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Flavor mixing transformations for a uniformly accelerated observer

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Abstract. We study flavor mixing transformations for a uniformly accelerated observer (Rindler observer) in the simplest case of two charged scalar fields with different masses. This is obtained starting from an analysis of boson field mixing in inertial frame (Minkowski frame) in the hyperbolic basis. As a consequence of the non-trivial interplay between the Bogolubov transformation related to flavor mixing and the one arising from the Rindler spacetime structure, the Fulling-Rindler condensate of the inertial vacuum gets deeply modified, losing its original thermal nature.

1. Introduction

Since Pontecorvo's pioneering work [1], the issue of flavor mixing has turned out to be one of the hottest topics in particle-physics phenomenology. Despite the great progress coming from the theoretical and experimental developments of the two last decades, anyway, it remains one of the most puzzling phenomena. Many features, in fact, are still intriguing, among which, for instance, its origin within Standard Model, the related problem of the generation of neutrino masses and the non-trivial condensate structure exhibited by the vacuum for the mixed fields. The latter aspect, in particular, has been extensively explored, starting from the initial observation of the unitary inequivalence between mass and flavor vacua in QFT [2, 3]. In particular, oscillation formulas have been derived, exhibiting corrections with respect to the usual ones [4] and the entanglement properties of mixed particles have been recently investigated [5].

An equally important question dealing with flavor mixing and oscillations is the way in witch they are affected by a gravitational field [6, 7, 8, 9]. In this paper, we do a first step in this direction, by analyzing the QFT of two mixed scalar fields from the viewpoint of the Rindler observer, i.e. a uniformly accelerated observer in the Minkowski spacetime. It is worth stressing that till now the QFT of mixed fields has been studied only in inertial frame and in the plane-wave basis. Here we focus on the hyperbolic representation, where the boost generator is diagonal, which is a convenient starting framework for the analysis in the accelerated frame. The formalism we elaborate may serve as basis for studying flavor oscillations in curved background: once extended to neutrino fields, indeed, it can be compared with the other approaches existing in literature, such as the WKB approximation [6], the plane-wave method [7] or geometric treatments [8].

Analyzing field mixing in an accelerated frame, anyway, is not just a tool for studying the influence of gravitation on flavor oscillation formulas. There are, in fact, a number of theoretical problems appearing in such a context. For instance, it has been recently highlighted that the inverse β-decay rates of accelerated protons in the inertial and comoving frames disagree when neutrino mixing is taken into account [10]. A more detailed analysis of this and other aspects will be discussed elsewhere [11].

The paper is organized as follows: in Section 2 the standard QFT of two mixed scalar fields in the plane-wave expansion is briefly reviewed. Section 3 is devoted to the comparison of such a framework with the alternative hyperbolic basis; in particular, the physical equivalence between the two representations is shown. In Section 4, after a short discussion about the Rindler spacetime structure, the Rindler-Fulling quantization scheme is analyzed, both for free and mixed fields. Conclusions are summarized in the last Section.

2. Field mixing for an inertial observer: plane-wave representation

Let us start by discussing some of the major results about flavor mixing in the simplest case of two complex scalar fields. We review, in particular, the quantization of mixed fields for an inertial observer in the usual plane-wave basis. Such a topic has been widely analyzed in the last two decades, first in the case of Dirac fermions [2, 12] and later for other fields [3].

As it is well known, mixing transformations in a simplified two flavor model are defined according to the following equations

$$\phi_A(x) = \phi_1(x)\,\cos\theta + \phi_2(x)\,\sin\theta, \tag{1}$$

$$\phi_B(x) = -\phi_1(x)\,\sin\theta + \phi_2(x)\,\cos\theta, \tag{2}$$

where $\phi_j(x)$, $j = 1, 2$ are two free charged scalar fields of masses m_j, $\phi_\chi(x)$, $\chi = A, B$ are the mixed fields of "flavor" χ and θ is the mixing angle. In terms of the complete sets of plane-waves $\left\{U_{\mathbf{k},j}\,, U^*_{\mathbf{k},j}\right\}$, the expansion of $\phi_j(x)$ reads[1]:

