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Quantum statistical gravity: time dilation due to local information in many-body quantum systems

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Abstract. We propose a generic mechanism for the emergence of a gravitational potential that acts on all classical objects in a quantum system. Our conjecture is based on the analysis of mutual information in many-body quantum systems. Since measurements in quantum systems affect the surroundings through entanglement, a measurement at one position reduces the entropy in its neighbourhood. This reduction in entropy can be described by a local temperature, that is directly related to the gravitational potential. A crucial ingredient in our argument is that ideal classical mechanical motion occurs at constant probability. This definition is motivated by the analysis of entropic forces in classical systems.

1. Introduction

After almost one century of coexistence, the relation between Einstein's theory of general relativity and quantum physics is still not well understood. At the same time, the precise relation between microscopic quantum physics and macroscopic classical physics has not been completely demystified. There are some suggestions that these two problems are related [1, 2, 3, 4, 5].

Two fundamentally different strategies are used to relate the quantum to the classical. The first one is based on the wave-particle duality and is most succinctly expressed in the path integral formulation. The correspondence between the classical and the quantum is here mathematically very direct, because it is the same action that appears in both theories. It can therefore be used to go from the classical to the quantum and vice versa. A familiar example is electromagnetism: Maxwell's equations are derived from the quantummechanical path integral by means of the stationary phase approximation.

The second strategy, statistical physics, is fundamentally different. From a quantum theory, thermodynamic relations can be computed, but those thermodynamic relations cannot be quantised. Still, it is the only method suited for the description of complex macroscopic systems, about which we only have thermodynamic and hydrodynamic information [6, 7].

The main efforts to find a quantum mechanical description of gravity have been based on the first method [8, 9] but also the second strategy has been explored [10, 11, 12]. The latter efforts go under the name of thermodynamic or entropic gravity. Conceptually, this strategy seems preferable, since the physics of gravity deals with macroscopic objects, that have nonzero entropy and that are coupled to environments [13, 14]. In thermodynamic gravity, the gravitational interaction is seen as emergent rather than as an explicit ingredient in the microscopic theory.

Gravity being the most universal force in the universe, it would ideally emerge in any complex quantum theory. In this paper, we will argue that this might be the case.

The first ingredient in our argument is that the presence of matter at some place in the universe constitutes information, defined as missing entropy [15, 16]. Within the framework of quantum mechanics, when knowledge about a particular realisation of the system is available, the incompatible part of the wave function has to be projected out. When the quantum system has entanglement, a local projection also influences the probability distribution in its vicinity. We will show that the effect of local information on its surroundings can be described by a position dependent 'entanglement' temperature [17, 18], defined by approximating the local reduced density matrix by a Gibbs state. We then demonstrate that the inhomogeneity of the entanglement temperature is reflected in a spatial variation of the magnitude of the energy fluctuations.

This mechanism is most easily illustrated in the EPR setting. When the first qubit of a Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$ is not measured, the reduced density matrix of the second qubit is maximally mixed, equivalent to infinite temperature. On the other hand, when the first one is measured in the $\uparrow, \downarrow$ basis, the second qubit is in a pure state, which corresponds to a zero temperature density matrix.

It is actually well known that entanglement allows to make the connection between pure quantum states and statistical density matrices [19, 20]. Tracing over environment degrees of freedom leaves the system typically in a canonical Gibbs state, even if the composite state is pure [21]. We make a natural extension of this work by considering what happens if the environment is not fully traced over.

The main assumption in our quantum statistical approach to gravity is that the magnitude of energy fluctuations in a region sets the energy scale for the local physics. This follows from the fundamental characterisation of phenomena by their probability. We will demonstrate that it is intimately related to the equivalence principle. 'Gravitational red shifts' then appear because the entanglement temperature is lower closer to a source of information. We will also show that a position dependent entanglement temperature, implies the acceleration of semi-classical wave packets.

In our scenario, gravitational forces emerge as an interplay between quantum measurement and the statistical meaning of classical mechanics. Usually, we do not think about mechanical systems in terms of probability distributions, except in systems where 'entropic' forces occur [22, 23]. We therefore start our paper with a short description of entropic forces in Sec. 2. It is illustrated that the entropic attraction between two objects is due to correlations in their joint probability distribution.

