

# Violation of Bell's inequality: Must the Einstein locality really be abandoned?

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## Abstract.

Since John Bell has established his famous inequality and several independent experiments have confirmed the distinct polarization correlation of entangled photons predicted by quantum mechanics it is evident that quantum mechanics cannot be explained by local realistic theories. Actually, the observed polarization correlation can be deduced from wave optical considerations. The correlation has its origin in the phase coupling of the two circularly polarized wave packets leaving the photon source simultaneously. The experimental results violate Bell's inequality although no non-local interactions have to be assumed. In consequence the principle of locality remains valid in the scope of quantum mechanics. However, the principle of realism has to be replaced by the less stringent principle of contextuality.

## 1. Introduction

In 1964 John Bell [1] presented an inequality allowing physicists to discriminate between quantum mechanics and local realistic theories. Bell showed that spin correlations of entangled particles derived from local realistic theories must be less pronounced than predicted by quantum mechanics. When experiments especially those with entangled photons confirmed the quantum mechanical results [2, 3, 4] local realistic theories were ruled out.

When A. Khrennikov [5] discusses the dilemma whether locality or objectivity should be abandoned he comes to the conclusion that quantum mechanics is nonobjective in the sense that the values of physical observables cannot be assigned to a quantum system before the measurement process has taken place. In Khrennikov's opinion objectivity has to be replaced by contextuality.

B.-G. Englert concluded in his article on quantum theory [6] from considerations on statistical operators that quantum mechanics does not require any non-local action in order to reproduce the polarization correlations of entangled photons. Therefore it might be wise to query the concept of realism or the closely related concept of objectivity. Neither Khrennikov nor Englert propose a mechanism leading to the observed polarization correlations.

The best evidence for the violation of Bell's inequality has been achieved by the measurements of Aspect *et al* [2, 3] and Weihs *et al* [4]. In the experiments of Aspect and coworkers the linear polarizations of the entangled photons are correlated. In contrast, in the experiment of Weihs *et al* the linear polarizations are anti-correlated. In this article mainly the experiments of Weihs *et al* will be discussed. The experiments of Aspect *et al* [2, 3] will be only shortly touched upon at the end.



With respect to the experiment of Weihs *et al* not only the experimentally approved polarization correlations on singlet states will be deduced from wave optical considerations but also polarization correlations of entangled photon pairs in triplet configuration. When future experiments with down conversion sources will confirm the predicted results for triplet states it will be a strong argument in favour of the wave optical considerations presented in this article.

## 2. Symmetries of photon source and detector systems

By means of an optically nonlinear BBO crystal Weihs *et al* [4] convert ultraviolet photons into entangled pairs of green photons. The two electromagnetic wave packets associated with the photons are simultaneously emitted in (nearly) the same direction. The senses of rotation of the photons leaving such a parametric down conversion source are statistically distributed. Photon pairs in triplet configuration are always rotating in the same sense and photon pairs in singlet configuration are always rotating in opposite sense.

After the photons have passed a half wave plate in order to exchange the vertically polarized ordinary and the horizontally polarized extra-ordinary beam the two wave packets traverse separate compensating BBO plates which can be tilted with respect to a horizontal axis perpendicular to the propagation direction of the photons. The compensating plates serve three purposes. They cancel to a great extent the undesired phase shift between ordinary and extra-ordinary beam summed up in the primary birefringent BBO plate.

By tilting the compensating plates the phase of ordinary and extra-ordinary beam can be shifted with respect to each other. The relative phase of the two linearly polarized waves has to be fine-tuned to be  $\pm \pi/2$  thus forming circularly polarized waves. If the probability of finding single photons with a given linear polarization does no longer depend on the polarization direction the pulses are really circularly polarized. This fine-tuning has to be done in both photon channels.

By a larger change of the tilting angle the sense of rotation of the photons passing through the compensating plate can be reverted. If the sense of rotation is reverted in one of the two photon channels a triplet configuration is converted into a singlet configuration or vice versa.