$$\phi_j(x) = \int d^3\mathbf{k}\left\{a_{\mathbf{k},j}\,U_{\mathbf{k},j}(x) + \bar{a}^\dagger_{\mathbf{k},j}\,U^*_{\mathbf{k},j}(x)\right\}, \quad j = 1, 2, \tag{3}$$

where

$$U_{\mathbf{k},j}(x) = \left[2\omega_{\mathbf{k},j}(2\pi)^3\right]^{-\frac{1}{2}} e^{i\left(\mathbf{k}\cdot\mathbf{x}-\omega_{\mathbf{k},j}t\right)} \tag{4}$$

are solutions of the Klein-Gordon equation in the Minkowski coordinates $(t, x^1, \vec{x})$ for the field of mass m_j. It is not difficult to show that these mode functions are orthonormal with respect to the KG product

$$(\phi_\alpha, \phi_\beta) = i\int d^3\mathbf{x}\left[\phi^*_\beta(x)\,\overleftrightarrow{\partial_t}\,\phi_\alpha(x)\right], \tag{5}$$

where the integration is performed on a hypersurface of constant t.

The ladder operators $a^\dagger_{\mathbf{k},j}$ and $a_{\mathbf{k},j}$ ($\bar{a}^\dagger_{\mathbf{k},j}$ and $\bar{a}_{\mathbf{k},j}$) in Eq.(3) create and annihilate a particle (antiparticle) of momentum $\mathbf{k}$ and frequency $\omega_{\mathbf{k},j} = \sqrt{m_j^2 + \mathbf{k}^2}$, respectively. They are assumed to satisfy the canonical commutation relations. In addition, since we are dealing with two free fields, the vacuum state they annihilate is accordingly defined as $|0_M\rangle \equiv |0_M\rangle_1 \otimes |0_M\rangle_2$, where $|0_M\rangle_j$ is the vacuum for the field of mass m_j. We will refer to such a state as the "Minkowski inertial vacuum".

[1] The metric $\eta_{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)$ and natural units will be used throughout all the paper. Moreover, the following shorthand notation will be adopted for 4-, 3- and 2-vectors: $x \equiv (t, x^1, x^2, x^3)$, $\mathbf{x} \equiv \{x^1, x^2, x^3\}$ and $\vec{x} \equiv \{x^2, x^3\}$.

The completeness of plane-waves Eq.(4) allows us to adopt the following free fields-like expansions for the "flavor fields"

$$\phi_\ell(x) = \int d^3\mathbf{k}\Big\{a_{\mathbf{k},\ell}(t)\, U_{\mathbf{k},s}(x) + \bar{a}^\dagger_{\mathbf{k},\ell}(t)\, U^*_{\mathbf{k},s}(x)\Big\}, \tag{6}$$

where $(\ell, s) = (A, 1), (B, 2)$. The time dependent flavor operators $a_{\mathbf{k},\ell}$ are given by[2]

$$a_{\mathbf{k},\ell} = (\phi_\ell, U_{\mathbf{k},s})\,. \tag{7}$$

By using Eq.(1), for instance, we obtain for $a_{\mathbf{k},A}$ the following transformation

$$a_{\mathbf{k},A} = \cos\theta\, a_{\mathbf{k},1} + \sin\theta\, \left(\rho^{\mathbf{k}\,*}_{12}\, a_{\mathbf{k},2} \,+\, \lambda^{\mathbf{k}}_{12}\, \bar{a}^\dagger_{-\mathbf{k},2}\right), \tag{8}$$

where the Bogolubov coefficients $\rho^{\mathbf{k}}_{12}$ and $\lambda^{\mathbf{k}}_{12}$ are such that

