

Frequency-based redshift for cosmological observation and Hubble diagram from the 4-D spherical model in comparison with observed supernovae

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Abstract. According to the formerly reported 4-D spherical model of the universe, factors on Hubble diagrams are discussed. The observed redshift is not the prolongation of wavelength from that of the source at the emission but from the wavelength of spectrum of the present atom of the same element. It is equal to the redshift based on the shift of frequency from the time of emission. We demonstrate that the K-correction corresponds to conversion of the light propagated distance (luminosity distance) to the proper distance at present (present distance). Comparison of the graph of the present distance times $1+z$ versus the frequency-based redshift with the reported Hubble diagrams from the Supernova Cosmology Project, which were time-dilated by $1+z$ and K-corrected, showed an excellent fit for the Present Time (the radius of 4-D sphere) being c.a. 0.7 of its maximum.

1. Introduction

The author previously reported the light propagated distance and the redshift derived from the 4-D spherical model of the universe [1]. However, in actual cosmological observation, the redshift is based on the wavelength prolongation from the spectrum of the present atom but not from the spectrum at the time of emission at the light source. Under varying light speed along with the cosmological expansion, the redshift based on wavelength is different from that based on frequency. In this paper, we will discuss the two kinds of redshift, and provide the Hubble diagram expected from the 4-D spherical model. Subsequently, we will induce the proper distance at present (present distance) as a function of redshift or time of emission, and compare its Hubble diagram with those reported by the Supernova Cosmology Project (SCP).

According to the 4-D spherical model [1–4], the space energy spreads with expansion in a 3-D surface of a 4-D sphere as shown in Fig.1. Light and any other energy we detect in the 3-D space is a vibration of the intrinsic space energy in the 3-D space. The radius x of the 4-D sphere for the space energy distribution is our Observed Time (shown by Time or T), which we feel passing constantly and commonly for all of us [5]. Listed below definitions and abbreviations of key terms for the model.



x (Radius of universe):	Radius of the 4D sphere, equal to the Observed Time (Time or T)
CU (Cosmic Unit):	Unit of x and T , being one at its maximum. (Time-related variables are expressed in the CU in this article.)
T_E (Time of Emission):	Time when the light was emitted.
T_P (Present Time):	Present Time of universe, when the light reaches us.
T_{ER} (Relative Time of Emission):	Relative ratio of T_E to T_P . $T_{ER} \equiv T_E/T_P$
T_B (Back in Time):	Back in Time from present when the light was emitted. $T_B \equiv T_P - T_E$
T_{BR} (Relative Back in Time):	Relative ratio of T_B to T_P . $T_{BR} \equiv T_B/T_P$
T_C (Time Clear):	Time when the space became transparent to light.
T_{CR} (Relative Time Clear):	Relative ratio of T_C to T_P . $T_{CR} \equiv T_C/T_P$

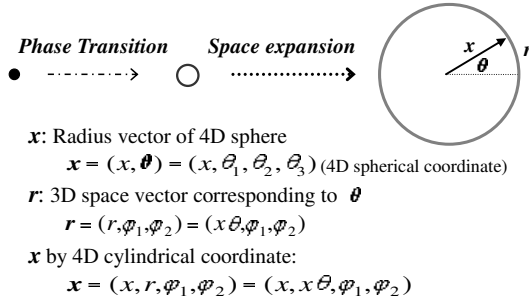


Figure 1. 4-D spherical model of the universe. The space energy spreads with expansion in a 3-D surface of a 4-D sphere.

2. Redshift for cosmological observation

Under the varying light speed $C(T)$, redshift based on wavelength is different from that based on frequency. Light of $\nu_0(T_E)$, $\lambda_0(T_E)$ released at T_E reaches us now at T_P exhibiting $\nu(T_P)$, $\lambda(T_P)$. $\nu_0(T_P)$, $\lambda_0(T_P)$ is the spectrum of the present atom. $\nu_0(T_E)$ is equal to $\nu_0(T_P)$ because the electric orbital energy gap of the atom is same for both.

$$C(T_E) = \nu_0(T_E) * \lambda_0(T_E) \quad \rightarrow \quad C(T_P) = \nu(T_P) * \lambda(T_P) \quad (1)$$

$$\text{Present atom spectrum : } C(T_P) = \nu_0(T_P) * \lambda_0(T_P) \quad (2)$$

