

Schrödinger equation in (pre-Planckian?) space–time early universe, with minimal distance, and minimum effective graviton mass

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Abstract. We review Vuille's generalized Schrödinger equation with its Ricci scalar inclusion, in curved space–time. This has a simplified version in the pre-Planckian regime, which leads to comparing a resultant admissible wave function with Bohmian reformulations of quantum physics, a radial distance given by a modified Poisson's equation and a minimal graviton mass. Finally, we look if Bohmian mechanics has a role in our formulation.

Keywords. Ricci tensor, Schrödinger equation, modified Poisson's equation, massive gravity, inflaton physics, Bohmian mechanics.

1. Introduction, organization of the paper

In Section 2, we develop an initial Schrödinger equation for curved space–time. In Eq. (5), there is a simple modified planar wave with a restricted curvature term in the phase. Later in Section 2, using a modified Poisson's equation, we present a scheme for a minimal radial component to the phase term in the modified Schrödinger equation. Section 3 introduces an early-universe modified version of the Heisenberg uncertainty principle (HUP), and Section 4 shows how this HUP affects the radial contribution in Eq. (5). Section 5 concludes with reflections on LIGO, and the Compton wavelength that need to be taken into consideration.

2. Vuille's treatment of Schrödinger equation for curved space–time

Here, we bring up [1] and a reset of the Schrödinger equation in early space–time, with curvature. From [1], we start with

$$ih\partial_t\Psi = -\frac{h^2}{2m}\nabla^2\Psi + mc^2\Psi \quad (1)$$

and apply several substitutions:

$$\begin{aligned} \nabla^2 &\rightarrow \left(\frac{p^\alpha p^\beta}{m^2 c^2} - g^{\alpha\beta}\right) \nabla^\alpha \nabla^\beta \\ p^\alpha &= ih \cdot g^{\alpha\beta} \nabla_\beta \end{aligned}$$



$$p^\beta = i\hbar \cdot \nabla^\beta$$

$$mc^2 = \frac{g^{\alpha\beta} p^\alpha p^\beta}{m}. \quad (2)$$

Then, after more derivation and by using the Ricci tensor, R_a^c , Vuille [1] obtained [2]

$$\nabla^a \nabla_a \left(\nabla^b \nabla_b + \frac{m^2 c^2}{\hbar^2} \right) \Psi + \nabla^a R_a^c \nabla_c \Psi = 0. \quad (3)$$

3. Simplifying Eq. (3) in (pre-Planckian?) space-time

We will rewrite Eq. (3), with the result that, in pre-Planckian space-time,

$$\left(\partial_r^4 + \frac{m^2 c^2}{\hbar^2} \partial_r^2 \right) \Psi + R_r^r \partial_r^2 \Psi = 0. \quad (4)$$

The Ricci tensor, R_a^c , in this setting becomes a constant, and is part of how the wave function evolves. Our candidate wave functional takes the form

$$\Psi \propto \Psi_{\text{initial}} e^{Nr} \approx \Psi_{\text{initial}} \exp \left(i \sqrt{\frac{m^2 c^2}{\hbar^2} + R_r^r \cdot r} \right). \quad (5)$$

This wave function, as a spatial variable, r , which we will delineate by the treatment involving Eq. (6) below. To do this we use a modified Poisson's equation as given below.

4. What is important about the modified Poisson's equation [2]?

We first refer to two necessary and sufficient conditions for the onset of a massive graviton given in [2] and combine them with Padmanabhan's work [3]. We will redo the reference calculations in [2] with

$$\left\{ \nabla^2 + \left[\lambda^{-2} = \left(\frac{m_{\text{grav}} c}{\hbar} \right)^2 \right] \right\} \left\{ U = \frac{Gm}{r} \cdot \exp \left[\left(-\frac{r}{\lambda} \right) = \left(\frac{r \cdot m_{\text{grav}} c}{\hbar} \right) \right] \right\} = -4\pi G\rho. \quad (6)$$

