelerating expansion. I then conclude that the evolutions obtained here can alleviate the cosmic coincidence problem while in some aspects the fine-tuning problem arises.

1. Introduction

In the world of physics, gravity is one of the most interesting topics as being one of the four forces in nature. To gain more understanding on nature, physicists were searching for and formulating theories of gravity for centuries. One of the most successful gravity theory was proposed by Einstein, formally known as general relativity. General relativity has brought useful insights to the gravitational physics especially in the aspects of the solar system, e.g. it has proved itself useful in the context of the Mercury's trajectory. Despite its advantages, the situation is totally different on the larger scales. For the scale of galaxies general relativity fails to describe galaxies' rotations without introducing the invisible "dark matter" while general relativity needs the existence of the mysterious "dark energy" to be able to predict the ongoing cosmic accelerating expansion. On one side, it is important to seek for descriptions for these dark contents as ingredients of physics beyond the standard model. On the other side, it is also reasonable to think that general relativity may not be accurate enough to describe the larger systems. The latter idea has brought the field of modified gravity to life.

One of the remarkable models of modified gravity comes from a very simple attempt to extend the gravity theory. In the context of particle physics, gravitational force is carried by a force carrier known as a graviton. The graviton that carries the force in general relativity actually has no mass. In this sense, one can think of a simple generalization of the gravity theory by introducing nonzero mass to the graviton. This idea is the essence of the massive gravity theories. Massive gravity started its story in 1939 when Fierz and Pauli proposed a linear theory of massive gravity [1]. Though it is actually a direct generalization from the linearized general relativity, it was proved by van Dam, Veltman, and Zakharov that the Fierz-Pauli theory cannot



recover the linearized general relativity [2, 3]. To cure this problem, Vainshtein proposed that this problem can be lifted if nonlinearities are included into the theory [4]. Furthermore, the nonlinearities required in this aspect must be chosen very carefully, pointed out by Boulware and Deser, to avoid the ghost instability, and instability where one of the degrees of freedom has negative kinetic energy [5]. Embracing those requirements, de Rham, Gabadadze, and Tolley proposed a nonlinear theory of massive gravity in 2010, dubbed dRGT massive gravity [6, 7]. This nonlinear theory has given opportunities to cosmologists to study our universe in the context of massive gravity. In particular, this massive gravity theory can describe the cosmic accelerating expansion in terms of the nonvanishing graviton mass.

Though the dRGT massive gravity provides a possible answer to the cosmic acceleration, it faces a serious problem. In particular, the corresponding cosmological solution is unstable in the context of the dRGT massive gravity [8]. This means that we cannot trust the cosmological solution obtained from the theory. To fix this problem, there are a lot of attempts to modify the dRGT massive gravity. One of those attempts is to let the graviton mass to be a varying function of a scalar field, dubbed mass-varying massive gravity [9, 10, 11, 12, 13]. This model, however, implies that the graviton mass, as a varying function, keeps shrinking its mass as the universe expands. With hopes of fixing this problem, in this work we allow the graviton mass to vary due to not only a scalar field but also its kinetic term. Moreover, the scalar field is now governed by the k-essence lagrangian. We then investigate this proposed model in terms of its cosmology. We found that such an allowance can cause modifications on the gravity sector as if there effectively exists the dark matter. As a result, not only that the graviton mass contributes as the effective dark energy but also it gives rise to the effective dark matter. Furthermore, since this model can provide descriptions of both dark energy and dark matter in terms of varying graviton mass, we also investigate this model in the context of the cosmic coincidence problem. Particularly, the cosmic coincidence problem is a “coincidence” of the universe to have dark matter as much as dark energy. In the aspects of the well-known Λ CDM model, dark energy and dark matter vary in a very drastic way over cosmological time such that the fact that the universe constituted of dark matter as much as dark energy, as observed nowadays, appears as a coincidence. We then attempted to explain this coincidence in the context of mass-varying massive gravity. In this work, we review the key contents of this mass-varying massive gravity presented in Ref. [13].