The observed redshift is not the prolongation of wavelength from that of the source at the emission but from the wavelength of spectrum of the present atom of the same element. It is equal to the redshift based on the shift of frequency from the time of emission.

$$z + 1 = \frac{\lambda(T_P)}{\lambda_0(T_P)} = \frac{\nu_0(T_P)}{\nu(T_P)} = \frac{\nu_0(T_E)}{\nu(T_P)} = z_\nu + 1 \quad (3)$$

Wavelength-based redshift z_λ : Wavelength prolongation from T_E to T_P is the space expansion ratio n times the light speed ratio $C(T_P)/C(T_E)$ as formerly reported [1].

$$z_\lambda + 1 \equiv \frac{\lambda(T_P)}{\lambda_0(T_E)} = n \times \frac{C(T_P)}{C(T_E)} = \frac{T_P}{T_E} \times \frac{C(T_P)}{C(T_E)} = \frac{1}{T_{ER}} \times \frac{C(T_P)}{C(T_E)} \quad (4)$$

Frequency-based redshift z_ν (observed redshift):

$$z_\nu + 1 \equiv \frac{\nu_0(T_E)}{\nu(T_P)} = \frac{\lambda(T_P)}{\lambda_0(T_P)} = \frac{\lambda(T_P)\lambda_0(T_E)}{\lambda_0(T_E)\lambda_0(T_P)} = n \times \frac{C(T_P)}{C(T_E)} \times \frac{C(T_E)}{C(T_P)} = n = \frac{1}{T_{ER}} \quad (5)$$

Hereafter we use simply z for z_ν .

3. Factors affecting the brightness of a star

3.1. Factor by wavelength prolongation

Along with the space expansion, the wavelength is stretched by $n \equiv T_P/T_E = 1/T_{ER}$. While the energy of a single photon $h\nu$ decreases to $1/n$ times, the number of photons increases to n -fold to offset and preserve the total energy. As far as we use the luminosity as energy per unit time, we do not have to consider the factor by wavelength prolongation.

3.2. Factor by scattering

I previously proposed the following light speed equation, where x is the radius of 4-D sphere equal to our Observed Time that we feel passing constantly and commonly for all of us [1].

$$C(x) = K * f_D * f_{EM} = K * \frac{1}{x\sqrt{1-x}} * \left(1 - \frac{T_C^3}{x^3}\right) \quad (6)$$

The Electromagnetic interaction factor f_{EM} is effect of scattering. There are plural Time Clear T_C values. One is for the Cosmic Microwave Background (CMB) radiation [6] about 380 thousands years after the Big Bang when the universe cleared up after completion of atom formation from plasma. Later, hydrogen atoms started to form stars and radiate light, which re-ionized interstellar hydrogen atoms [7]. Then, the space became opaque again. Once the number of stars reached roughly plateau, the factor f_{EM} immediately increased and approached to one due to rapid decrease of the star density and the hydrogen atom density in the space. The third type of T_C -like is local one for light propagation in substances such as glass, water and air on the earth, and dust and gas in galaxies, whereas the optical path in the substance replaces the radius x in the f_{EM} formula. T_C of a substance and its density ρ give the absorbance. If the density is constant, the transmittance becomes as follows, where x is the optical depth.

$$I(x) = I_0 \left(1 - \int (T_C \rho)^3 dx\right) = I_0(1 - kx) \quad (7)$$

Stellar light emitted during the reionization period, said to be roughly from 150 million years to one billion years after the Big Bang [7], does not reach us or is darken by scattering. For stellar light emitted later, corresponding to $T_{ER} > 0.072$ or $z < 12.8$, the factor of scattering by reionization is negligible. Observed one is extinction due to absorption and scattering by dust and gas [8]. Atmospheric extinction on the earth is depending on location and altitude of the observing device. Galactic extinctions by our galaxy or host galaxy also dim the brightness a little, depending on directions of light lines. Light of shorter wavelength is dimmed more than light of longer wavelength, resulting in reddening of observed light. Corrections in brightness for extinctions are made from color excess functions of individual objects using respective standard extinction curves.

