

Generalized Three-Form Field

Pitayuth Wongjun^{1,2}

¹ The institute for fundamental study, Naresuan University, Phitsanulok 65000, Thailand

² Thailand Center of Excellence in Physics, Ministry of Education, Bangkok 10400, Thailand

E-mail: pitayuthw@nu.ac.th

Abstract. A generalized three-form field is an extended version of the canonical three-form field by considering a Lagrangian of the generalized three-form field as a function of the kinetic and the mass terms. In this work, we investigated cosmological models due to this generalized three-form field. It is found that one can use the three-form field to interpret the non-relativistic matter without the caustic problem. Moreover, by analyzing the dynamical system, a viable model of dark energy due to the generalized three-form field can be obtained.

1. Introduction

It is well-known that there are two accelerated expansion phases of the Universe: the inflationary phase, taking place during the very early era of the Universe, and the dark energy phase, corresponding to the expansion nowadays. A scalar field model is one of simple models to describe these phenomena [1, 2, 3, 4]. Beside the cosmological models due to the scalar field, a three-form field can be successfully used to describe both inflationary phase and dark energy phase [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]. At the perturbation level, it is obvious to see that the three-form field can generate intrinsic vector perturbations while it is not possible for the scalar field. In the present work, by mimicking the k-essence scalar field, we investigate a generalized version of the three-form field by considering a Lagrangian as a function of kinetic and mass terms [18]. We briefly review the important ingredients of the three-form field including energy momentum tensor and the equation of motion in covariant form. The cosmological solution of the model is investigated. A constant equation of state parameter yields the power law of both kinetic and mass terms in the Lagrangian so that one can use the three-form field to interpret the non-relativistic matter without caustic problem which is a problem found in k-essence scalar field model. This issue is discussed in section 2 in the present work. In section 3, we consider the non-constant case of the equation of state parameter and then use the dynamical system to analyze the behavior of the Universe. We found that it is possible to obtain a viable model of dark energy due to the generalized three-form field. Finally, the results are summarized and discussed in section 4.

2. Generalized Three-Form Filed

In this section, we will review the generalized three-form field model by considering a Lagrangian of the three-form field, $A_{\alpha\beta\gamma}$, as a function of the kinetic and mass terms as follows, [18]

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R + P(K, y) \right], \quad (1)$$



where the kinetic term and scalar quantity of the three-form field are expressed as

$$K = -\frac{1}{48}F_{\alpha\beta\gamma\delta}F^{\alpha\beta\gamma\delta}, \quad y = \frac{1}{12}A_{\alpha\beta\gamma}A^{\alpha\beta\gamma}, \quad F_{\mu\nu\rho\sigma} = \nabla_{[\mu}A_{\nu\rho\sigma]}. \quad (2)$$

From this action, the equations of motion and the energy momentum tensor of the three-form can be written as

$$E_{\alpha\beta\gamma} = \nabla_{\mu} \left(P_{,K} F^{\mu}_{\alpha\beta\gamma} \right) + P_{,y} A_{\alpha\beta\gamma} = 0, \quad (3)$$

$$T_{\mu\nu} = \frac{1}{6}P_{,K}F_{\mu\rho\sigma\alpha}F_{\nu}{}^{\rho\sigma\alpha} - \frac{1}{2}P_{,y}A_{\mu\rho\sigma}A_{\nu}{}^{\rho\sigma} + Pg_{\mu\nu}. \quad (4)$$

where the notation with subscript $P_{,x}$ denotes $P_{,x} = \partial_x P$. The energy momentum tensor is conserved up to the equation of motion as $6\nabla_{\mu}T^{\mu}_{\nu} = F_{\nu\alpha\beta\gamma}E^{\alpha\beta\gamma} = 0$.