Here, we will be using in the pre-Planckian potential as the inputs from the data usually associated with [3]:

$$a \approx a_{\text{min}} t^\gamma \Leftrightarrow \phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right] \Leftrightarrow V \approx V_0 \exp \left\{ -\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi(t) \right\}. \quad (7)$$

In other words, we will be using the inflation given by

$$\phi \approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right]. \quad (8)$$

If this is accurate, then our approximation is to call the potential in Eq. (7) the same as U in Eq. (6). Then, with rearrangements, we arrive at

$$\left[\frac{d^2}{dr^2} + \left(\frac{m_{\text{grav}} c}{\hbar} \right)^2 \right] \cdot \left[\frac{r^{-1} \alpha \cdot (3\alpha - 1)}{32\pi^2} \right] = G \cdot \rho. \quad (9)$$

Then, after algebra, the quadratic equation this engenders is

$$m_{\text{grav}}^2 \approx \left[\frac{32\pi^2 r \cdot h^2 \cdot G \cdot \rho}{c^2 \cdot \alpha \cdot (3\alpha - 1)} - \frac{16\pi^2 r^{-1} \cdot h^2 \cdot G}{c^2 \cdot \alpha \cdot (3\alpha - 1)} \right] \Rightarrow r^2 - \frac{r \cdot m_{\text{grav}}^2}{\frac{32\pi^2 h^2 \cdot G \cdot \rho}{c^2 \cdot \alpha \cdot (3\alpha - 1)}} - \frac{1}{2\rho} = 0. \quad (10)$$

A candidate for the density function comes next, and how we obtain a critical value for r .

5. Density function inserted into Eq. (10)

In [4] we assume

$$V_{\text{pre-Planckian}} \approx \left(\Delta E \approx \frac{h}{\delta t \cdot a_{\text{min}}^2 \phi_{\text{inf}}} \right). \quad (11)$$

As far as applications to [3],

$$r^2 - \frac{r^4 \cdot m_{\text{grav}}^2 a_{\text{min}}^2 \cdot \delta t \cdot \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma-1)}} \cdot t \right]}{\frac{32\pi^2 h^2 \cdot G}{c^2 \cdot \alpha \cdot (3\alpha-1)}} - \frac{r^3 a_{\text{min}}^2 \delta t \cdot \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma-1)}} \cdot t \right]}{2} = 0. \quad (12)$$

Then, if we use Eq. (12) and look at extremely small times in the inflaton, the above becomes

$$r^2 + \frac{32\pi^2 \cdot h^2 \cdot G}{c^2 \cdot \alpha \cdot (3\alpha-1)} \cdot r + \frac{32\pi^2 h^2 \cdot G}{m_{\text{grav}}^2 a_{\text{min}}^2 \cdot \delta t \cdot |\phi|} = 0. \quad (13)$$

If r is very small, we can eliminate the graviton mass; if r is not so small, our answer will contain the graviton mass. We claim that, warts and all, this is a first-order approximation of the distance, from the moment of creation, for definitive acquisition for a ‘mass’ to a massive graviton. Also, it is a definitive restraint.

6. Conclusion. An initial radial condition depending upon $m_{\text{grav}} \neq 0$?

To incorporate some sort of measurement protocol, Eq. (13) would have to be the absolute magnitude. This has yet to be determined in our discussions with members of the Chongqing University HFGW, but what is noticeable is that the inflaton equation as given by Padmanabhan [3] hopefully will not be incommensurate with the physics of the Corda Criteria given in the gravity’s breath document [5]. Also, [6] gives $N_{\text{e-foldings}} \geq 65$. Furthermore, we should keep in mind the physics incorporated in [7, 8]. That is, as to the work of LIGO. it is important to keep in mind that, in addition, [9] has confirmed that a subsequent analysis of the event GW150914 by the LSC constrained the graviton Compton wavelength of those alternative theories of gravity in which the graviton is massive and placed a 90% confidence on the upper bound of e^{-13} km for the Compton wavelength of the graviton. We assume that this may be changed considerably to a much smaller value. This will in its own way subsequently add additional rigor to the analysis of the graviton mass stated in Eq. (10) and is in accord with [10] as well as the Compton wavelength lower bound as given in Eq. (6) above. Finally, we leave open the possibility of refining Eq. (3) and Eq. (4) based on later review of [11]. The implications of having a radial value dependent upon graviton mass, if r not so small, are astounding and need clarification. That plus a review of [11] for possible refinement of Eq. (3) and Eq. (4) will be controversial. This needs much study.