2. The model and the equations of motion

In this section we explicitly expressed the model action and discuss about important features through the equations of motion. Following the previously mentioned concepts, one can construct the corresponding action as follows [13],

$$S = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R[g] + V(X, \phi) (\mathcal{L}_2[g, f] + \alpha_3 \mathcal{L}_3[g, f] + \alpha_4 \mathcal{L}_4[g, f]) + P(X, \phi) \right], \quad (1)$$

where R is a Ricci scalar corresponding to the physical metric $g_{\mu\nu}$, $V(X, \phi)$ corresponds to the varying graviton mass which is a function of the scalar field ϕ and its kinetic term $X \equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$, $P(X, \phi)$ is the k-essence lagrangian governing the scalar field ϕ , and the

terms denoted by $\mathcal{L}_i$'s are expressed as follows,

$$\mathcal{L}_2[g, f] = \frac{1}{2} ([\mathcal{K}]^2 - [\mathcal{K}^2]), \quad (2)$$

$$\mathcal{L}_3[g, f] = \frac{1}{3!} ([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]), \quad (3)$$

$$\mathcal{L}_4[g, f] = \frac{1}{4!} ([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4]), \quad (4)$$

where the square brackets $[\cdot]$ denote the trace of an object inside them and we have defined a building-block tensor $\mathcal{K}^\mu_\nu$ as

$$\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \left(\sqrt{g^{-1}f} \right)^\mu_\nu. \quad (5)$$

Here $f_{\mu\nu}$ is a reference metric which allows us to construct the nontrivial and nonlinear massive gravity. Moreover, the introduction of this reference metric $f_{\mu\nu}$ helps in the sense that the general covariance can now be equipped into the massive gravity via the following expression,

$$f_{\mu\nu} = \partial_\mu \varphi^\rho \partial_\nu \varphi^\sigma \tilde{f}_{\rho\sigma}, \quad (6)$$

where φ^μ 's are known as the Stückelberg fields which, since they transform as scalars, then implement the general covariance into the reference metric. For simplicity, we can choose the form of $\tilde{f}_{\mu\nu}$ to be of the same form as the physical metric in order to ease our calculation.

Since we focus on cosmological implications, we then consider this model with the physical metric of the Friedmann-Lemaître-Robertson-Walker (FLRW) form. In particular, we use the following ansatz for the physical metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -N(t)^2 dt^2 + a(t)^2 \Omega_{ij}(x) dx^i dx^j, \quad (7)$$

$$\Omega_{ij}(\varphi) = \delta_{ij} + \frac{k \delta_{ia} \delta_{jb} \varphi^a \varphi^b}{1 - k \delta_{lm} \varphi^l \varphi^m}, \quad (8)$$

where N is a lapse function that allows the time reparameterization, a is a scale factor denoting the overall scale of the 3-dimensional space, Ω_{ij} is the 3-dimensional spatial metric which classifies the spatial geometry to be open, flat, or closed one. For the reference metric, it is chosen to be of the same form as the FLRW metric as

$$\tilde{f}_{\mu\nu} d\varphi^\mu d\varphi^\nu = -n(\varphi^0)^2 (d\varphi^0)^2 + \alpha(\varphi^0)^2 \Omega_{ij}(\varphi) d\varphi^i d\varphi^j, \quad (9)$$

where $n(\varphi^0), \alpha(\varphi^0)$ are a lapse function and a scale factor in the reference sector.

From the action in Eq. (1) we can find the equations of motion, assuming the unitary gauge $\varphi^\mu = x^\mu$ and the spatially flat geometry $k = 0$, as follows,

$$M_p^2 \left(3H^2 + 3\frac{k}{a^2} \right) = -3VF + \rho_X - P, \quad (10)$$

$$M_p^2 \left(\frac{2\dot{H}}{N} + 3H^2 + \frac{k}{a^2} \right) = -3VF + VF_{,\bar{X}} (\bar{X} - \eta) - P, \quad (11)$$

$$\frac{\dot{V}}{V} = NH (1 - h\bar{X}) \frac{F_{,\bar{X}}}{G}, \quad (12)$$

$$Na^3 (3V_{,\phi} (F - G\eta) + P_{,\phi}) = \frac{d}{dt} \left[\frac{a^3}{\sqrt{2\bar{X}}} \rho_X \right], \quad (13)$$