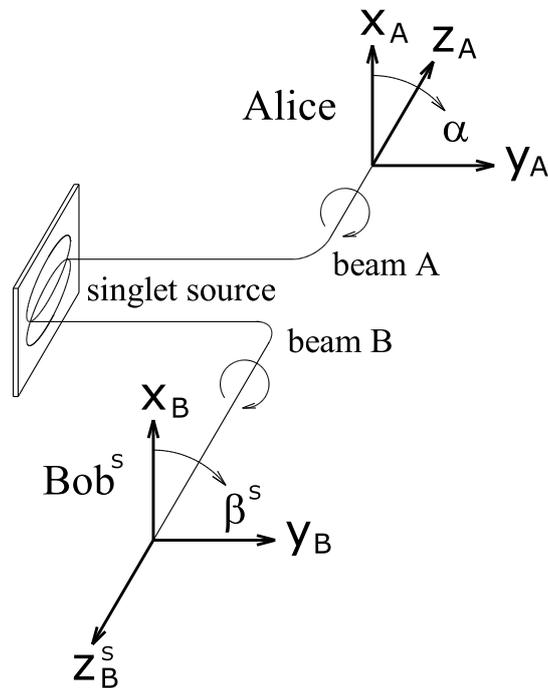
When the coordinate systems of the two observers are suitably chosen the polarization correlation of entangled photons exclusively depends on  $\alpha - \beta$  where  $\alpha$  and  $\beta$  are the polarization angles looked for by Alice and Bob. If the circular polarizations of the two photons are opposite to each other (singlet state) the coordinate systems of the two observers must have opposite handedness. If the circular polarizations of the two photons are equal (triplet state) the coordinate systems of the observers must have equal handedness. That means the symmetries of source and detector systems have to match. In figures 1 and 2 appropriate coordinate systems are shown for studying singlet and triplet states.

In the figures the observer Bob, the corresponding  $z$ -axis and the angle  $\beta$  are distinguished by a superscript indicating that the coordinate systems of Bob are different when the photon pairs are in singlet (S) or triplet (T) configuration. One has to bear in mind that the angles  $\beta^S$  and  $\beta^T$  are opposite to each other, i.e.  $\beta^S = -\beta^T$ . In the past mainly entangled photon pairs in singlet configuration have been studied with the coordinate system depicted in figure 1. Therefore the angle  $\beta$  traditionally used in the literature is identical with  $\beta^S$ .

The singlet case has been exhaustively investigated by Weihs *et al* [4] using the above described down conversion source. Although the triplet configuration has only fragmentarily studied by Gregor Weihs [7] with the same down conversion source it will be discussed at first because the argumentation is simpler than in the singlet case which will be treated afterwards.

## 3. Polarization correlation of photon pairs in triplet configuration

Circularly polarized waves can be understood as the superposition of two equally large linearly polarized waves. The phase shift of these two linearly polarized waves with orthogonal



**Figure 1.** Coordinate systems of two observers with a down conversion source emitting photon pairs in singlet configuration. The coordinate systems of Alice and Bob are right-handed and left-handed, respectively. The angles  $\alpha$  and  $\beta^S$  indicate the orientation of the linear polarization looked for by Alice and Bob<sup>S</sup>. The thin lines between source and observers indicate optical fibres.

polarization directions must be  $\pi/2$ . The absolute orientation of the linearly polarized waves is not relevant as long as they remain orthogonal to each other.

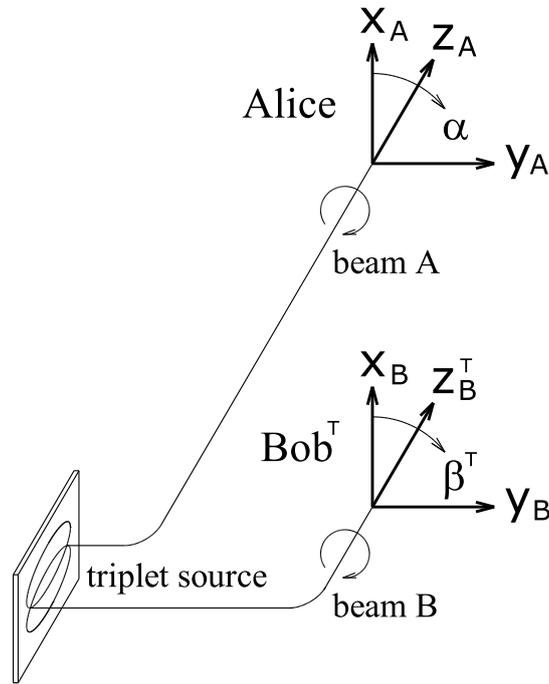
The fragmentation of the circularly polarized wave into two equally sized linearly polarized waves is a convenient method to separate photons into two commensurate disjunct groups. Photons initially embedded in one of the two linearly polarized wave components will always stay in this wave component. The two groups of photons do not intermix. This conservation law even holds when the polarization character of the waves is temporarily changed by the use of birefringent plates or electro-optical modulators. Thus photons detected by Alice or Bob behind polarizing filters were already contained in the corresponding linearly polarized wave components just at the source.