Finally, we should consider, for future work, a Bohmian interpretation of Eq. (5) while comparing the results of a nonzero volume, $\Delta V^{(3)}$, in a (pre-Planckian) wave functional for Eq. (14) as given by Rubakov [12]. We then get

$$\frac{M}{2} \frac{dq}{d\tau} = \frac{Mc}{2} = V(q) \Rightarrow \Psi_{\text{initial}} \propto \Psi_0^+ \approx \exp \left(- \int_0^{q^+} M \sqrt{c} dq \right) \xrightarrow{c=1} \Psi_0^+ \approx e^{-Mq^+}. \quad (14)$$

Below, we outline the De Broglie–Bohmian path of a wave functional according to [13]. The second part of our conclusion regards examining if we can compare a (pre-Planckian?) wave function against the construction given in [13]. According to [13], we would have

$$\Psi(x, t) = \Psi(x_0, t_0) \exp \left\{ \left[\frac{i}{\hbar} \int_{x_0, t_0}^{x, t} \left(\frac{\nabla S_{\text{action}}^2}{2m} - (Q + V) \right) dt \right] - \int_{x_0, t_0}^{x, t} \frac{\nabla S_{\text{action}}^2}{2m} dt \right\}. \quad (15)$$

Here, S_{action} would be the same as Eq. (16), given below, In order to do this, it may be useful to look at the classical degeneracy argument for forming Ψ_{initial} , and the reference by Rubakov [12] may be useful for this. That is, using a false vacuum analogy, we can write, if q is a generalized space–time unit of “length,” and we examine a quartic potential

$$V(q) = -\frac{\mu^2 \cdot q^2}{2} + \frac{\tilde{\lambda}q^4}{4} \ \& \ S_{\text{action}} = \int_{q_0^-}^{q_0^+} \sqrt{2MV(q)} \, dq$$

$$\Rightarrow \Psi_{\text{initial}} \approx \Psi_0^+ \approx \exp\left(-\int_0^{q^+} \sqrt{2MV(q)} \, dq\right), \quad (16)$$

Whereas we have V as given by Eq. (15), and then we have, by [13],

$$R = \text{Re } \Psi(x, t)$$

$$Q = -\frac{\hbar^2 \nabla^2 R}{2mR}$$

$$\nabla \xrightarrow{\text{pre-Planckian}} \partial_r. \quad (17)$$

Here, if we use $\hbar = c = 1$ and have $\Psi(x_0, t_0)$ as given by Eq. (15), we find

$$Q = -\frac{\hbar^2 \nabla^2 R}{2mR} \xrightarrow{\text{pre-Planckian}} \frac{1}{2M} \cdot (m_g^2 + R_r^r). \quad (18)$$

Here, M would be given by Eq. (19) as given below: If we use Ng infinite quantum statistics, [14], and a nonzero massive graviton mass (massive gravity) [10],

$$E = M = [S_{\text{entropy}} \approx n_{\text{grav}}] \cdot m_{\text{grav}}. \quad (19)$$

That is, if V were given by Eq. (18), we would find to a point that

$$\Psi(x, t) \approx \Psi(x_0, t_0) \exp\left\{\frac{i}{\hbar} \int_{x_0, t_0}^{x, t} \left[\frac{\nabla S_{\text{action}}^2}{2m} - (Q + V)\right] dt\right\}. \quad (20)$$

That is, the term $\Psi(x_0, t_0)$ would likely be the same, but, interestingly enough, $\int_{x_0, t_0}^{x, t} \frac{\nabla S_{\text{action}}^2}{2m} dt$ would likely be almost zero, not contributing at all. We will be examining if the following true. This has to be confirmed rigorously.