$$\rho^{\mathbf{k}}_{12} \equiv \int d^3\mathbf{k}'\left(U_{\mathbf{k},1}, U_{\mathbf{k}',2}\right) = |\rho^{\mathbf{k}}_{12}|\, e^{i\left(\omega_{\mathbf{k},2}-\omega_{\mathbf{k},1}\right)t}, \qquad \lambda^{\mathbf{k}}_{12} \equiv \int d^3\mathbf{k}'\left(U^*_{\mathbf{k}',2}, U_{\mathbf{k},1}\right) = |\lambda^{\mathbf{k}}_{12}|\, e^{i\left(\omega_{\mathbf{k},1}+\omega_{\mathbf{k},2}\right)t}, \tag{9}$$

with

$$|\rho^{\mathbf{k}}_{12}| \equiv \frac{1}{2}\left(\sqrt{\frac{\omega_{\mathbf{k},1}}{\omega_{\mathbf{k},2}}} + \sqrt{\frac{\omega_{\mathbf{k},2}}{\omega_{\mathbf{k},1}}}\right), \qquad |\lambda^{\mathbf{k}}_{12}| \equiv \frac{1}{2}\left(\sqrt{\frac{\omega_{\mathbf{k},1}}{\omega_{\mathbf{k},2}}} - \sqrt{\frac{\omega_{\mathbf{k},2}}{\omega_{\mathbf{k},1}}}\right) \tag{10}$$

(a similar expression can be derived for $a_{\mathbf{k},B}$). Starting from Eqs.(8) and (9), one can easily prove that flavor operators, just as the corresponding mass operators, obey the canonical commutation relations (at equal times).

The flavor vacuum $|0(\theta,t)\rangle_{A,B}$ at time t can be now defined as

$$a_{\mathbf{k},\chi}(t)\; |0(\theta,t)\rangle_{A,B} = \bar{a}_{\mathbf{k},\chi}(t)\; |0(\theta,t)\rangle_{A,B} = 0\,, \qquad \chi = A, B\,. \tag{11}$$

By inverting the set of equations for the flavor operators with respect to the mass ones, it is possible to show that the vacuum Eq.(11) exhibits a condensate structure of particle-antiparticle pairs, both with the same and different masses. The condensation density for any t is given by

$${}_{A,B}\langle 0(\theta,t)|a^\dagger_{\mathbf{k}',j}\, a_{\mathbf{k},j}|0(\theta,t)\rangle_{A,B} = \sin^2\theta\, \left|\lambda^{\mathbf{k}}_{12}\right|^2 \delta^3(\mathbf{k}-\mathbf{k}'), \qquad j = 1, 2\,. \tag{12}$$

It is worth noting that, in the infinite volume limit, flavor and mass vacua are orthogonal to each other due to the infinite number of degrees of freedom. This shows the fundamental result that the Hilbert space for the mixed field states is unitarily inequivalent to the Hilbert state space where the free-field operators are defined (see Ref.[2, 3] for the details).

3. Field mixing for an inertial observer: hyperbolic representation

The foregoing discussion is completely within the usual plane-wave basis. According to Lorentz covariance, anyway, there is no reason to prefer such a representation to those which diagonalize the other generators of the Lorentz group. We may wonder, for instance, how the above formalism appears if we quantize the fields in the hyperbolic representation, that is, the representation in which the Lorentz boost generator is diagonal[3]. The introduction of the latter quantization scheme, however, is not a purely mathematical exercise of change of basis. As we

[2] For simplicity, from now on we will omit the time dependence of flavor operators and vacuum when it is not strictly necessary.

[3] We will consider, in particular, the generator of a Lorentz boost along the x^1-axis.

shall see in the next Section, indeed, the hyperbolic expansion turns out to be an useful tool for analyzing the field quantization from the viewpoint of an accelerated observer [13].