In a quantum system, the role of the probability distribution of local states is played by the reduced density matrix. Correlations between regions are reflected in the quantum mutual information [24, 25]. An entropy and a temperature can be associated to the reduced density matrix. We show that local measurements can affect the temperature of the regions that are correlated with it. Our general arguments about local information in quantum systems are illustrated by calculations on a one-dimensional non-interacting fermion model in Sec. 3. From the requirement of probability conservation of a wave packet, we show in Sec. 4 that a temperature gradient leads to acceleration.

2. Classical entropic forces

Theoretical classical mechanics deals with isolated systems, that satisfy energy conservation. This excludes systems that are coupled to baths and therefore it is only an approximate description of real physical systems, where friction due to coupling with an environment is always present. In addition to friction, environments can also exercise forces. These are known as entropic forces [23] and are essential in e.g. depletion forces between colloidal particles in

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fluids, osmotic pressure and the elasticity of polymers.

Let us start with an elementary analysis of entropic forces. It is the simplest setting in which one can see that probability conservation is a generalisation of energy conservation. We consider the classic example of two spheres immersed in a fluid. Due to repulsive interactions between the sphere and the fluid, there is a depletion region around each sphere. The presence of the spheres therefore lowers the entropy of the fluid (it reduces the available volume for each molecule). When the depletion regions of the two spheres overlap, the entropy of the fluid increases. The probability in terms of their positions $\mathbf{x}_{1,2}$ and momenta $\mathbf{p}_{1,2}$ reads

$$p = \frac{1}{Z} e^{S(\mathbf{x}_1 - \mathbf{x}_2) - \frac{p_1^2}{2m_1 T} - \frac{p_2^2}{2m_2 T}}, \tag{1}$$

where T is the temperature (we set $k_B = 1$) and Z a normalisation constant. In the absence of the entropic term S in the probability (1), energy conservation is equivalent to probability conservation and the temperature T is irrelevant.

In the presence of S on the other hand, probability and energy conservation are no longer the same. Energy conservation remains unaltered, but probability conservation requires

$$-T \ln p = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} - TS(\mathbf{x}_1 - \mathbf{x}_2) \tag{2}$$

to be constant. A term $-TS$ is added to the energy, which gives rise to 'entropic forces'

$$\frac{d\mathbf{p}_i}{dt} = T \nabla_{\mathbf{x}_i} S. \tag{3}$$

The reason why the temperature usually does not appear in classical mechanics is that one implicitly requires that the mechanical energy E is much larger than the thermal energy. This means that classical mechanics is only concerned with statistically unlikely events, with $p \sim e^{-E/T} \ll 1$. This may seem contradictory to the standard classical to quantum correspondence, where the classical paths are the most likely, but corresponds to classical objects carrying a large amount of information. The meaning of the deterministic motion is that, conditional on being at position x at time t, there is unit probability to be at a place x' at time t'. In quantum physics, where probabilities are the most elementary quantities, it is then natural to elevate these implicit aspects to the definition of a classical mechanical object: it is an unlikely excitation that evolves at constant probability.

3. Local information and temperature in quantum systems

After these preliminary remarks on classical mechanics, we turn our attention to quantum systems. The central object in quantum mechanics of closed systems is its wave function. We will consider quantum systems that are defined on a lattice. A typical example is the Hubbard model. At first sight, one would have little hope for the emergence of relativistic physics out of such systems, but Lieb and Robinson showed in their seminal work that causality emerges in such systems under very general conditions [26].

In order to make our discussion more concrete, we perform some calculations on a specific model. The simplest systems from a theoretical point of view are the ones with quadratic Hamiltonians, that describe free quasi-particles. We will here consider the ground state of the fermionic hopping Hamiltonian of the form

$$\hat{H} = -\sum_i \left[\left(t\hat{c}_{i+1}^\dagger \hat{c}_i + h.c. \right) - \mu \hat{c}_i^\dagger \hat{c}_i) \right], \tag{4}$$

where t is the hopping amplitude and μ is the chemical potential. Apart from a Hamiltonian, we need to specify the quantum state of the system. Here, we will consider the ground state of the Hamiltonian, but the analysis could be extended to excited states as well.

