A fluid/gravity prescription of the post-Newtonian parameter

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Abstract. The KSS bound for the shear viscosity-to-entropy density ratio is employed in the fluid/gravity correspondence setup, to provide a theoretical bound for the post-Newtonian parameter in the Casadio-Fabbri-Mazzacurati black string solution that matches the observational bound [1]. This derived bound is here shown to vary according to the black string temperature, and, although the post-Newtonian parameter reaches bigger values as the temperature increases for fixed black string masses, it is still into the observational/experimental bound range. The Casadio-Fabbri-Mazzacurati-AdS black brane is shown to converge to the Schwarzschild-AdS black brane, for sufficiently high order expansion of the dual boundary energy-momentum tensor.

1. Introduction

Extra-dimensional generalisations of the general relativity comprise extended models with unexpected consequences in the physics of black hole [2, 3] and ramifications into astrophysics [4, 5]. Particularly, black strings represent physical solutions of the brane effective Einstein equations [6, 7, 8, 9]. On the other hand, fluid dynamics can be thought of describing QFT, in the long wavelength regime [10]. At low energies, the associated hydrodynamics describe physical systems near the equilibrium and the transport parameters, for example conductivity and viscosity, encrypt the information on the perturbations and their propagation along the fluid. Such a paradigm relies on the AdS/CFT correspondence setup [11, 12], whose long wavelength regime establishes the interplay between black holes solutions in (asymptotically) anti-de Sitter (AdS) spacetime and hydrodynamics on the boundary, associated to a field theory in a strongly coupled regime [13, 14]. A prominent feature in fluid/gravity correspondence is given by the shear viscosity-to-entropy density ratio [10], whose universality, under certain conditions, has been explored in various contexts [13, 14, 15, 16, 17, 18].

Black strings and black branes, thus, have hydrodynamical aspects beyond entropy and temperature, encoded by transport coefficients as vorticity, viscosity, shear, diffusion parameters and rates. Black strings and black branes solutions have the shear viscosity-to-entropy density ratio η/s that is bounded [15] [1],

$$\frac{\eta}{s} \gtrapprox \frac{1}{4\pi} \,, \tag{1}$$

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[1] Units with $\hbar = c = k_{\mathrm{B}} = 1$ shall be employed; spacetime 4D indexes run in the range $\mu = 0, 1, 2, 3$ and 5D indexes run in the range $A = 0, 1, 2, 3, 4$.

which is called the Kovtun-Starinets-Son (KSS) result [15]. In the case of Einstein gravity or any theory in the limit $N_c \to \infty$, the KSS result is an equality $\eta/s = 1/4\pi$. This result is derived in the context of SUGRA in a $\mathcal{N} = 4$ SUSY $SU(N_c)$ gauge theory, in the large 't Hooft coupling constant regime $g^2 N_c$ [17]. The AdS bulk dynamics corresponds to vacuum states of the boundary Yang-Mills setup, implying that AdS-Schwarzschild solutions are equivalent to boundary gauge theory states [19]. Other solutions of the Einstein equations [20, 1] correspond to a different thermal state in the boundary gauge theory.

The fluid/gravity correspondence was used to bound the post-Newtonian parameter via the black string [1], using the CFM black strings [4] associated with the effective Casadio-Fabbri-Mazzacurati (CFM) solutions [21, 22]. CFM metrics represent solutions of the bulk Einstein equations projected onto the brane, having a parametrised post-Newtonian parameter β corresponding to the Eddington-Robertson-Schiff parameter, bounded by experimental/observational data [23, 24]. In general, one can construct a dual fluid stress tensor by solving the bulk Einstein equations in asymptotically AdS spacetime and taking the long wavelength limit, but there are only a few black hole solutions that admit such a hydrodynamic dual on the boundary, even asymptotically. CFM solutions were studied in the context of the fluid/gravity correspondence [1]. A 5D CFM black string was proposed to have a 4D dual fluid dynamical prescription, and the fluid/gravity correspondence can impart a bound for the parametrised post-Newtonian parameter, through the universal KSS bound (1). After studying the bound for β, derived by the KSS result using the CFM-AdS black string, here such bound shall be further analysed, and its variation according to the black brane temperature shall be scrutinised.

The procedure to reported in this paper is, after reviewing the CFM solutions, to use the Green-Kubo formula to bound the parametrised post-Newtonian parameter through the shear viscosity-to-entropy density ratio by including up to the second-order hydrodynamics terms, discussing the unicity of the AdS-Schwarzschild solution.