In order to answer our question, let us expand the free fields $\phi_j(x)$, $j = 1,2$ in terms of the complete and orthonormal sets of boost modes $\left\{\widetilde{U}^{(\sigma)}_{\kappa,j}\,,\,\widetilde{U}^{(\sigma)*}_{\kappa,j}\right\}$, related to the plane-waves by[4]

$$\widetilde{U}^{(\sigma)}_{\kappa,j}(x) \equiv \int_{-\infty}^{+\infty} dk_1\, p^{(\sigma)*}_{\Omega,j}(k_1)\, U_{\mathbf{k},j}(x) = \frac{e^{\sigma\pi\Omega/2}}{2\sqrt{2}\,\pi^2}\, K_{i\sigma\Omega}(\mu_{\vec{k},j}\xi)\, e^{i\left(\vec{k}\cdot\vec{x}-\sigma\Omega\eta\right)}, \tag{13}$$

where the hyperbolic coordinates (η, ξ) are defined according to the following equations

$$t = \xi \sinh\eta\,, \qquad x^1 = \xi\cosh\eta. \tag{14}$$

The subscript κ in Eq.(13) stands for $(\Omega, \vec{k})$, $K_{i\sigma\Omega}$ is the modified Bessel function of second kind and imaginary order and $\mu_{\vec{k},j} = \sqrt{m_j^2 + |\vec{k}|^2}$. The convolution function $p^{(\sigma)}_{\Omega,j}(k_1)$ is given by

$$p^{(\sigma)}_{\Omega,j}(k_1) = \frac{1}{\sqrt{2\pi\omega_{\mathbf{k},j}}}\left(\frac{\omega_{\mathbf{k},j}+k_1}{\omega_{\mathbf{k},j}-k_1}\right)^{i\sigma\Omega/2}, \quad j = 1,2. \tag{15}$$

It is easy to show that the sets of these functions are both complete and orthonormal (see Ref.[14] for the details).
Strictly speaking, Eq.(13) holds only in the region $x^1 > |t| \cup x^1 < -|t|$. The correct global functions, namely the Gerlach's Minkowski Bessel modes, can be obtained by analytically continuing the solutions Eq.(13) across $x_1 = \pm t$ [15]. For our purpose, nevertheless, it is enough to consider the modes as above defined.

The free fields $\phi_j(x)$, $j = 1,2$ can be thus expanded as follows

$$\phi_j(x) = \sum_{\sigma}\int_0^{+\infty} d\Omega \int d^2\vec{k}\, \left\{ d^{(\sigma)}_{\kappa,j}\, \widetilde{U}^{(\sigma)}_{\kappa,j}(x) + \bar{d}^{(\sigma)\dagger}_{\kappa,j}\, \widetilde{U}^{(\sigma)*}_{\kappa,j}(x)\right\}, \tag{16}$$

where the role of the integration on k_1 in the plane-wave expansion Eq.(3) has been now replaced by the sum on σ and the positive integration on Ω. It naturally arises the question how the "hyperbolic" ladder operators in Eq.(16) are related to the corresponding ones in the plane-wave representation Eq.(3). In this regard, by equating the two alternative field expansions Eqs.(3) and (16) on a hyperplane of constant η and forming the KG inner product of both the sides with the boost mode $\widetilde{U}^{(\sigma)}_{\kappa,j}$, one immediately obtains

$$d^{(\sigma)}_{\kappa,j} = \int d^3\mathbf{k}' \left\{ a_{\mathbf{k}',j}\left(U_{\mathbf{k}',j}, \widetilde{U}^{(\sigma)}_{\kappa,j}\right) + \bar{a}^{\dagger}_{\mathbf{k}',j}\left(U^*_{\mathbf{k}',j}, \widetilde{U}^{(\sigma)}_{\kappa,j}\right)\right\}, \quad j = 1,2. \tag{17}$$

Since the boost modes $\widetilde{U}^{(\sigma)}_{\kappa,j}$ are linear combinations of positive frequency plane-waves alone (see Eq.(13)), the coefficient of $\bar{a}^{\dagger}_{\mathbf{k}',j}$ in Eq.(17) vanishes. Therefore, by exploiting the orthonormality condition of $\left\{U_{\mathbf{k},j}\,,U^*_{\mathbf{k},j}\right\}$, $j = 1,2$, it follows that (similarly for $\bar{d}^{(\sigma)}_{\kappa,j}$)

$$d^{(\sigma)}_{\kappa,j} = \int_{-\infty}^{+\infty} dk_1\, p^{(\sigma)}_{\Omega,j}(k_1)\, a_{\mathbf{k},j}. \tag{18}$$

[4] The physical meaning of $\sigma = \pm 1$ and of the positive parameter Ω in Eq.(13) will be clear once the Rindler-Fulling quantization scheme for an accelerated observer is introduced.