3.3. Factor by variation of the space energy density

Along with the space expansion, the density of the space energy (medium) decreases and causes light speed alteration. I formerly proposed the energy density factor of light speed f_D [1,3] as

$$f_D = \frac{1}{x\sqrt{1-x}}. \quad (8)$$

As an example of varying density of a wave medium, think of sound propagation. The sound propagates faster in a higher density medium. The intensity of volume is greater in a medium of higher density for same distance, that is, the sound propagates louder in the water than in the air. This phenomenon should be same for variation of the space energy density.

The brightness is usually expressed as energy per unit time, that is, luminosity as energy per unit time and flux as energy per unit time per unit area. The flux is shown as follows, where L is the luminosity and r is the distance from the light source.

$$F = \frac{L}{4\pi r^2} \quad (9)$$

Under constant light speed, the luminosity would be invariant by time. However, it varies depending on the light speed at the time of detection because it is a per unit-time value of energy, while the total energy from the source is invariant. Therefore, the luminosity is a function of the time of detection $T = x$.

$$F(x) = \frac{L(C(x))}{4\pi r^2} = \frac{L(x)}{4\pi r^2} \quad (10)$$

For our observation, the variable x is fixed to the Present Time T_P . $L = L(T_P)$ and $F = F(T_P)$. Therefore, we do not have to consider influence of light speed variation by space expansion for our observation of brightness whenever light was emitted.

4. Magnitude of flux and distance

Light of luminosity L emitted at T_E reaches us now at T_P . We observe now the flux $F(T_E)$ after correction of observational biases. $LD(T_E)$ is the light propagated distance equal to the luminosity distance. Define the magnitude of the flux as follows.

$$F(T_E) = \frac{L}{4\pi LD(T_E)^2} \quad (11)$$

$$m(T_E) \equiv -2.5 \lg F(T_E) \quad (12)$$

Define the relative magnitude to the reference of the same luminosity exhibiting redshift 0.05 as the “magnitude of LD to $z = 0.05$, $DM_{0.05}$ ”. It is a distance modulus. $z = 0.05$ corresponds to $T_{ER} = 1/(1+z) = 1/1.05$.

$$\begin{aligned} DM_{0.05} &\equiv m(T_{ER}) - m(1/1.05) \\ &= 5 \lg LD(T_{ER}) - 5 \lg LD(1/1.05) \end{aligned} \quad (13)$$

From eq.6 of light speed, the light propagated distance is given as follows [1], where f_{EM} is negligible if $T_{ER} > 0.072$ or $z < 12.8$.

$$\begin{aligned} LD(T_E) &= \int_{T_E}^{T_P} C(x) dx = \int_{T_E}^{T_P} \frac{K}{x\sqrt{1-x}} \left(1 - \frac{T_C^3}{x^3}\right) dx \approx \int_{T_E}^{T_P} \frac{K}{x\sqrt{1-x}} dx \\ &= K \left(\log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \right) - \log \left(\frac{1 - \sqrt{1-T_E}}{1 + \sqrt{1-T_E}} \right) \right) \end{aligned} \quad (14)$$

$$LD(T_{ER}) = K \log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P T_{ER}}}{1 - \sqrt{1-T_P T_{ER}}} \right) \quad (15)$$

We get the following equation for the magnitude of LD to $z = 0.05$.

$$\begin{aligned} DM_{0.05}(T_{ER}) &= 5 \lg \left(\log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P T_{ER}}}{1 - \sqrt{1-T_P T_{ER}}} \right) \right) \\ &\quad - 5 \lg \left(\log \left(\frac{1 - \sqrt{1-T_P}}{1 + \sqrt{1-T_P}} \cdot \frac{1 + \sqrt{1-T_P/1.05}}{1 - \sqrt{1-T_P/1.05}} \right) \right) \end{aligned} \quad (16)$$

Fig.2 shows graphs of $DM_{0.05}(T_{ER})$ with parameter T_{ER} for T_P values 0.6, 0.7 and 0.8 in dual logarithmic scale. (a) is versus the frequency-based redshift $z = 1/T_{ER} - 1$. (b) is versus the Relative Back in Time $T_{BR} = 1 - T_{ER}$, which times T_P signifies the light travel time since the emission. The dotted line in black is a reference for constant light speed based on the distance $LD_C(T_{ER}) = c(T_P - T_E) = c \cdot T_P(1 - T_{ER})$.