All expectation values within a subregion A are described by the reduced density matrix $\rho_A = \mathrm{Tr}_{S\setminus A}\rho$, where the trace is over all sites except the ones in A, which is of the form $\hat{\rho}_A = \exp(-\hat{H}_A)$, where the 'modular' or 'entanglement' Hamiltonian can be written as

$$\hat{H}_A = \sum_{i,j} h_{ij}\hat{a}_i^\dagger \hat{a}_j. \tag{5}$$

An example for $N_A = 10$ sites is shown in Fig. 1. The structure of the original Hamiltonian is clearly visible in the entanglement Hamiltonian. When comparing entanglement Hamiltonians for different subsystem sizes, one finds that the main effect is that the magnitude of the matrix elements increases. We thus find that the entanglement Hamiltonian is of the form $h_A = \beta_A \tilde{h}_A$. We will call this scale dependent temperature T_A the 'entanglement' temperature [17, 18]. Fig. 1b shows the dependence of β_A on system size, where $\tilde{h}_A$ is normalised by the largest tunneling matrix element. A linear increase of the effective inverse temperature with system size is apparent, in agreement with analytical calculations exploiting the adS/CFT correspondence [27].

This behavior can be understood from the fact that, unlike the entropy of a Gibbs state, the entropy is not extensive. For 1-D free fermions it is well know that the ground state entanglement entropy is $S = 1/3\log(L)$. The entanglement temperature must thus decrease such that the entropy of a Gibbs state of the local (smooth) Hamiltonian equals the entanglement entropy.

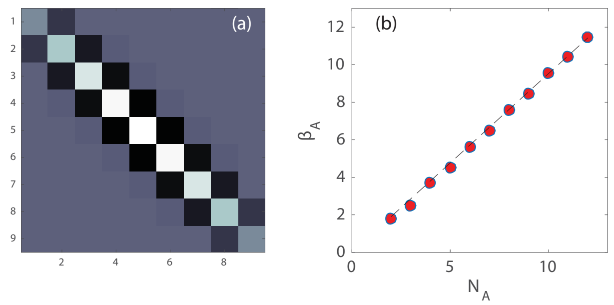

Figure 1. (a) The entanglement Hamiltonian h_{ij} (left) for the fermionic Hamiltonian (4) on a subregion of 10 sites. The structure of the original Hamiltonian is clearly visible, with hopping matrix elements next to the diagonal. (b) The entanglement temperature increases as a function of the system size.

Let us now consider a subregion AB that consists of two disconnected parts A and B, separated by a distance R. In terms of the entanglement Hamiltonian, there is a difference between h_{AB} and the direct sum $h_A \oplus h_B$. Because more information is present in the compound subsystem AB, larger matrix elements are found in h_{AB}: the joint system is at a lower temperature than the individual systems. Physically, this means that the uncertainty about the state A is reduced when information is obtained about system B.

It is instructive to look at the limiting cases $R = \infty$ and $R = 0$ when A and B have the same number of sites. When A and B are infinitely far apart they should not be entangled and the total entanglement Hamiltonian is just the direct sum entanglement Hamiltonian, hence the entanglement temperature is the same as for the individual systems. However, when the two systems touch they simple form one system that is twice as big. It was already shown above that the entanglement temperature is only half of that of the separate subsystems.

This is illustrated in Fig. 2, where we show the entanglement Hamiltonian consisting of two subregions of 10 sites that are separated by 15 sites. The left hand panel shows the full matrix, where one can clearly identify the direct sum of uncoupled entanglement Hamiltonians and some off-diagonal couplings. The right hand panels show the tunneling matrix elements and compares them to the case of a single subregion (dashed lines). Two features stand out. First, the tunneling matrix elements are larger, corresponding to a larger inverse temperature: $\beta_{AB} > \beta_A$. Secondly, there is also an asymmetry in the matrix elements, which could be interpreted as a temperature gradient.