In Ref.[14] such operators are explicitly shown to diagonalize the x^1-boost generator.
From Eq.(18) we find out that the vacuum annihilated by d-operators is the same as the usual Minkowski vacuum $|0_M\rangle$. Additionally, by using the orthonormality and completeness conditions of the sets $\left\{p^{(\sigma)}_{\Omega,j}\right\}$ and the commutation relations of $a_{\mathbf{k},j}$ and $a^\dagger_{\mathbf{k},j}$, $j=1,2$, it is possible to show that the transformation Eq.(18) is canonical. Combining both these results, we can definitely state that the quantum formalisms above introduced, the plane-wave and the hyperbolic constructions, are totally equivalent from the point of view of free fields.

To understand if such an equivalence still holds when mixing is involved, let us extend the hyperbolic quantization to the flavor fields Eqs.(1) and (2). Retracing the same steps of the plane-wave representation, we expand the above fields as follows

$$\phi_\ell(x)=\sum_\sigma\int_0^{+\infty}d\Omega\int d^2\vec{k}\,\left\{d^{(\sigma)}_{\kappa,\ell}\,\widetilde{U}^{(\sigma)}_{\kappa,s}(x)+\bar{d}^{(\sigma)\dagger}_{\kappa,\ell}\,\widetilde{U}^{(\sigma)*}_{\kappa,s}(x)\right\}, \tag{19}$$

where $(\ell,s)=(A,1),(B,2)$ and $d^{(\sigma)}_{\kappa,\ell}$ are the flavor operators in the hyperbolic basis.
In order to establish whether the quantization formalism of mixed fields is somehow affected by such a change of basis, let us focus on flavor vacuum in this new representation. For this purpose, following the approach used in Ref.[13], we could express flavor d-operators in Eq.(19) in terms of the corresponding mass operators in Eq.(16). Exploiting Eq.(18), the relations thus obtained could be recast in terms of mass a-operators and, in the last step, of flavor a-operators by using the inverse of Eq.(8). Such a procedure, however, would be rather laborious. Here we propose a more straightforward approach, yielding the same result.

By directly comparing Eqs.(6) and (19) for the flavor fields and performing the same calculations as in absence of mixing, we readily obtain

$$d^{(\sigma)}_{\kappa,\ell}=\int_{-\infty}^{+\infty}dk_1\,p^{(\sigma)}_{\Omega,s}(k_1)\,a_{\mathbf{k},\ell}, \tag{20}$$

where $(\ell,s)=(A,1),(B,2)$. Therefore, the relation Eq.(18) between d- and a- mass operators also holds for the corresponding flavor operators[5].
Eq.(20) shows a significant result: since flavor d-annihilators are linear combinations of flavor a-annihilators alone, the change of representation from plane-wave to hyperbolic modes does not affect the flavor vacuum $|0(\theta,t)\rangle_{A,B}$ in Eq.(11) and, consequently, the Hilbert space structure for the mixed fields.

4. Field mixing for a uniformly accelerated observer: Fulling-Rindler quantization

In this Section we show how the hyperbolic quantization previously introduced turns out to be an helpful construction in the extension of QFT to an accelerated frame of reference.
As it is well known, the most suitable framework to describe the motion of an accelerated observer in the Minkowski spacetime is the Rindler metric[6]. The analysis of flavor mixing from the viewpoint of such an observer, therefore, requires a preliminary discussion about the properties of this background. In doing so, we also briefly review the Rindler-Fulling scheme [16], which is the natural way for quantizing the fields in an accelerated frame.