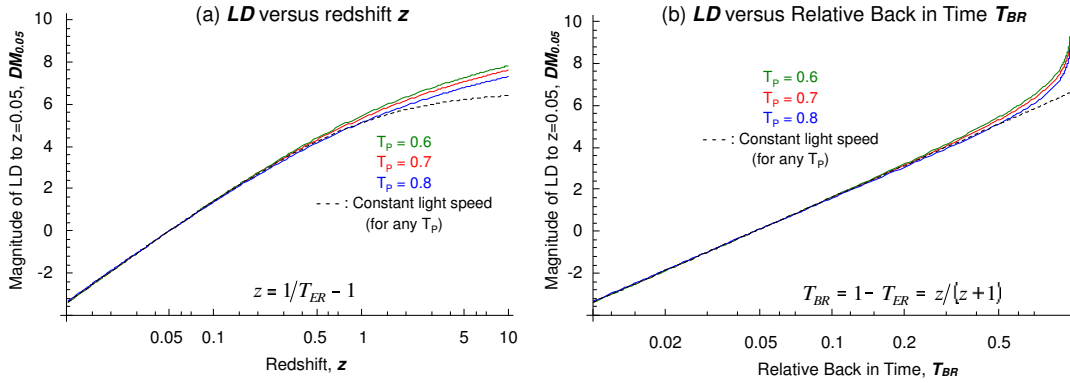


Figure 2. Magnitude of light propagated distance LD to $z = 0.05$ versus (a) redshift or (b) Relative Back in Time. T_P is Present Time. The dotted line in black is a reference for constant light speed.

5. Hubble diagram

According to the 4-D spherical model, the expansion speed of the 3-D space by our Observed Time is constant for a given angle θ as shown by

$$\frac{dr}{dT} = \frac{dr}{dx} = \frac{d(x\theta)}{dx} = \theta. \quad (17)$$

r is a distance between any two points in the 3-D space. θ is the angle between the two positions from the center of 4-D sphere. For variable θ at a given x , the recessional velocity is in proportion to θ and the distance $r = \theta \cdot x$. This is the Hubble's law. The said velocity is of the proper distance. Refer the proper distance at present to as the "present distance, PD ".

5.1. LD-PD conversion

Take a case shown in Fig.3. Light emitted at A_E (3-D location A , time T_E) reaches us at B_P (B , T_P). The light propagated distance LD is the 3-D space component of the line $A_E B_P$, equal to the line CB_P . n is the space expansion ratio given by $n = A_P B_P / A_E B_E = T_P / T_E = 1 / T_{ER}$. As we see in Fig.3, the ratio of CB_P to $A_E B_E$ is

$$\frac{CB_P}{A_E B_E} = 1 + \frac{1}{2}(n - 1) = \frac{n + 1}{2}. \quad (18)$$

The ratio of converting the light propagated distance LD to the present distance PD becomes as follows.

$$\frac{PD}{LD} = \frac{A_P B_P}{A_E B_E} \cdot \frac{A_E B_E}{CB_P} = n \cdot \frac{2}{n + 1} = \frac{2(z + 1)}{z + 2} = \frac{2}{1 + T_{ER}} \quad (19)$$

We get the following "magnitude of PD to $z = 0.05$, $DM_{0.05}^{PD}$ ".

$$\begin{aligned} DM_{0.05}^{PD}(T_{ER}) &= DM_{0.05}(T_{ER}) + 5 \lg(2 / (1 + T_{ER})) - 5 \lg(2 / (1 + 1/1.05)) \\ &= DM_{0.05}(T_{ER}) - 5 \lg(1 + T_{ER}) + 5 \lg 2.05 - 5 \lg 1.05 \end{aligned} \quad (20)$$

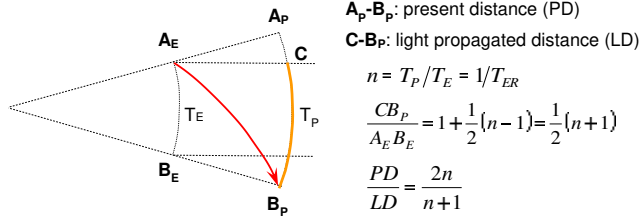


Figure 3. Light line in the 4-D coordinate. Light emitted at $A_E(A, T_E)$ reaches us at $B_P(B, T_P)$. The light propagated distance LD is the 3-D space component of the line $A_E B_P$ equal to the line CB_P .