where the dot represents the time derivative and we have used the following definitions,

$$\begin{aligned}
 H &\equiv \frac{\dot{a}}{aN}, & h &\equiv \frac{H_\alpha}{H}, & H_\alpha &\equiv \frac{\dot{\alpha}}{\alpha n}, \\
 \eta &\equiv \frac{n}{N}, & \rho_X &\equiv 6XV_{,X}(F - G\eta) + 2XP_{,X}, \\
 F &\equiv \left(2 + \frac{4}{3}\alpha_3 + \frac{1}{3}\alpha_4\right) - (3 + 3\alpha_3 + \alpha_4)\bar{X} + (1 + 2\alpha_3 + \alpha_4)\bar{X}^2 - (\alpha_3 + \alpha_4)\frac{\bar{X}^3}{3}, \\
 G &\equiv \frac{1}{3}(3 + 3\alpha_3 + \alpha_4) - (1 + 2\alpha_3 + \alpha_4)\bar{X} + (\alpha_3 + \alpha_4)\bar{X}^2 - \alpha_4\frac{\bar{X}^3}{3}, \\
 \bar{X} &\equiv \frac{\alpha}{a}.
 \end{aligned} \tag{14}$$

Note that since ρ_X lies on the right-hand side of the Friedmann equation in Eq. (10), we can view ρ_X as an energy density of an effective matter. For simplicity, we may assume that both the graviton mass function and the k-essence lagrangian are functions of the kinetic term X only,

$$V(X, \phi) = V(X), \quad P(X, \phi) = P(X). \tag{15}$$

From this assumption, Eq. (13) can be rewritten as

$$\frac{d}{dt}\rho_X + 3HN\rho_X = \frac{\dot{X}}{2X}\rho_X. \tag{16}$$

This equation can be thought of as a continuity equation of a nonrelativistic matter of energy density ρ_X which can flow to other sectors due to the nonzero term on the right-hand side. In other words, this effective matter corresponding to ρ_X is not conserved due to the flow. Consequently, we can imply that this model of mass-varying massive gravity, with some specific assumptions, can give rise to effective dust which from now on will be thought of as effective dark matter of energy density ρ_X .

Since we have seen that this model can give rise to the effective dark matter, it is important to investigate whether this model can provide a description of the dark energy responsible to the cosmic accelerating expansion or not. To this end, we may perform a simple analysis by first defining the following definitions,

$$\rho_g \equiv -3VF + 6XV_{,X}(F - G\eta), \quad p_g \equiv 3VF - VF_{,\bar{X}}(\bar{X} - \eta). \tag{17}$$

Note that ρ_g can be viewed as contributions arising from the graviton mass alone which can be seen from the Friedmann equation in Eq. 10 that ρ_g is the term involving the graviton mass function V on the right-hand side of the equation, and p_g is the corresponding pressure. In the following calculation we will investigate whether the matter described by the energy density ρ_g and the pressure p_g can be a candidate for the dark energy. We assume that the equation of state parameter $w_g \equiv \frac{p_g}{\rho_g}$ of this matter satisfies $w_g = -1$. By equating w_g to -1 we assume that this matter content shares the same properties as the cosmological constant in Λ CDM model (see Ref. [14] for a review). As a consequence, we obtain the following condition,

$$6XV_{,X}(F - G\eta) = VF_{,\bar{X}}(\bar{X} - \eta). \tag{18}$$

In a very simple case where $F, G, \bar{X}, \eta, F_{,\bar{X}}$ are constants, we obtain a very simple condition for V as

$$V \propto X^{\lambda/2}, \quad \lambda = \text{constant}. \tag{19}$$

This form of V indicates that the system in consideration is in the dark energy phase where the cosmic expansion is driven as if there is the cosmological constant. This encourages us to utilize this form of V in the following calculations.