As a first step, let us recall that the line element $ds^2=\eta_{\mu\nu}dx^\mu dx^\nu$ in the hyperbolic coordinates Eq.(14) takes the form

$$ds^2=(dt)^2-\left(dx^1\right)^2-\sum_{j=2}^{3}\left(dx^j\right)^2\ \underset{hyperb.\ coord.}{\longrightarrow}\ ds^2=\xi^2d\eta^2-d\xi^2-\sum_{j=2}^{3}\left(dx^j\right)^2. \tag{21}$$

[5] This implies that the flavor operators $d^{(\sigma)}_{\kappa,\chi}$, $\chi=A,B$, just as the corresponding operators in the plane-wave basis, obey the canonical commutation relations (at equal times).
[6] As usual, we will refer to such an observer as the “Rindler observer”.

The world line of a uniformly accelerated observer with proper acceleration a is thus given by

$$\xi(\tau) = \text{const} \equiv a^{-1}\,, \quad \vec{x}(\tau) = \text{const}\,, \tag{22}$$

where τ is the proper time measured along the line. From Eq.(21), we have $\eta(\tau) = a\tau$, that is, the proper time τ measured by an observer with proper acceleration a is the same as the Rindler time η, up to the scale factor a.

By using the Minkowski coordinates, one can easily verify that the line Eq.(22) describes an hyperbola in the (t, x^1) plane with asymptotes $t = \pm x^1$. Such a trajectory is parametrized by the proper acceleration a: for $a > 0$, the observer is confined within the right wedge $R_+ = \{x|x^1 > |t|\}$, for $a < 0$, instead, his motion occurs in the left wedge $R_- = \{x|x^1 < -|t|\}$. The two regions R_+ and R_-, moreover, are causally separated from each other; there is no way for an accelerated observer in R_+ to exchange information with one in R_- [17].

Keeping these properties in mind, the Rindler-Fulling procedure is now introduced as opposed to the more familiar Minkowski quantization. To this purpose, following the approach used in Ref.[14], we expand the free fields as follows

$$\phi_j(x) = \sum_\sigma \int_0^{+\infty} d\Omega \int d^2\vec{k} \left\{ b^{(\sigma)}_{\kappa,j}\, u^{(\sigma)}_{\kappa,j}(x) + \bar{b}^{(\sigma)\dagger}_{\kappa,j}\, u^{(\sigma)*}_{\kappa,j}(x) \right\}, \quad j = 1, 2. \tag{23}$$

The Rindler modes $u^{(\sigma)}_{\kappa,j}$ of positive frequency Ω with respect to the time η are defined by

$$u^{(\sigma)}_{\kappa,j}(x) = \theta(\sigma\xi)\left[2\Omega(2\pi)^2\right]^{-\frac{1}{2}} h^{(\sigma)}_{\kappa,j}(\xi)\, e^{i\left(\vec{k}\cdot\vec{x} - \sigma\Omega\eta\right)}\,, \tag{24}$$

where $\sigma = \pm 1$ refers to the right/left wedges $R_\pm$. They are solutions of the KG equation in Rindler coordinates for the field of mass m_j (see Ref.[14] for the mathematical details). As in Eq.(13), the function $h^{(\sigma)}_{\kappa,j}$ is given by the modified Bessel mode of second kind and imaginary order, up to a normalization factor. According to our previous considerations, the Heaviside step function $\theta(\sigma\xi)$ has been inserted in Eq.(24) in order to restrict such modes to only one of the two causally separated wedges (R_+ for $\sigma = +1$ and R_- for $\sigma = -1$). In addition, by expressing the KG inner product Eq.(5) in the Rindler coordinates and using Eq.(24), it is possible to show that the modes above defined form a complete orthonormal set.
As it is known, the ladder operators $b^{(\sigma)\dagger}_{\kappa,j}$ and $b^{(\sigma)}_{\kappa,j}$ ($\bar{b}^{(\sigma)\dagger}_{\kappa,j}$ and $\bar{b}^{(\sigma)}_{\kappa,j}$) in Eq.(23) can be interpreted as creators and annihilators of a Rindler particle (antiparticle), respectively. They are assumed to satisfy the canonical commutation relations. The Rindler vacuum is accordingly defined as $|0_R\rangle \equiv |0_R\rangle_1 \otimes |0_R\rangle_2$, where $|0_R\rangle_j$ is the vacuum for the field of mass m_j.