$$\int_{x_0, t_0}^{x, t} \frac{\nabla S_{\text{action}}^2}{2m} dt \xrightarrow{\text{pre-Planckian}} \epsilon^+ \approx 0. \quad (21)$$

This value, and if it exists or is not viable, will be contrasted with a radial distance value of approximately

$$r = -\frac{1}{2} \cdot \frac{32\pi^2 \hbar^2 \cdot G}{c^2 \cdot \alpha \cdot (3\alpha - 1)} \cdot \frac{1}{2 \cdot m_{\text{grav}}^2}$$

$$\cdot \left\{ 1 - \sqrt{1 + 16c \cdot (\Delta t) \cdot \left[\sqrt{2} V_0 \left(\frac{\gamma(3\gamma - 1)}{8\pi G V_0} \right)^{\frac{1}{\sqrt{2}}} \frac{(c \cdot m_{\text{grav}})^2 c^{1+\sqrt{2}}}{(\Delta t)^{1+\sqrt{2}}} \right]^2} \right\}. \quad (22)$$

See the appendix for why this was chosen.

This is the future work of our coming investigations: linking Bohmian mechanics with Eq. (5). If Eq. (21) holds, then Eq. (20) is very close to Eq. (5) and Bohmian mechanics, with some semiclassical interpretations is close to our results based on [1].

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References

- [1] Vuille C 1999 Schrödinger's equation in general relativity. *General Relativity and Relativistic Astrophysics, Eighth Canadian Conference* ed C P Burgess and R C Myers (New York: Melville) p 243
- [2] Poisson E and Will C 2014 *Gravity, Newtonian, Post Newtonian, Relativistic* (Cambridge, England: Cambridge University Press)
- [3] Padmanabhan T 2005 Understanding our universe, current status and open issues. *100 Years of Relativity, Space-Time Structure: Einstein and Beyond* ed A Ashatekar (Singapore: World Scientific) p 175 arxiv:0503107 [gr-qc]
- [4] Beckwith A 2016 Gedanken experiment for refining the Unruh metric tensor uncertainty principle via Schwarzschild geometry and Planckian space-time with initial nonzero entropy and applying the Riemannian–Penrose inequality and initial kinetic energy for a lower bound to graviton mass (massive gravity). *J High Energy Phys., Gravit. Cosmology* **2** 106 doi:10.4236/jhepgc.2016.21012
- [5] Corda C 2012 Gravity's primordial breath. *Elect. J. Theo. Phys.* **9** 1 <http://www.ejtp.com/articles/ejtpv9i26p1.pdf>
- [6] Freese K 1992 Natural inflation. *Particles, Strings, and Cosmology* ed P Nath and S Recucroft (Singapore: World Scientific) p 408
- [7] Abbott B, the LIGO Scientific Collaboration and the Virgo Collaboration 2016 Observation of gravitational waves from a binary black hole merger *Phys. Rev. Lett.* **116** 061102
- [8] Abbott B, the LIGO Scientific Collaboration and the Virgo Collaboration 2016 GW151226: Observation of gravitational waves from a 22-solar-mass binary black hole coalescence. *Phys. Rev. Lett.* **116** 241103
- [9] . Abbott B et al. 2016 Tests of general relativity with GW150914 *Preprint* arxiv:1602.03841
- [10] Goldhaber A and Nieto M 2010 Photon and graviton mass limits *Rev. Mod. Phys.* **82** 939 (*Preprint* arxiv:0809.1003)
- [11] Carmelli M 1982 *Classical Fields, General Relativity and Gauge Theory* (New York: Wiley Interscience)
- [12] Rubakov V 2002 *Classical Theory of Gauge Fields* (Princeton, NJ: Princeton University Press)
- [13] Licata I and Fiscaletti D 2014 *Quantum Potential: Physics, Geometry, and Algebra* (New York: Springer-Verlag)
- [14] Ng Y J 2008 Spacetime foam: From entropy and holography to infinite statistics and nonlocality. *Entropy* **10** 441 doi:10.3390/e10040441