At this point we have seen roughly that in this model of massive gravity the gravity can be modified as if there is the effective dust in the system. Moreover, the model can also describe the cosmic accelerating expansion, at least in the cosmological constant manner. These features encourage us to investigate in the next section in terms of the cosmic coincidence problem. This cosmic coincidence problem is a problem arising from the fact that we observed possible amounts of dark energy and dark matter in our universe. In particular, we found that there exists as much dark energy as there is dark matter in our universe (those amounts are in the same order of magnitude). This is somehow unusual in the context of Λ CDM model since each matter content varies so drastically over cosmological time in the Λ CDM universe (see Ref. [15]). Thus, the fact that the universe contains dark matter and dark energy, each of the same order of magnitude, seems to be a “coincidence” in the Λ CDM context. Since this mass-varying massive gravity model has a possibility of being able to explain the coexistence between dark matter and dark energy, it is worthwhile to consider the model in the aspect of the cosmic coincidence problem.

3. Dynamical Analysis & Cosmic Coincidence Problem

In order to investigate the model in the aspect of the cosmic coincidence problem, since the equations of motion we found in Eq. (10) - Eq. (13) are cumbersome, we can use the methods of dynamical analysis to extract some useful information out of those equations. We first use the specific form of V we obtained earlier and we also assume the form of P as follows,

$$V(X) = V_0 X^{\lambda/2}, \quad P(X) = P_0 X^{\gamma/2}, \quad (20)$$

where λ, γ are constants. We then define density parameters for each matter content in the right-hand side of the Friedmann equation in Eq. (10) as follows,

$$x \equiv -\frac{FV}{M_p^2 H^2}, \quad y \equiv \frac{\rho_X}{3M_p^2 H^2}, \quad z \equiv -\frac{P}{3M_p^2 H^2}. \quad (21)$$

These density parameters are governed by the following dynamical equations,

$$x' = 3x \left(y + sx - \frac{s}{r} \right), \quad (22)$$

$$y' = 3y \left(y + sx - 1 - \frac{s}{\lambda r} \right), \quad (23)$$

$$1 = x + y + z, \quad (24)$$

$$y = -\lambda x(1 - r) - z\gamma, \quad (25)$$

where the prime denotes the derivative with respect to $\log a$ and

$$r \equiv \frac{G\eta}{F}, \quad s \equiv \frac{F_{,\bar{X}}(\bar{X} - \eta)}{3F}. \quad (26)$$

We can also find an effective equation of state parameter for the entire system to be

$$w_{eff} = -1 + y + xs. \quad (27)$$

From these dynamical equations one can find fixed points of the system at which the configuration of the system will stay as it is or, in other words, the system will be stationary if its configuration corresponds to any of these points. All of the fixed points are shown in Table 1.

Table 1. Summary of the properties of the fixed points.

Name	x	y	z	w_{eff}	existence	stability
(a)	0	0	1	-1	$\gamma = 0$	$\frac{s}{r} \geq 0$
(b)	$\frac{1}{r}$	0	$1 - \frac{1}{r}$	$-1 + \frac{s}{r}$	$\gamma = \lambda$	$\frac{\lambda}{1-\lambda} \leq \frac{s}{r} < 0$
(c)	0	$1 + \frac{s}{\lambda r}$	$-\frac{s}{\lambda r}$	$\frac{s}{\lambda r}$	$\gamma = 1 + \frac{\lambda r}{s}$	$\frac{\lambda}{1-\lambda} < \frac{s}{r} < -1$
(d)	$\frac{1}{1+\lambda(r-1)}$	$\frac{\lambda(r-1)}{1+\lambda(r-1)}$	0	$\frac{1}{\lambda-1}$	$\lambda = \frac{s}{s-r}$	$0 < \lambda < 1$
(e)	$\frac{1+(\lambda-1)z_0}{1+\lambda(r-1)}$	$-\frac{\lambda(1-r(z_0+1))}{1+\lambda(r-1)}$	z_0	$\frac{1}{\lambda-1}$	$\lambda = \gamma = \frac{s}{s-r}$	$0 < \lambda < 1$