We are now interested in the relation between the free-field quantization in the inertial frame and the Rindler-Fulling scheme. To this end, let us remind that, as shown in the previous Section, the plane-wave and the hyperbolic constructions are completely equivalent from the viewpoint of an inertial observer. Therefore, in our comparison, we can equivalently consider the field expansions Eqs.(3) or (16) for such an observer. To simplify the calculations, we choose the latter. In such a way, by matching Eqs.(16) and (23) on a hypersurface of constant η and multiplying both the sides for the Rindler mode $u^{(\sigma)}_{\kappa,j}$, $j = 1, 2$ it follows that

$$b^{(\sigma)}_{\kappa,j} = \sqrt{1 + N_R(\Omega)}\, d^{(\sigma)}_{\kappa,j} + \sqrt{N_R(\Omega)}\, \bar{d}^{(-\sigma)\dagger}_{\widetilde{\kappa},j}, \quad j = 1, 2, \tag{25}$$

where $\widetilde{\kappa}$ stands for $(\Omega, -\vec{k})$ and

$$N_R(\Omega) = \frac{1}{e^{2\pi\Omega} - 1} \tag{26}$$

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is the Bose-Einstein condensate of the Minkowski vacuum $|0_M\rangle$[7]

$$\langle 0_M|b^{(\sigma)\dagger}_{\kappa,j}\, b^{(\sigma)}_{\kappa',j}|0_M\rangle = N_R(\Omega)\,\delta^3(\kappa-\kappa'), \quad j=1,2. \tag{27}$$

From Eq.(27) we find out that the inertial vacuum appears to be not empty from the viewpoint of an accelerated observer; just as the flavor vacuum with respect to free fields, indeed, it exhibits a condensate structure in terms of the Rindler particles. The crucial point is that such a non-trivial result, both for mixing and Rindler condensation densities, mathematically arises from a Bogolubov transformation involving two sets of creators and annihilators: mass and flavor ladder operators in the former case (see Eq.(8)), Minkowski and Rindler ones for the latter effect (Eq.(25)).

Up to now we have separately analyzed the physics hiding under such Bogolubov transformations. It arises thus the natural question how the thermal condensate in Eq.(26) gets modified if we extend the Rindler-Fulling scheme to the mixed fields Eqs.(1) and (2).
For this purpose, all we need is to recast the mixing transformation Eq.(8) in terms of the Rindler b-operators. Once again, by exploiting the completeness of the Rindler modes in Eq.(24), we take for the flavor fields the following expansion

$$\phi_\ell(x) = \sum_\sigma \int_0^{+\infty} d\Omega \int d^2\vec{k}\left(b^{(\sigma)}_{\kappa,\ell}\; u^{(\sigma)}_{\kappa,s}(x) + \bar{b}^{(\sigma)\dagger}_{\kappa,\ell}\; u^{(\sigma)*}_{\kappa,s}(x)\right), \tag{28}$$

where $(\ell,s)=(A,1),(B,2)$ and $b^{(\sigma)}_{\kappa,\ell}$ are the flavor operators for the Rindler observer.
The interplay between the mixing and thermal Bogolubov transformations can be now easily investigated by comparing Eqs.(19) and (28). By multiplying both the sides for the Rindler mode $u^{(\sigma)}_{\kappa,j}$, $j=1,2$, one can show that the relation Eq.(25) between b- and d- mass operators also holds for the corresponding flavor operators [13]

$$b^{(\sigma)}_{\kappa,\chi} = \sqrt{1+N_R(\Omega)}\; d^{(\sigma)}_{\kappa,\chi} + \sqrt{N_R(\Omega)}\; \bar{d}^{(-\sigma)\dagger}_{\tilde{\kappa},\chi}, \quad \chi=A,B. \tag{29}$$