In the following it is assumed that the Wollaston prisma of Alice is orientated in direction  $\alpha$ . Thus one of Alice's detectors looks for photons polarized in the directions  $\alpha$  and  $\alpha + 180^\circ$ . The other one looks for photons polarized in the directions  $\alpha - 90^\circ$  and  $\alpha + 90^\circ$ .

One can assume that the light source contains two sets of linear oscillators. The oscillators  $A_\alpha$  and  $A_{\alpha+90^\circ}$  send circularly polarized light towards Alice. The oscillators  $B_\alpha^T$  and  $B_{\alpha+90^\circ}^T$  send circularly polarized light to Bob<sup>T</sup>. The subscripts indicate the orientation of the linear oscillators.

In the experiments with the BBO down conversion source the linear polarizations are anti-correlated. That means at the source the electric fields of the two clockwise or anti-clockwise rotating wave packets are always orthogonal to each other or in other words the circularly polarized wave packets are phase shifted by  $\Delta\varphi = +\pi/2$  or  $-\pi/2$ .

Four configurations of the simultaneously emitted wave packets have to be distinguished.



**Figure 2.** Coordinate systems of two observers with a down conversion source emitting photon pairs in triplet configuration. The coordinate systems of Alice and Bob are both right-handed. The angles  $\alpha$  and  $\beta^T$  indicate the orientation of the linear polarization looked for by Alice and Bob<sup>T</sup>. The thin lines between source and observers indicate optical fibres.

The time dependences of the four linear oscillators are given in table 1 on the premise that the time dependence of oscillator  $A_\alpha$  is arbitrarily chosen to be  $+\cos \omega t$ . All four cases lead to the same polarization correlation because the transmission probability of the analyzers is periodic with a period of  $180^\circ$ . Thus the signs of the time dependences are not relevant.

Linearly polarized wave packets simultaneously emitted from the oscillators  $A_\alpha$  and  $B_{\alpha+90^\circ}^T$  are equal or opposite in phase and thus induce a maximum coincidence rate. The same is true for the wave packets simultaneously emitted by the oscillators  $A_{\alpha+90^\circ}$  and  $B_\alpha^T$ . The coincidence rates in the two channels with matching (or opposite) phases must be equal because of symmetry reasons. Each channel provides half of the total coincidence rate  $I_0$  obtained when both polarizing filters are removed. The channel  $A_\alpha$  and  $B_\alpha^T$  and the channel  $A_{\alpha+90^\circ}$  and  $B_{\alpha+90^\circ}^T$  do not provide coincidences because the two simultaneously emitted wave packets have a phase shift of  $\pi/2$  at the source and thus are orthogonal to each other.

As has been explained above the orientation of the linearly polarized waves can be arbitrarily chosen. Therefore the coincidence rates found in the four possible coincidence channels are for each angle  $\alpha$

$$\begin{aligned}
 I(\alpha, \beta^T = \alpha) &= 0 \\
 I(\alpha, \beta^T = \alpha + 90^\circ) &= I_0/2 \\
 I(\alpha + 90^\circ, \beta^T = \alpha + 90^\circ) &= 0 \\
 I(\alpha + 90^\circ, \beta^T = \alpha) &= I_0/2.
 \end{aligned} \tag{1}$$

For  $\beta^T = \alpha \pm 90^\circ$  the photon pairs are fully correlated. Assuming both detectors have

perfect efficiency the correlation holds for each individual pair of entangled photons. If one of the photons is detected at Alice by the detector looking for photons polarized in the direction  $\alpha$  the entangled photon will be detected at Bob<sup>T</sup> by the detector looking for the polarization direction parallel to  $\beta^T = \alpha + 90^\circ$ .