The fixed point (a) and (b) correspond to the situations found in Λ CDM model and the dRGT massive gravity respectively. The point (c) can be a great candidate for the dust domination period since the configuration for y can be nonzero. If this is the case, then we should have a very large value of λ for the effective equation of state parameter to approach zero. For the cosmic acceleration period, it is reasonable to think that one of the point (d) or (e) can represent this period since w_{eff} can be close to -1 when $\lambda \rightarrow 0$. If these points represent dynamics of the universe which evolves from the dust dominated era to the dark energy/dark matter era, despite being constant, λ must be very large during the dust domination while it must be very small at the time of cosmic acceleration. This suggests that λ must change in time or be a varying function. By promoting λ to be a function, λ is governed by the following dynamical equation,

$$\lambda' = \frac{6s}{r} \left(\frac{\lambda}{2} - (1 + \Gamma) \right), \quad (28)$$

where $\Gamma \equiv XV_{,XX}/V_{,X}$. We then compute numerical solutions of the evolutions of each matter content, including that of radiation, in order to verify the idea that the system can evolve from the dust dominated era to the dark energy/dark matter era. To obtain the numerical solutions, we first set an initial condition at $\log a = 0$ corresponding to the (present time) so that ratio of the amount of dark energy to that of dark matter at that time is 7 : 3, which is chosen for simplicity to represent the observed amounts of the dark energy and the dark matter. Note that the amount of radiation is also included which is set to be of order 10^{-5} to ensure that there is only small portion of radiation at the present time. Moreover, we also choose appropriate values of λ and s and let $\lambda = \gamma$ at $\log a = 0$ to ensure that the configuration at the present time corresponds to either point (d) or point (e) and the equation of state parameter is close to -1 . Furthermore, we choose Γ to be a small negative number so that λ grows slowly as the time ($\log a$) goes backward. This parameter setup is equivalent to setting specific values to all of the model parameters, including α_3 and α_4 . According to their characteristics, the solutions are classified into three following kinds,

- (i) **Massive Gravity Domination** This solution is explicitly shown in Figure 1 where the system evolves without the presence of k-essence lagrangian, in other words $z = 0$ all the time. Unfortunately, this solution only tells us that without the presence of nonzero z we cannot have a universe filled with both dark energy and dark matter at the same time as the universe will eventually evolve towards a full domination of dark energy (blue-dashed line, representing x) where no dark matter is presented (red-dotted line, representing y).
- (ii) **Late-time Universe 1** This solution, shown in Figure 2, shows us the possibility to solve the cosmic coincidence problem. In other words, the universe at late-time is filled with comparable amounts between dark energy (black-solid line, representing $x + z$) and

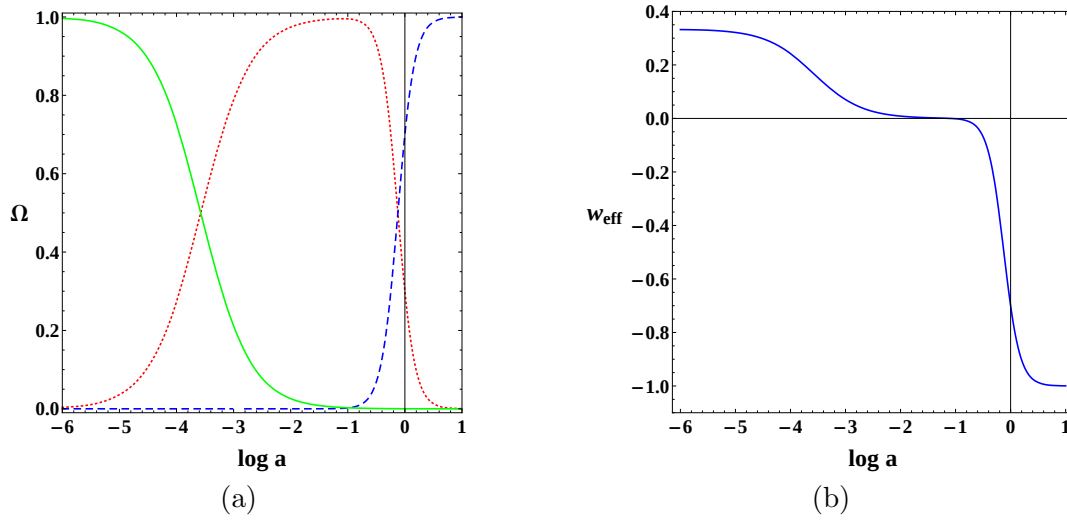


Figure 1. The plot (a) shows evolutions of various kinds of matter and energy. The green-solid line corresponds to the evolution of radiation, the red-dotted line corresponds to the evolution of the effective dark matter (y), and the blue-dashed line corresponds to that of the effective dark energy (x). The plot (b) shows the corresponding effective equation of state parameter throughout the evolution. Note that at late time the system is filled mostly with the effective dark energy.

dark matter (red-dotted line, representing y). Note that in this case the dark energy is represented by the quantity $x + z$ rather than that in the previous case which is represented only by x (because $z = 0$). The effective equation of state parameter at the dark energy phase is significantly less than -1 .