By considering a flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric, the components of the three-form field, $A_{\alpha\beta\gamma}$, can be written as $A_{0ij} = 0, A_{ijk} = a^3 \varepsilon_{ijk} X(t)$ where ε_{ijk} is the three-dimensional Levi-Civita symbol with $\varepsilon_{123} = 1$. As a result, the energy density and pressure of the three-form field can be expressed as

$$\rho_X = 2KP_{,K} - P, \quad (5)$$

$$p_X = -\rho_X - 2yP_{,y}. \quad (6)$$

For the equation of state parameter, $w_X = p_X/\rho_X = -1 - 2yP_{,y}/\rho_X$, a partial differential equation of P can be written as

$$2yP_{,y} + (1 + w_X)2KP_{,K} = (1 + w_X)P. \quad (7)$$

By taking the equation of state parameter to be a constant, one can solve this equation to obtain the exact solution as $P = P_0 K^{\nu} y^{\mu}$, where P_0 is an integration constant and μ, ν are the exponent constants. As a result, the equation of state parameter can be expressed in terms of the constants as

$$w_X = -1 + \frac{2\mu}{1 - 2\nu}, \quad \nu \neq \frac{1}{2}. \quad (8)$$

From this expression, the description of non-relativistic matter can be obtained by choosing a proper parameters to yield $w_X = 0$, for example, $\mu = 1, \nu = -1/2$. Therefore, the Lagrangian of the three-form is still finite and then the model can avoid the caustic problem which is found in the k-essence scalar field case [18]. Moreover, one can relax the assumption of constant w by assuming $w_X = w_X(y)$. As a result, one of the solutions can be written as $P = P_0 K^{\nu} e^{\frac{(1-2\nu)}{2}\lambda y}$ [18] where $w_X = -1 + \lambda y$ and λ is a constant. From this solution, we found that it is possible to obtain the solution which admits the dark energy model. We will investigate this possibility in detail by using the dynamical system in the next section.

3. Dynamical system of the dark energy model

In this section, the dynamics of the Universe will be explored by using the dynamical system. By including the non-relativistic matter and radiation into the generalized three-form field model in Eq. (1), non-zero components of Einstein equation can be written as

$$1 = \Omega_X + \Omega_m + \Omega_r, \quad (9)$$

$$\frac{2H'}{3H^2} = -1 - w_X \Omega_X - \frac{1}{3}\Omega_r, \quad (10)$$

where Ω is the density parameter, $H = \dot{a}/a$ is the Hubble parameter, prime denotes the derivative with respect to $N = \ln a$ and the subscripts m and r represent the quantity corresponding to matter and radiation respectively. The autonomous system is obtained by using the conservation of energy momentum tensor for each species together with the Einstein equations, Eq. (9) and Eq. (10). As a result, the autonomous system corresponding to the generalized three-form field can be written as

$$\Omega'_X = -3\Omega_X \left(w_X(1 - \Omega_X) - \frac{1}{3}(1 - \Omega_X - \Omega_m) \right), \quad (11)$$

$$\Omega'_m = 3\Omega_m \left(w_X\Omega_X + \frac{1}{3}(1 - \Omega_X - \Omega_m) \right), \quad (12)$$

$$w'_X = (1 + w_X)\Gamma \frac{y'}{y}, \quad \Gamma = 1 - \frac{yQ_{,y}}{Q} + \frac{yQ_{,yy}}{Q_{,y}}, \quad (13)$$