The analyzer on Alice's side oriented along  $\alpha$  lets pass 100% of the linearly polarized wave emitted by oscillator  $A_\alpha$ . If the analyzer at Bob<sup>T</sup> is oriented along  $\beta^T \neq \alpha \pm 90^\circ$  it lets pass only a fraction

$$\cos^2(\alpha - \beta^T \pm 90^\circ) = \sin^2(\alpha - \beta^T) \quad (2)$$

of the wave packets send out by the linear oscillator  $B_{\alpha+90^\circ}^T$ . Bob's detector also lets pass linearly polarized light emitted by oscillator  $B_\alpha$ . But this light does not provide additional coincidences because the oscillations of  $A_\alpha$  and  $B_\alpha$  are orthogonal to each other. Therefore the coincidence rate is

$$I(\alpha, \beta^T) = I_0 \sin^2(\alpha - \beta^T)/2. \quad (3)$$

The complementary coincidence rate is consequently

$$I(\alpha, \beta^T \pm 90^\circ) = I_0 \sin^2(\alpha - \beta^T \mp 90^\circ)/2 = I_0 \cos^2(\alpha - \beta^T)/2. \quad (4)$$

The two coincidence rates  $I(\alpha, \beta^T)$  and  $I(\alpha, \beta^T + 90^\circ)$  add up to  $I_0/2$ .

The considerations are symmetric with respect to the role of Alice and Bob<sup>T</sup>. If the two sets of oscillators are oriented along  $\beta^T$  and  $\beta^T + 90^\circ$  the same polarization correlation is obtained.

The polarization correlation can be derived more formally by determining the overlap integral of the two linearly polarized waves looked for by Alice and Bob<sup>T</sup> just at the source.

The time dependences of the linear oscillators  $A_\alpha$ ,  $A_{\alpha+90^\circ}$  and  $B_\beta^T$  are given in table 2 again on the premise that the time dependence of oscillator  $A_\alpha$  is chosen to be  $+\cos\omega t$ . The time dependences of  $B_\beta^T$  in the four rows can be summarized in the form

$$\pm \sin(\omega t \pm (\alpha - \beta^T)). \quad (5)$$

The overlap integral of the two normalized functions  $\sqrt{2} \cos\omega t$  and  $\pm\sqrt{2} \sin(\omega t \pm (\alpha - \beta^T))$  is

$$\begin{aligned} & \pm \frac{\omega}{2\pi} \int_0^{2\pi/\omega} 2 \cos\omega t \sin(\omega t \pm (\alpha - \beta^T)) dt = \\ & \pm \left( \frac{\omega}{2\pi} \int_0^{2\pi/\omega} 2 \cos\omega t \sin\omega t dt \cos(\alpha - \beta^T) \right) \\ & \pm \frac{\omega}{2\pi} \int_0^{2\pi/\omega} 2 \cos\omega t \cos\omega t dt \sin(\alpha - \beta^T) = \pm \sin(\alpha - \beta^T). \end{aligned} \quad (6)$$

**Table 1.** Time dependences of the linear oscillators  $A_\alpha$ ,  $A_{\alpha+90^\circ}$ ,  $B_\alpha^T$ , and  $B_{\alpha+90^\circ}^T$  simultaneously emitting two circularly polarized waves in (nearly) the same direction towards Alice and Bob<sup>T</sup>. The two circularly polarized photons form a triplet pair. At the source the polarization directions of the two waves are always perpendicular to each other. The two circularly polarized wave packets are twisted by  $\Delta\varphi = +90^\circ$  or  $-90^\circ$  with respect to each other.

	Rotation	$\Delta\varphi$	$A_\alpha$	$A_{\alpha+90^\circ}$	$B_\alpha^T$	$B_{\alpha+90^\circ}^T$
A	clockwise	$+90^\circ$	$+\cos\omega t$	$+\sin\omega t$	$+\sin\omega t$	$-\cos\omega t$
B	countercl.	$+90^\circ$	$+\cos\omega t$	$-\sin\omega t$	$-\sin\omega t$	$-\cos\omega t$
C	clockwise	$-90^\circ$	$+\cos\omega t$	$+\sin\omega t$	$-\sin\omega t$	$+\cos\omega t$
D	countercl.	$-90^\circ$	$+\cos\omega t$	$-\sin\omega t$	$+\sin\omega t$	$+\cos\omega t$

The coincidence rate is proportional to the square of the overlap amplitude of the two wave packets looked for by Alice and Bob<sup>T</sup> just at the source. Thus the coincidence rate is

$$I(\alpha, \beta^T) = I_0 \sin^2(\alpha - \beta^T)/2. \quad (7)$$

in agreement with equation (3).