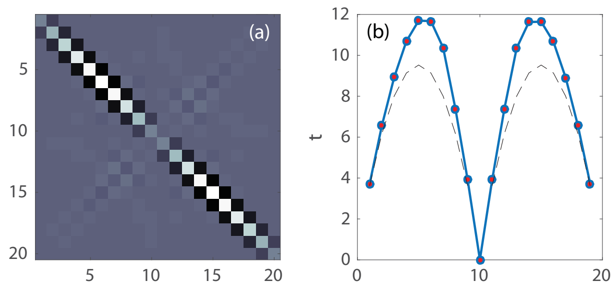

Figure 2. (a) The entanglement Hamiltonian h_{ij} for a subsystem consisting of two disjoint regions (10 sites each) separated by a distance of 15 sites. (b) Tunneling matrix elements (first off-diagonal) are compared with the case of a single region of 10 sites (dashed lines).

The entropy of the total system AB is also different from the sum of the entropies of A and B. The difference between the two is the mutual information

$$I_{AB} = S_A + S_B - S_{AB}, \tag{6}$$

which is always positive. Fig. 3 shows the mutual information as a function of the distance for our free fermion toy system (blue line). The mutual information is seen to decay slowly, with a power law behavior at large distance. This can be attributed to the fact that the system is gapless [28]. The red line shows the effective temperature β_{AB} of the joint density matrix $\hat{\rho}_{AB}$, which shows the same behavior at large distances. This shows a clear connection between the increase in mutual information and the decrease in effective temperature when the two subregions are brought closer together.

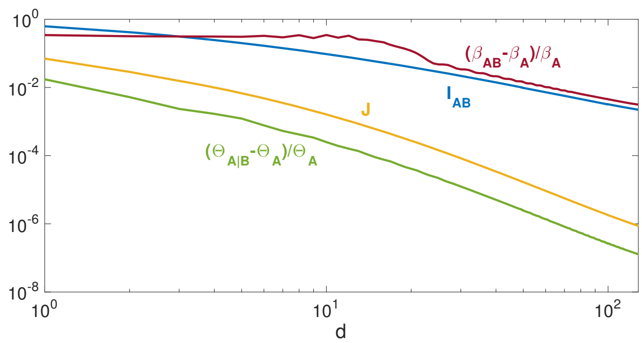

Figure 3. The mutual information I_{AB} as a function of the distance between two subregions of sizes $N_A = N_B = 10$ (blue), the effective temperature of the density matrix $\hat{\rho}_{AB}$ (red), the average conditional entropy J for projection on the Schmidt basis in region A (orange) and the energy average fluctuation width after projection on the Schmidt basis (green).

For classical systems, one can rewrite the mutual information in terms of conditional entropies as:

$$I_{AB} = S(A) - \sum_{x_B} p(x_B) S(A|x_B), \tag{7}$$

where the conditional entropy $S(A|x_B)$ is defined in terms of the conditional probability distribution as $S(A|x_B) = -\sum_{x_A} p(x_A|x_B) \ln p(x_A|x_B)$.

For quantum systems unfortunately, the situation is more complicated, because conditional probabilities are no longer simply defined in terms of joint probabilities, but by means of

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projection operators. It has been found that for quantum systems, the second definition of the mutual information is always smaller than the first one. This has led Zurek to introduce the notion of 'quantum discord' as [29]

$$D = I_{AB} - \max_{\Pi_j^A} J_{\Pi_j^A}(\rho), \tag{8}$$

where J is the conditional entropy that depends on the set of projective measurements Π_j^A. Unfortunately, the optimisation problem over measurements has been proven to be NP complete [30].

If one is interested in the physics of a region A, one would want to know the reduced density matrix ρ_A. Imagine now that a measurement in region B is performed. When there are correlations (quantified by the mutual information) between these two regions, the result of this measurement will affect the density matrix in region A. In general, the conditional entropy $S_{A|B}$ will be lower than S_A, with the mutual information as an upper bound on the average entropy reduction for a certain type of measurement. In view of the relation between entropy and temperature, this means that the temperature of the region A depends on information about region B. In other words, a measurement in B results in a position dependent temperature in its surroundings. Regions close to B will be more affected than regions farther away, as indicated by the behavior of the mutual information.

The simplest class of projections to be performed on this system is a projection on the Schmidt basis in region B. While this is clearly not the best basis in the sense of Eq. (8), it gives at least a lower bound on the average amount of information that can be gained with a projective measurement. From Fig. 3, it is seen that conditional entropy J is lower than the mutual information and decays faster as a function of the distance, but it still shows a power law type decay.