However, since our final aim is to calculate the Rindler distribution of mixed particles in the vacuum $|0_M\rangle$, it is useful to recast Eq.(29) in terms of the ladder operators $a_{\mathbf{k},j}$ and $\bar{a}_{\mathbf{k},j}$ in Eq.(3). The spectrum of mixed Rindler particles in the inertial vacuum takes thereby the form

$$\begin{aligned}\langle 0_M|\, b^{(\sigma)\dagger}_{\kappa,\chi}\, b^{(\sigma)}_{\kappa',\chi}\,|0_M\rangle \; &= \\ N_R(\Omega)\,\delta(\kappa-\kappa') \; + \; \sin^2\theta &\Bigg[\left(\sqrt{N_R(\Omega)\,N_R(\Omega')} \; + \; \sqrt{(1+N_R(\Omega))\,(1+N_R(\Omega'))}\right)N_{\lambda\lambda} \\ &+ \sqrt{1+N_R(\Omega)}\,\sqrt{N_R(\Omega')}\,N_{\rho\lambda} \; + \; \sqrt{N_R(\Omega)}\,\sqrt{1+N_R(\Omega')}\,N^*_{\rho\lambda}\Bigg]\delta^2(\vec{k}-\vec{k}')\end{aligned} \tag{30}$$

where $\chi=A,B$ and

$$N_{\lambda\lambda} = \int_{-\infty}^{+\infty} dk_1\; p^{(\sigma)*}_{\Omega',j}(k_1)\, p^{(\sigma)}_{\Omega,j}(k_1)\,|\lambda^{\mathbf{k}}_{12}|^2, \quad N_{\rho\lambda} = \int_{-\infty}^{+\infty} dk_1\; p^{(\sigma)*}_{\Omega',j}(k_1)\, p^{(\sigma)}_{\Omega,j}(-k_1)\,\lambda^{\mathbf{k}\,*}_{12}\,\rho^{\mathbf{k}}_{12}, \tag{31}$$

with $\rho^{\mathbf{k}}_{12}$ and $\lambda^{\mathbf{k}}_{12}$ defined in Eq.(9).

[7] One can recognize in Eqs.(25) and (26) the well-known Unruh effect: a uniformly accelerated observer feels in the Minkowski vacuum a heat bath with a temperature proportional to the magnitude of his acceleration [18].

Therefore, for mixed fields, the "usual" Rindler condensate of the inertial vacuum Eq.(26) gets non-trivially modified, losing its original thermal nature. Anyway, it can be readily seen that the standard value is recovered for $\theta \to 0$ and in the limit $m_1 \to m_2$ (see Eq.(10)), as one would expect in absence of mixing.
It is worth noting that the expectation value Eq.(30) is non-diagonal with respect to the parameters Ω and Ω'. Since such a result has been obtained without using any approximation, we can state that the factorization of the hyperbolic Bogolubov coefficients into the product of a Dirac delta distribution of Ω and Ω' with suitable functions adopted in Ref.[13] is not quite correct. It is our purpose to discuss this and other aspects in a forthcoming paper.

5. Conclusions

The issue of flavor mixing from the point of view of a uniformly accelerated observer has been discussed in the case of two complex scalar fields with different masses. In spite of such a minimal setting, the Bogolubov transformation built in field mixing has been found to non-trivially combine with the thermal Bogolubov transformation associated to the Rindler spacetime structure, leading to a meaningful modification of the "usual" Fulling-Rindler spectrum. The result obtained Eq.(30), in fact, shows that the particle distribution detected by an accelerated observer in the inertial vacuum loses its thermal nature when mixing is taken into account.

We stress that the formalism developed in this paper represents an useful springboard for studying flavor oscillations in the context of Quantum Field Theory on curved background. Indeed, once extended to the fermionic case and, in particular, to neutrino fields, the results derived within this framework can be compared with those obtained through the other approaches dealing with such a problem [6, 7, 8]. Additionally, it may be taken as starting point for investigating Lorentz invariance breakdown in the context of mixed neutrinos [19]. More work is inevitably required along these lines.

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