The predicted results should be carefully tested in future experiments. This means among others that quarter wave plates in front and at the rear of the electro-optical modulators should be introduced. By means of a Poincaré sphere Gregor Weihs describes at large in his doctor thesis [7] that an electro-optical modulator only twists a linear polarization into another linear polarization when the modulator is sandwiched between two quarter wave plates with appropriate orientations of the optical axes. In the experiment of Weihs *et al* [4] these quarter wave plates have been omitted. Only for the angles 0°, 90°, 180° and 270° Alice and Bob really look for linearly polarized light. For the pretended angles 45°, 135°, 225° and 315° the observers look for circularly polarized waves. For all other angles Alice and Bob look for elliptically polarized light.

In the triplet configuration the entangled photons are anti-correlated with respect to the linear polarization and correlated with respect to the circular polarization. Therefore the omission of the quarter wave plates would lead to fallacious angular dependences of the coincidence rate.

#### 4. Polarization correlation of photon pairs in singlet configuration

The argumentation to derive the polarization correlation in the singlet case is less obvious than in the triplet case because the electric fields of the associated wave packets rotate in opposite directions. In fact, it is not so difficult to define a constant phase as could have been assumed at first sight. In the experiment of Weihs *et al* [4] this counter-rotation has been achieved by switching the tilting angle of the compensating plate on Bob's side and thus reverting the sense of rotation of the photons propagating towards Bob. A constant phase difference of the two counter-rotating waves can be defined when the sense of rotation of the photons on Bob's side is once more reverted by going over from the right-handed coordinate system of Bob<sup>T</sup> (Fig. 1) to the left-handed coordinate system of Bob<sup>S</sup> (Fig. 2).

The two reversions cancel each other when the associated mirror planes coincide. In the experiment of Weihs *et al* [4] this condition is fulfilled because both mirror planes are oriented along  $\beta = 0^\circ$ .

Consequently the argumentation used in the previous section can be applied analogously. Thus the coincidence rate is

$$I(\alpha, \beta^S) = I_0 \sin^2(\alpha - \beta^S)/2 \quad (8)$$

**Table 2.** Time dependences of the linear oscillators  $A_\alpha$ ,  $A_{\alpha+90^\circ}$  emitting circularly polarized waves towards Alice. The oscillators  $B_\beta^T$  and  $B_{\beta+90^\circ}^T$  send circularly polarized waves towards Bob. The time dependence of the oscillator  $B_\beta^T$  is given in the last column. The four rows for clockwise and anticlockwise rotation and the phase shift  $\Delta\varphi = +90^\circ$  or  $-90^\circ$  are listed in the same order as in table I

	Rotation	$\Delta\varphi$	$A_\alpha$	$A_{\alpha+90^\circ}$	$B_\beta^T$
A	clockwise	$+90^\circ$	$+\cos\omega t$	$+\sin\omega t$	$+\sin(\omega t + (\alpha - \beta^T))$
B	countercl.	$+90^\circ$	$+\cos\omega t$	$-\sin\omega t$	$-\sin(\omega t - (\alpha - \beta^T))$
C	clockwise	$-90^\circ$	$+\cos\omega t$	$+\sin\omega t$	$-\sin(\omega t + (\alpha - \beta^T))$
D	countercl.	$-90^\circ$	$+\cos\omega t$	$-\sin\omega t$	$+\sin(\omega t - (\alpha - \beta^T))$

as has been confirmed by Weihs *et al* [4].

As has already been mentioned in the previous section, the quarter wave plates before and behind the electro-optical modulators have been omitted in the experiment of Weihs *et al* [4]. This experimental laxness has no consequences for the coincidence rates because in the singlet configuration the entangled photons are not only anti-correlated with respect to the linear polarization but also with respect to the circular polarization. In fact, for the sake of clarity the missing quarter wave plates should be introduced in future experiments in order to really prove the intended polarization correlations.

### 5. Polarization correlation of entangled photons emitted by doubly excited atoms

Aspect, Grangier and Roger could show in 1981 that polarization correlations of entangled photons significantly violate Bell's inequality. They studied circularly polarized photon pairs emitted by doubly excited calcium-40 atoms in a  $(4p^2\ ^1S_0) \rightarrow (4s4p\ ^1P_1) \rightarrow (4s^2\ ^1S_0)$  decay cascade. The short living intermediate  $^1P_1$  state has a life time of 5 *ns*. Only entangled photon pairs leaving the atom in opposite directions have been detected by the two observers. The two photons form a singlet state because the two wave packets are rotating in opposite directions with respect to their propagation directions.