5.2. Time dilation

Any model of universe including space expansion requires the so-called time dilation [9]. Take a time interval of two events of light emission. Light emits at location A, time T_E , and then reaches B at T_P . Another light dispatches A at $T_E + \Delta t$ and arrives at B at $T_E + \Delta t'$. Due to the space expansion by $1 + z$ from T_E to T_P , the time interval of the events increases by $1 + z$ for light detection at T_P .

$$\Delta t' = (1 + z)\Delta t \quad (21)$$

This time dilation was verified by observation on the light curve width of supernovae. Observed supernovae exhibited a uniform light curve width after divided by $1 + z$ along with normalization of maximum luminosities [10, 11].

However, this is not a real time dilation but the distance $c\Delta t$ expands during the light travels from A to B. In Hubble diagrams including those by the SCP [12], the luminosity distance is multiplied by $1 + z$ as the time dilation. A local light-path near T_E expands by $1 + z$ at T_P . However, the expansion ratio of later path becomes smaller and approaches one for path around T_P . The expansion ratio of the whole light propagated distance to the proper distance at T_E is given by

$$\frac{CB_P}{A_E B_E} = \frac{n+1}{2} = 1 + \frac{z}{2} \quad (22)$$

, which is not $1 + z$. The dilation $1 + z$ is not the LD-PD conversion, either. The time duration $T_P - T_E$ is not dilated whereas is given as a function of the redshift. What does $(1 + z)LD$ signify? Provide tentatively that the light speed has been constant from the emission to the present. The light propagated distance LD_C from T_E to T_P is given as follows.

$$LD_C = c(T_P - T_E) = c \cdot T_P(1 - T_{ER}) = c \cdot T_P \cdot z/(1 + z) \quad (23)$$

Multiply the both sides by $1 + z$, we get the following equation.

$$(1 + z)LD_C = c \cdot T_P \cdot z \equiv k \cdot z \quad (24)$$

The time duration of light traveling is dilated by $1 + z$, and becomes $T_P \cdot z$ equal to the duration when light travels from the present location A at T_P to our location B in future at $T_P(1 + z)$. An important feature of this modification is that the $(1 + z)$ -dilated LD becomes proportional to the redshift z if light speed has been constant from the emission. This could be a reason why time-dilated luminosity distance versus redshift graphs are often used as a Hubble diagram, in addition to the time interval dilation seen on the light curve width of supernovae [10, 11].

5.3. K-correction

The K-correction converts a measurement of an object at a redshift z to an equivalent measurement in the rest frame of the object at $z = 0$ [13–16]. $f(\lambda)$ is the flux density by spectral energy distribution (SED) at wavelength λ of an object 10 parsecs away (energy per unit time per unit area per unit wavelength). $S_i(\lambda)$ is the effective transmission of band i . $Z_i(\lambda)$

is the flux density of a zero-magnitude standard star. Virtually consider a stationary universe without expansion. The apparent magnitude in band x becomes as follows, where DM^0 is the distance modulus defined by $DM^0 = 5 \lg (D_L^0/10pc)$, where D_L^0 is the luminosity distance.

$$m_x^0 = M_x + DM^0 = -2.5 \lg \left(\frac{\int f(\lambda) S_x(\lambda) d\lambda}{\int Z_x(\lambda) S_x(\lambda) d\lambda} \right) + DM^0 \quad (25)$$

In a space-expansion model of the universe, the SED is redshifted at the observer. If the SED is redshifted by z in the above case with the same flux density function and distance, the magnitude in band y becomes as follows because the wavelength is dilated by $1+z$.

$$m_y = -2.5 \lg \left(\frac{\frac{1}{1+z} \int f \left(\frac{\lambda}{1+z} \right) S_y(\lambda) d\lambda}{\int Z_y(\lambda) S_y(\lambda) d\lambda} \right) + DM^0 \quad (26)$$