- (iii) **Late-time Universe 2** This solution is just another possibility to solve the cosmic coincidence problem. From Figure 3, the entire evolution is restricted by the condition $\gamma = \lambda$. The result of such a condition is that it allows the universe filled with comparable amounts of dark energy (black-solid line, representing $x + z$) and dark matter (red-dotted line, representing y). However, this solution requires a fine-tuning of the initial conditions in the numerical computations. This poses another problem into this solution which is known as the fine-tuning problem.

4. Discussion

We have seen various features of this model of mass-varying massive gravity where the graviton mass is promoted to be a function of a scalar field and also its kinetic term. Such an allowance, as far as its cosmology is concerned, can modify the gravity in this theory as if there is the effective dark matter in the system of interest. We then investigate in a very simple way a possibility of this model, while the effective dark matter is presented, to enter the phase of the cosmic accelerating expansion and we obtain the following suitable form of the graviton mass function $V(X) \propto X^{\lambda/2}$. Such a structure is investigated whether it can provide a meaningful cosmology or not via the method of dynamical analysis. Through it we found five fixed points, one corresponds to the Λ CDM cosmology, one is similar to the cosmology in dRGT massive gravity, one can be thought of as the dust dominated period, and other two have a chance to solve the cosmic coincidence problem since they can provide the cosmic acceleration while the effective dark matter is presented. The points of our interest are the dust dominated point and the cosmic accelerating expansion points. If these points are to explain each phase of the

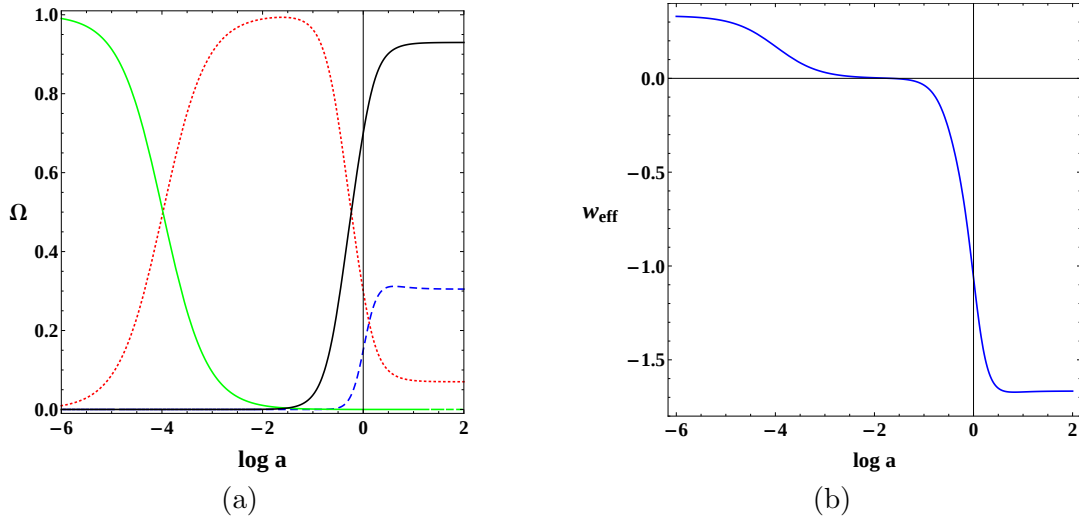


Figure 2. The plot (a) shows evolutions of various kinds of matter and energy. The green-solid line corresponds to the evolution of radiation, the red-dotted line corresponds to the evolution of the effective dark matter (y), the blue-dashed line corresponds to the quantity x , and the black-solid line corresponds to the “total” dark energy (which is $x + z$). The plot (b) shows the corresponding effective equation of state parameter throughout the evolution. Note that at late time the system is filled with both effective dark matter and effective dark energy, accompanying with the effective equation of state parameter being significantly less than -1 .