Appendix

Fixing a closed form of the radial distance, based upon [3] and [4]:

$$F_{\text{grav}} = -\nabla V_{\text{potential}} = \sqrt{2}V_0 \left[\frac{\gamma(3\gamma - 1)}{8\pi GV_0} \right]^{\frac{1}{\sqrt{2}}} \frac{c^2}{r^{1+\sqrt{2}}}. \quad (\text{A1})$$

Now, the time derivative of energy is [4]

$$\frac{dE}{dt} = v_{\text{velocity}} \cdot (m_{\text{mass}} \cdot a_{\text{accel}} - F_{\text{force}}). \quad (\text{A2})$$

Then, if the derivative $\frac{dE}{dt}$ becomes instead incremental,

$$\Delta E \Delta t = c(\Delta t)^2 \cdot \left\{ m_{\text{mass}} \cdot a_{\text{accel}} - \sqrt{2}V_0 \left[\frac{\gamma(3\gamma - 1)}{8\pi GV_0} \right]^{\frac{1}{\sqrt{2}}} \frac{c^2}{r^{1+\sqrt{2}}} \right\}. \quad (\text{A3})$$

Then, using $\Delta E \approx \frac{h}{\delta t \cdot a_{\text{min}}^2 \phi_{\text{inf}}}$,

$$\frac{h}{a_{\text{min}}^2 \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8\pi GV_0}{\gamma \cdot (3\gamma - 1)}} \cdot \delta t \right]} = c(\Delta t)^2 \left\{ m_{\text{mass}} \cdot a_{\text{accel}} - \sqrt{2}V_0 \left[\frac{\gamma(3\gamma - 1)}{8\pi GV_0} \right]^{\frac{1}{\sqrt{2}}} \frac{c^2}{r^{1+\sqrt{2}}} \right\}. \quad (\text{A4})$$

Now, set $\delta t = (\Delta t)$, and assume in the pre-Planckian regime that $m_{\text{mass}} \cdot a_{\text{accel}} \approx 0$ to get

$$\frac{h}{(\Delta t) \cdot a_{\text{min}}^2 \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left[\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot (\Delta t) \right]} = c(\Delta t) \left\{ -\sqrt{2} V_0 \left[\frac{\gamma(3\gamma - 1)}{8\pi G V_0} \right]^{\frac{1}{\sqrt{2}}} \frac{c^2}{r^{1+\sqrt{2}}} \right\}. \quad (\text{A5})$$

Hence, after algebra, we get

$$r = -\frac{1}{2} \cdot \frac{32\pi^2 h^2 \cdot G}{c^2 \cdot \alpha \cdot (3\alpha - 1)} \cdot \frac{1}{2 \cdot m_{\text{grav}}^2} \cdot \left\{ 1 - \sqrt{1 + 16c \cdot (\Delta t) \cdot \left[\sqrt{2} V_0 \left(\frac{\gamma(3\gamma - 1)}{8\pi G V_0} \right)^{\frac{1}{\sqrt{2}}} \frac{(c \cdot m_{\text{grav}})^2}{r^{1+\sqrt{2}}} \right]^2} \right\}. \quad (\text{A6})$$

which is further simplified by removing the spatial component on the right-hand side of Eq. (20), which is done via

$$r = -\frac{1}{2} \cdot \frac{32\pi^2 h^2 \cdot G}{c^2 \cdot \alpha \cdot (3\alpha - 1)} \cdot \frac{1}{2 \cdot m_{\text{grav}}^2} \cdot \left\{ 1 - \sqrt{1 + 16c \cdot (\Delta t) \cdot \left[\sqrt{2} V_0 \left(\frac{\gamma(3\gamma - 1)}{8\pi G V_0} \right)^{\frac{1}{\sqrt{2}}} \frac{(c \cdot m_{\text{grav}})^2 c^{1+\sqrt{2}}}{(\Delta t)^{1+\sqrt{2}}} \right]^2} \right\}. \quad (\text{A7})$$