The flux in the numerator is divided by $1+z$ because of the dilation of wavelength width for the flux density. The difference of the two magnitudes is the K-correction given as follows.

$$\begin{aligned} K_{xy} &\equiv m_y - m_x^0 = -2.5 \lg \left(\frac{\frac{1}{1+z} \int f \left(\frac{\lambda}{1+z} \right) S_y(\lambda) d\lambda}{\int Z_y(\lambda) S_y(\lambda) d\lambda} \right) + 2.5 \lg \left(\frac{\int f(\lambda) S_x(\lambda) d\lambda}{\int Z_x(\lambda) S_x(\lambda) d\lambda} \right) \\ &= -2.5 \lg \left(\frac{\int Z_x(\lambda) S_x(\lambda) d\lambda}{\int Z_y(\lambda) S_y(\lambda) d\lambda} \right) + 2.5 \lg(1+z) + 2.5 \lg \left(\frac{\int f(\lambda) S_x(\lambda) d\lambda}{\int f \left(\frac{\lambda}{1+z} \right) S_y(\lambda) d\lambda} \right) \end{aligned} \quad (27)$$

This formula is exactly the energy-based K-correction, which the SCP applied [14]. We can rewrite the definition of K-correction as

$$m_y = M_x + DM^0 + K_{xy} . \quad (28)$$

If we actually observe m_y with redshift z , it gives the following formula with the absolute magnitude in band y and the luminosity distance in the observed frame.

$$m_y = M_y + DM(z) . \quad (29)$$

Accordingly, we get the following relation for the K-correction.

$$K_{xy} = M_y - M_x + DM(z) - DM^0 \quad (30)$$

As shown in eq.30, the K-correction amounts to the sum of difference in absolute magnitude and that in distance modulus between the redshifted and rest frames. The difference $M_y - M_x$ is the cross-filter adjustment on absolute magnitude between x and y bands, an observational bias. The rest part

$$K_{xy}^D \equiv DM(z) - DM^0 \quad (31)$$

is the theoretical part of K-correction in distance. The luminosity distance $D_L(z)$ in the observed frame is equal to the light propagated distance $LD(z)$. The luminosity distance D_L^0 in the rest frame corresponds to the present distance PD of the light source. We can conclude that the theoretical part of K-correction for frame conversion exactly expresses the LD-PD conversion.

6. Comparison with Hubble diagrams from the Supernova Cosmology Project

The reported graphs of Hubble diagram by the SCP applied the time dilation by $1 + z$ and K-corrections [12, 17, 18]. For directly comparing the expected theoretical curves from the 4-D spherical model with the reported Hubble diagrams by the SCP, we multiply the PD by $1 + z$ because the theoretical part of K-correction corresponds to the LD-PD conversion.

$$(1 + z) \cdot PD = (1 + z) \cdot \frac{2(z + 1)}{z + 2} \cdot LD \quad (32)$$

$$\frac{1}{T_{ER}} \cdot PD = \frac{1}{T_{ER}} \cdot \frac{2}{1 + T_{ER}} \cdot LD \quad (33)$$

Refer the magnitude of this quantity to as the “adjusted magnitude of PD to $z = 0.05$, $DM_{0.05}^{adj}$ ”.

$$\begin{aligned} DM_{0.05}^{adj}(T_{ER}) &= 5(\lg LD(T_{ER}) + \lg(1/T_{ER}) + \lg(2/(1 + T_{ER})) - \lg LD(1/1.05) - \lg 1.05 - \lg(2 * 1.05/2.05)) \\ &= 5(\lg LD(T_{ER}) - \lg(T_{ER}) - \lg(1 + T_{ER}) - \lg LD(1/1.05) - 2 \lg 1.05 + \lg 2.05) \end{aligned} \quad (34)$$

It corresponds to the time-dilated by $1 + z$ and K-corrected magnitude for a common luminosity like type Ia supernovae in real observation, while respective zero points are different. Substituting eq.15 for LD in eq.34, we get the following final equation for comparison with observed data.