where we have chosen $P = P_0 K^\nu Q(y)$. From this system, one can see that the equation of w_X is decoupled from the others. Note that $\Gamma = 0$ for $Q \propto y^\mu$ which implies that $w'_X = 0$ as we expect for the case of constant w_X . From the autonomous system, there are three fixed points as follows; matter-dominated fixed point $(\Omega_X, \Omega_m, \Omega_r) = (0, 1, 0)$, radiation-dominated fixed point $(\Omega_X, \Omega_m, \Omega_r) = (0, 0, 1)$ and dark-energy-dominated fixed point $(\Omega_X, \Omega_m, \Omega_r) = (1, 0, 0)$. Considering equation of w_X in Eq. (13), it is convenient to choose the solution such that $y'/y = \alpha$ where α is a constant. This choosing is qualitatively similar to one in usual three-form field model [10, 11, 12]. Therefore, the equation for w_X in Eq. (13) has a fixed point at $w_X = -1$ and its dynamics and stability depend on the sign of $\alpha\Gamma$. If $\alpha\Gamma < 0$, where $w_X = -1$, the dark-energy-dominated fixed point will be a stable fixed point. By choosing Q to be a more general function than one in [18] as $Q = \exp\left(\frac{(1-2\nu)}{2}\lambda y^n\right)$, we obtain $\Gamma = n$. The conditions to avoid ghost and Laplacian instabilities have been investigated in [18] and then provide the conditions to the model parameter as $n(2\nu - 1) > 0$ and $n(2 + \lambda y^n) > 1$. The sufficient conditions for healthy theory are $n \geq 1/2, \nu > 1/2, \lambda > 0$. Hence, Γ must be always positive to avoid the instabilities. In order to obtain a viable model of dark energy, α must be negative. Moreover, the value of $|\alpha|$ must be sufficiently small to guarantee that w_X does not deviate too much over the history of the Universe. We numerically show this viable model by setting $\alpha = -0.1$ and $n = 1$ as found in Fig 1. Note that for $n \geq 2$, the result is not significantly different as long as $|\alpha|$ is sufficiently small enough.

4. Summary

In this paper, we investigated the generalized three-form field by considering a more general Lagrangian as $L = P(K, y)$ [18] where K is a canonical kinetic term and y is a standard mass term of the three-form field. From the energy momentum tensor of the three-form field, we found the relation between P and its partial derivatives via the equation of state parameter, w_X . By assuming constant w_X , the equation can be exactly solved and its solution can provide the description of non-relativistic matter without caustic problem as found in k-essence scalar field. For non-constant w_X , we specialize our attention in the case $w_X = w_X(y)$ and a suitable Lagrangian is chosen as $P = P_0 K^\nu \exp\left(\frac{(1-2\nu)}{2}\lambda y^n\right)$. By analyzing dynamical system, we found that it is possible to obtain a viable model of dark energy due to the generalized three-form field. We also confirm our analysis by using numerical method which can be found in Fig. 1.

Even though the model can be used to explain the cosmic acceleration of the Universe, it cannot provide the way to solve the coincidence problem. It is of interest to investigate dark energy and dark matter coupling to alleviate the coincidence problem in which both contents are constructed from the three-form fields. We leave this investigation for further work.

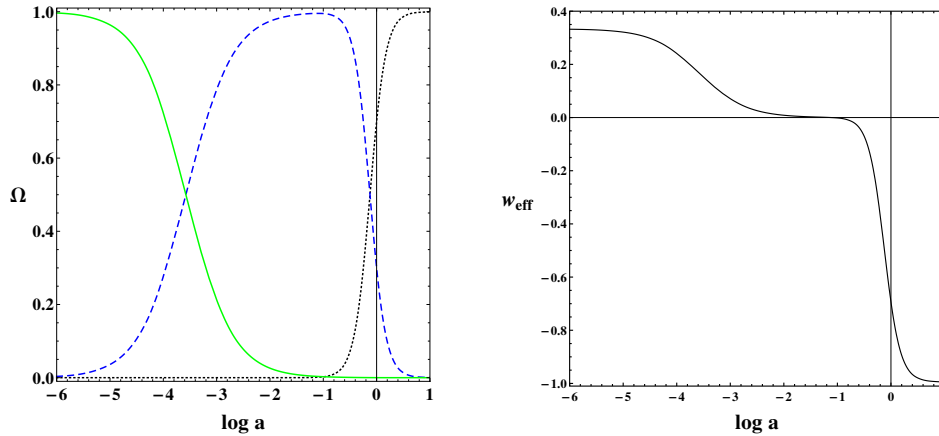


Figure 1. The left panel shows the evolution of the density parameters for three-form dark energy (black dotted line), matter (blue dashed line) and radiation (green solid line). The right panel shows the evolution of the effective equation of state parameter w_{eff} . The parameters are set as $\alpha = -0.1$ and $n = 1$.

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