A constant phase shift can be defined when the rotational frequencies of the two wave packets are equal. The two photons ( $\lambda_1 = 551.3\ nm, \lambda_2 = 422.7\ nm$ ) emitted in a decay cascade of calcium-40 have different energies. That means carrier and rotational frequencies cannot be equal for both photons. This particular feature should be systematically tested in future experiments.

The wave packets emitted by doubly excited atoms are in phase ( $\Delta\varphi = 0^\circ$ ) or opposite in phase ( $\Delta\varphi = 180^\circ$ ). That means the electric fields of the two wave packets are always parallel or antiparallel to each other at the source. In the wave optical model presented above the oscillators  $A_\alpha$  and  $B_\alpha$  are in phase. The same is true for the oscillators  $A_{\alpha+90^\circ}$  and  $B_{\alpha+90^\circ}$ . In consequence the coincidence rate is maximal when the two detectors are parallel and zero if the two detectors are orthogonal to each other. Thus the polarization correlation is given by

$$I(\alpha, \beta^S) = I_0 \cos^2(\alpha - \beta^S)/2 \quad (9)$$

The photons are correlated with respect to their linear polarization, but anti-correlated with respect to their circular polarization.

### 6. Concluding remarks

The derivation of the polarization correlation is based on the fact that circularly polarized wave packets associated with entangled photon pairs are phase shifted at the source. For such waves the information about the phase relation of the two associated wave packets at the source can be deduced from the difference angle of the polarization directions looked for by the two observers.

Elementary processes providing circularly polarized entangled photon pairs are generally characterized by a typical phase difference of the two simultaneously emitted wave packets. If the phase difference at the source is  $0^\circ$  (or  $180^\circ$ ) the linear polarizations are correlated [2, 3]. If the phase difference is  $90^\circ$  the linear polarizations are anti-correlated [4, 7].

When the polarization character of the wave packets is not changed the distances between source and observers have no influence on the experimentally found polarization correlations. Usually in optical fibers the polarization state is changed by spurious birefringent effects. Therefore Weihs *et al* [4, 7] have to recover the original polarization state by manually compensating the undesired phase shifts in both photon channels.

Initiated by the violation of Bell's inequality physicists investigating polarization correlations of entangled photons mostly assume that their must be a superluminal information transfer between the two observers. Salart *et al* [8] try to find a lower limit for the speed of spooky

action at a distance. Shalm *et al* [9, 10] even claim to have proven that spooky action at a distance is really real.

Following the wave optical considerations presented above the polarization correlations can be explained without assuming non-local interactions. Therefore it is no longer necessary to perform further experiments with enlarged distances [8] or with ultra-high efficiency detectors [9, 10] in order to close possible loopholes.

The contradictory statements on the occurrence of non-local interactions rest on different assumptions. Bell's inequality is derived from a purely particle based consideration. On the simplest guess it is purely accidental at which output of a Wollaston prism a circularly polarized photon is detected. Thus the coincidence rate is  $I_0/4$  for all angles  $\alpha$  and  $\beta$ .

Particle based considerations generally lead to wrong conclusions when quantum mechanical results depend on phase differences. For example the intensity ratio of the photons detected at the two outputs of a Mach-Zehnder interferometer can only be derived from the relative phase of the two partial waves reaching the second semi-permeable mirror. In the case of interferometers obviously only wave based considerations lead to proper results.

The wave optical explanation given above is purely wave based. The information contained in phase differences of distinct wave components is maintained. The polarization correlation determined by Alice and Bob is equal to the polarization correlation just at the source because the two photon groups contained in the two linearly polarized wave components do not intermix. Thus the principle of locality remains valid.

It is fully accidental through which output of a Wollaston prism the first photon leaves. For the second photon, however, the probability is well defined. There is no further evidence for any accidental behaviour. For  $\alpha = \beta$  the strict correlation even holds for each pair of entangled photons. Thereby it is not relevant which of the two photons is detected first.

On the other hand the principle of realism has to be reconsidered. The linear polarization component of a circularly polarized photon is not defined before the measurement has taken place. Nonetheless the polarization correlation of entangled photons measured at distant locations is well defined and agrees with the predictions of quantum mechanics. Obviously the principle of realism has to be replaced by the principle of contextuality. That means the polarization correlation is only well defined when the polarization directions looked for by the two observers are both fixed. But this assertion is trivial. If coincidence rates do not depend on  $\alpha$  as well as on  $\beta$  it would make no sense to measure coincidences at all.

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