$$\begin{aligned} DM_{0.05}^{adj}(T_{ER}) &= 5 \lg \left(\log \left(\frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}} \cdot \frac{1 + \sqrt{1 - T_P T_{ER}}}{1 - \sqrt{1 - T_P T_{ER}}} \right) \right) - 5 \lg(T_{ER}) - 5 \lg(1 + T_{ER}) \\ &\quad - 5 \lg \left(\log \left(\frac{1 - \sqrt{1 - T_P}}{1 + \sqrt{1 - T_P}} \cdot \frac{1 + \sqrt{1 - T_P/1.05}}{1 - \sqrt{1 - T_P/1.05}} \right) \right) - 10 \lg 1.05 + 5 \lg 2.05 \end{aligned} \quad (35)$$

As a reference, take $DM_{0.05}^{adj}$ based on LD_C presuming that the light speed had been constant since the emission to the present. From $LD_C = c \cdot T_P(1 - T_{ER})$, it becomes as follows.

$$\begin{aligned} DM_{0.05}^{adj}(T_{ER}) &= 5 \lg(1 - T_{ER}) - 5 \lg(1 - 1/1.05) - 5 \lg(T_{ER}) - 5 \lg(1 + T_{ER}) - 10 \lg 1.05 + 5 \lg 2.05 \\ &= 5 \lg(1 - T_{ER}) + 5 \lg 21 - 5 \lg(T_{ER}) - 5 \lg(1 + T_{ER}) - 10 \lg 1.05 + 5 \lg 2.05 \end{aligned} \quad (36)$$

Fig.4 shows the graphs of the adjusted magnitude of PD to $z = 0.05$, $DM_{0.05}^{adj}$ versus the redshift $z = 1/T_{ER} - 1$, respectively in a logarithmic scale and a uniform scale for z . The dotted line in black is the reference $DM_{0.05}^{adj}$ for constant light speed.

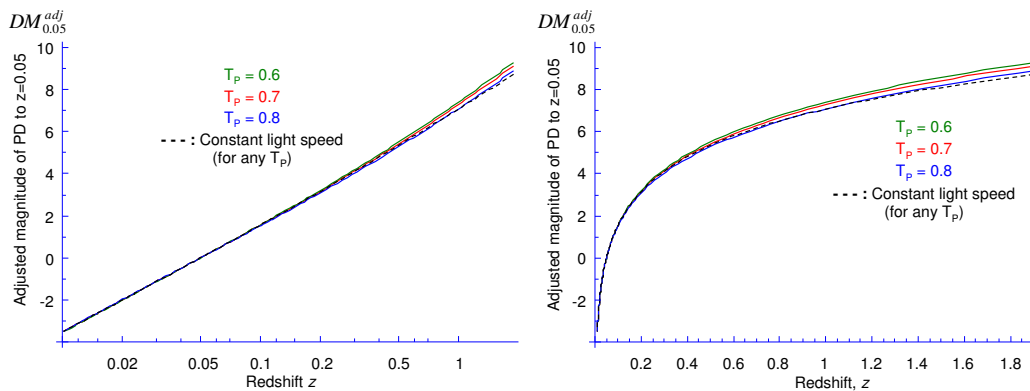


Figure 4. Adjusted magnitude of present distance PD to $z = 0.05$ versus redshift. z in logarithmic (left) and uniform (right) scales. The dotted line in black is a reference for constant light speed.

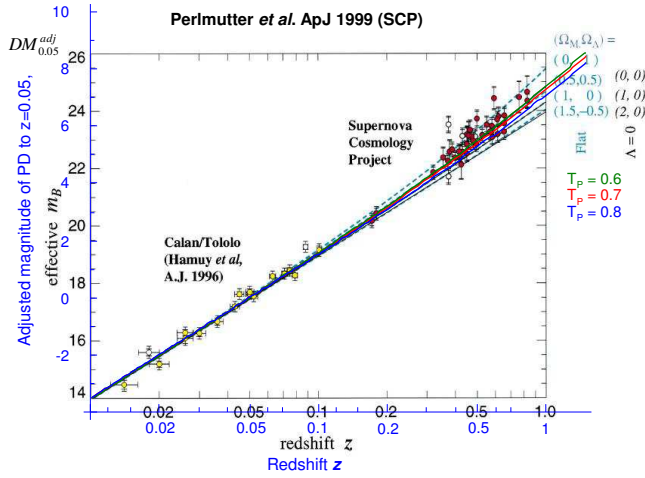


Figure 5. Superimposition of adjusted magnitude of present distance PD to $z = 0.05$ versus redshift graphs on the reported Hubble diagram from SCP (1999): z in a logarithmic scale.