4. Energy statistics and mechanical acceleration

In order to make contact with fundamental physics, one should think about the density matrix before measurement as the 'vacuum'. An unlikely measurement outcome then corresponds to the detection of matter. If matter is measured to be present in region B, this measurement then affects the statistics in region A, depending on the distance between A and B.

The principal ingredient that is responsible for the entropic force is the spatial variation of the energy fluctuations. As we have argued in the previous section, these can arise from local information about the system. As an example, we plot in Fig. 3 the energy variance after projection on the Schmidt basis (as for the computation of J) with a green line. The spatial dependence of the energy fluctuations follows the behavior of the conditional entropy, establishing a link between entropy and energy fluctuations. This suggests that the energy fluctuations Θ are proportional to the entanglement temperature. We will therefore call Θ the 'local temperature' in the following.

As in the classical case, we can now obtain the entropic force by requiring constant probability for fluctuations. A massive fluctuation is characterized by its internal energy E_{int}. When the local temperature Θ is position dependent, the internal energy has to vary in space in order to keep the ratio E_{int}/Θ constant, so to ensure a constant probability for the excitation. From this condition one finds immediately that there is a gradient in the internal energy, corresponding to a force

$$F = -\frac{dE_{int}}{dx} = -E_{int}\frac{d\ln\Theta}{dx}. \tag{9}$$

If we interpret the energy E_{int} in Eq. (9) as the relativistic rest energy $E_{int} = mc^2$ of the excitation in the sense that $p = mv$, we obtain the acceleration

$$a = -c^2\frac{d\ln\Theta}{dx}. \tag{10}$$

The local temperature can then be identified with the Newtonian potential as

$$\phi = c^2 \ln \Theta. \tag{11}$$

Let us recapitulate how in our view gravity emerges from quantum mechanics

(i) Local properties of quantum systems are described by a reduced density matrix, characterised by a local temperature.
(ii) Local information leads to a spatial dependent temperature.
(iii) In order to conserve their probability, excitations have to change their internal energy when they move to a region with a different temperature, which leads to acceleration.

Looking back at the ingredients that led to the (weak) equivalence principle, the most crucial step is the requirement that $E_{int}/\Theta(x)$ is constant, motivated by constant probability. This is our translation of the physics of ideal mechanical motion into the language of statistics.

Einstein translated the physics of ideal mechanical motion (free falling objects) into the language of classical field theory as ‘general covariance’: the form of the equations should be independent of the coordinates. This principle has turned out to be hard to implement in a quantum setting. The reason could be the fact that quantum mechanics is fundamentally a statistical theory, requiring a direct formulation of physical processes in a probabilistic language.

5. Conclusions and outlook

In the present work, we have suggested a mechanism for time dilation that emerges out of a quantum system through a statistical analysis. It is clear that a large effort will be needed before our suggestion can become a viable candidate for a description of real quantum gravity.

As we have discussed, information should be extracted from the quantum system through slow observables, such that the obtained information is stable. Technically, finding these observables is a difficult problem: the search takes place in an exponentially large Hilbert space.

A further issue concerns the range the gravitational potential. In the simple model that we investigated, we found a power law decay for the mutual information in one-dimensional noninteracting fermions. An obvious question is to find the conditions for which one obtains a $1/r$ gravitational potential at large distances in three dimensional systems.

So far, we have only provided arguments for one aspect of Einstein gravity to emerge from a statistical analysis of quantum systems, namely time dilation in a static situation. If gravity is really emerging according to our mechanism, one should be able to derive the full set of geodesic and Einstein equations from statistical considerations, with far richer phenomenology than Newtonian gravity. It should ultimately include cosmology from the perspective of closed quantum systems.

In conclusion, we conjecture that gravity appears as a deformation of statistics due to local information. This type of locally deformed quantum states are usually not considered in calculations on many body quantum systems, that are concerned with small numbers of excitations. It would actually seem implausible that large fluctuations in entangled quantum systems do not influence their surroundings. Two possibilities then remain if quantum mechanics is a fundamental theory. Either this influence is gravity or it is another, unknown effect. Our arguments are encouraging for the gravity scenario to be correct. The description of large fluctuations will be mathematically challenging, but a thorough understanding seems unavoidable if we want to grasp how the physical world is related to our quantum models.

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