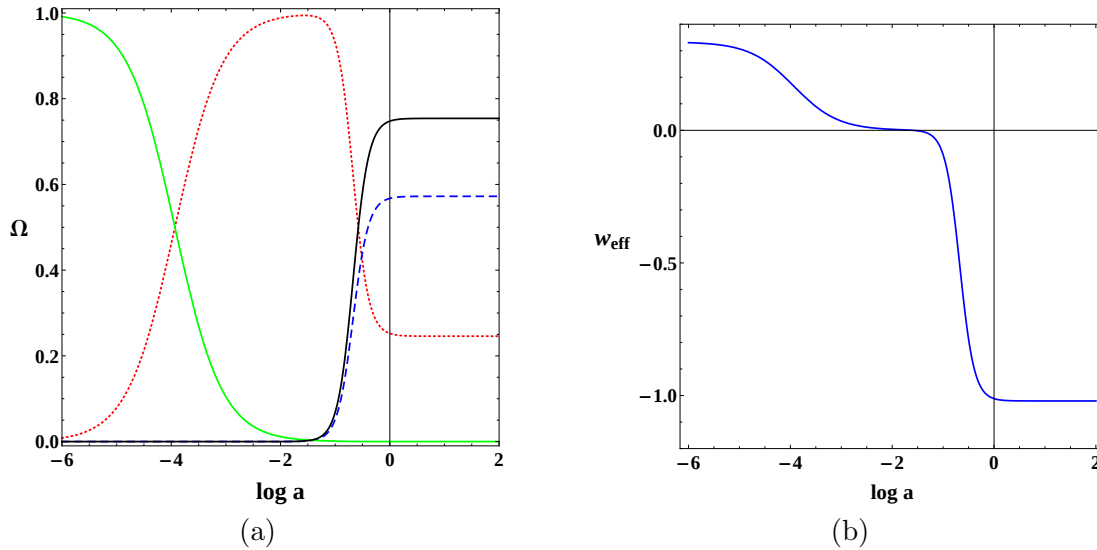


Figure 3. The plot (a) shows evolutions of various kinds of matter and energy. The green-solid line corresponds to the evolution of radiation, the red-dotted line corresponds to the evolution of the effective dark matter (y), the blue-dashed line corresponds to the quantity x , and the black-solid line corresponds to the “total” dark energy (which is $x + z$). The right plot shows the corresponding effective equation of state parameter throughout the evolution where the condition $\gamma = \lambda$ is satisfied. Note that at late-time the system is filled with both effective dark matter and effective dark energy, accompanying with the effective equation of state parameter being slightly less than -1 .

universe, the system should evolve from the dust dominated phase to the cosmic accelerating expansion phase. Such an evolution can be realized if the constant λ is allowed to be a function. We then show explicitly three possible evolutions of the system, each of them can explain the cosmic accelerating expansion, in Figure 1, 2, and 3. The first possibility corresponds to the situation where the k-essence lagrangian is not presented throughout the evolution, i.e. $z = 0$. However, the system of this configuration evolves towards a configuration corresponding to the fully dark energy dominated universe where the effective dark matter is almost absent. The second possibility is an example of the evolution which can solve the cosmic coincidence problem. The corresponding configuration at late time consists of comparable amounts between the effective dark matter and the effective dark energy. Moreover, the effective equation of state parameter at late-time is required to be significantly less than -1 . The third possibility corresponds to a constrained evolution which obeys $\gamma = \lambda$ throughout the evolution. The late-time configuration is capable of solving the cosmic coincidence problem while the effective equation of state parameter at late time is slightly less than -1 . However, this kind of evolution requires fine-tuning of the initial conditions. This then posts another problem on the configuration which is known as the fine-tuning problem. From our previous analysis, this model of mass-varying massive gravity can alleviate the cosmic coincidence problem, though in some aspects a new problem occurs, like the fine-tuning problem.

Acknowledgments

This work is a conference paper based on a work in Ref. [13]. Moreover, this work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIP) (No.2016R1C1B1010107).

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