Fig.5 is the superimposed results of Fig.4 on the Hubble diagram in a logarithmic scale for z , which Perlmutter et al reported in 1999 [12]. The vertical axis of their reported graph is the effective rest-frame B magnitude m_B^{eff} , which is defined as $m_B^{eff} = m_B + \alpha(s - 1)$ and $m_B = m_R - K_{BR} - A_R$, with time dilation by $1 + z$. The term $\alpha(s - 1)$ is the correction for stretch factor s . The stretch factor s is a ratio of each light-curve width to the mean after divided by $1 + z$ for each supernova. The average of s is one and the mean $\alpha(s - 1)$ is zero. A_R is the galactic extinction in band R. K_{BR} is the K-correction from observed frame in band R to rest-frame in band B.

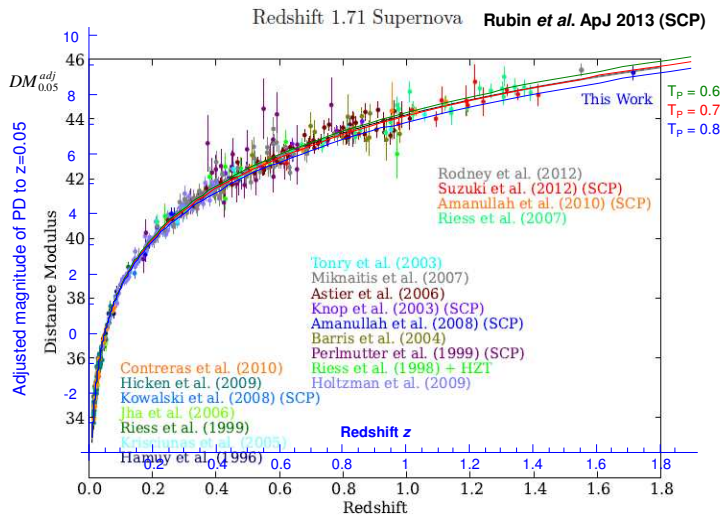


Figure 6. Superimposition of adjusted magnitude of present distance PD to $z = 0.05$ versus redshift graphs on the reported Hubble diagram from SCP (2013): z in a uniform scale.

Fig.6 is the superimposition of Fig.4 on the latest Hubble diagram from the SCP in a uniform scale for z , which Rubin et al reported in 2013 [18]. The vertical axis is the distance modulus, which is time-dilated and K-corrected m_B minus the absolute B-magnitude of supernovae Ia with other observational corrections for light curve width (stretch factor), color and host galaxy mass (extinctions). We find excellent fits to the observed data of supernovae for T_P around 0.7. In Fig.6, the graph for $T_P = 0.7$ very closely overlaps the line of flat $\Omega_m = 0.27$ Λ CDM universe that Rubin et al concluded as the best fit [18].

7. Conclusion

Before this article, I proposed the 4-D spherical model claiming a medium for light propagation called the space energy [1–4]. Light speed depends on the space energy density varying along with the space expansion. I demonstrated that Michelson-Morley type experiments [19–21] are incapable to detect a location shift of interference fringe while its brightness should vary [1]. Then, I proposed the acceleration factor of stationary waves for equation of motion instead of the Lorentz factor [2]. I also formerly reported the light speed equation, the light propagated distance and the redshift based on wavelength [1].

Under varying light speed, redshift based on wavelength is different from that based on frequency. Our observed redshift is the frequency-based redshift for comparison with initial value at the time of emission. In this article, we presented the Hubble diagram expected from the 4-D spherical model. The light propagated distance LD is the integration of the light speed by time from the emission to the present, and is equal to the luminosity distance. We showed the equation for converting the LD to the present distance PD (proper distance at present), which corresponds to the frame conversion by the K-correction. We compared the graph of magnitude of the PD times $1 + z$ versus the frequency-based redshift with the reported Hubble diagrams from the SCP, which were on the time-dilated by $1 + z$ and K-corrected luminosity distance versus the redshift. The superimposition of the graph from the model on the observed plots revealed an excellent fit for the Present Time being around 0.7 of its maximum.

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