2. A Casadio-Fabbri-Mazzacurati solution

In brane-world models, gravity can propagate into the bulk, originating a 5D Weyl fluid term that is projected onto the brane-world, via the Gauss-Codazzi method. Such a term must be taken into account in the Einstein effective equation. A by-product of the brane equations is the CFM metrics [22], which are originally vacuum solutions of the Einstein field equations, presenting a parametrised post-Newtonian parameter β. Ref. [21], for example, reported a specific bound for the parametrised post-Newtonian parameter, $\beta \approx 1$, by experimental/observational data [?]. Indeed, classical tests of general relativity provides the bound $|\beta - 1| \lesssim 0.003$ [23, 24], bringing data about the vacuum brane self-energy [2, 25, 26, 21].

The 4D Einstein effective equations can be derived when the 5D bulk Einstein equations are projected onto the 4D brane, by the Gauss-Codazzi method (the convention $8\pi G = c = 1 = \hbar$ is going to be fixed², where $G = \hbar c/M_{\rm pl}^2$ and $M_{\rm pl}$ denotes the Planck scale, and $\mu, \nu, \rho, \sigma = 0, 1, 2, 3$), yielding [27]

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} - \mathrm{T}_{\mu\nu} = 0, \tag{2}$$

where $R_{\mu\nu}$, R, and Λ are, respectively, the Ricci tensor, the Ricci scalar, and the 4D cosmological constant. The effective stress tensor in Eq. (2) can be split into a sum, $\mathrm{T}_{\mu\nu} = T_{\mu\nu} + \mathcal{E}_{\mu\nu} + \sigma^{-1} S_{\mu\nu}$, where the first component $T_{\mu\nu}$ denotes the brane matter stress tensor, $\mathcal{E}_{\mu\nu}$ inscribes high-energetic corrections from the 5D Weyl fluid. The tensor $S_{\mu\nu}$ encrypts Kaluza-Klein imprints from the bulk onto the brane [27, 2]. Static, spherically symmetric, compact distributions on the

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² Obviously, the precise values of all involved parameters shall be suitably taken into account, in the calculations in Sect. III.

brane, read

$$g_{\mu\nu}\, dx^\mu\, dx^\nu = -N(r)\, dt^2 + B(r)\, dr^2 + r^2\, d\Omega^2 \ . \tag{3}$$

The CFM metric is derived by dismissing the condition $N(r) = B^{-1}(r)$ [21]. The CFM I solution comprises Eq. (3), with

$$N(r) = 1 - \frac{2\,G\,M}{r} \qquad \text{and} \qquad B(r) = \frac{1 - \frac{3\,G\,M}{2\,r}}{\left(1 - \frac{2\,G\,M}{r}\right)\left[1 - \frac{G\,M}{2\,r}(4\beta - 1)\right]} \ . \tag{4}$$

The solution (4) relies on the parameter β, being the Minkowski solution recovered when $M \to 0$. The horizon $r = R$ on the brane is derived by solving the algebraic equation $B^{-1}(R) = 0$. The sign of $(\beta - 1)$ determines whether the CFM solution is either colder or hotter than the Schwarzschild solution [21]. Moreover, the 4D Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ diverges for $r = 0$ and $r = \frac{G\,M}{2}(4\beta - 1)$.

3. Fluid at the boundary, dual to the CFM black brane/black string solution

Hydrodynamics is the low-energy effective description for the low-momentum regime of correlation functions. Following the results derived in Ref. [1], the Green-Kubo formula plays a prominent role in the linear response theory, where the shear viscosity represents a fluid transport coefficient arising in response to a perturbation in the fluid stress-energy tensor.

The Kubo formula can be derived by coupling 4D gravity $g_{\mu\nu}$ to the fluid and determine the way how the fluid energy-momentum tensor responds to gravitational perturbations. In particular, for small metric perturbations which vary slowly in space and time, $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$ with $\|h_{\mu\nu}\| \ll 1$, the stress tensor can be expanded in the metric fluctuation $h_{\mu\nu}$ and in gradients [28]. This procedure was reported in Ref. [1], yielding

$$\begin{aligned}
\langle T^{\mu\nu}(x) \rangle &= \langle T^{\mu\nu} \rangle - \frac{1}{2}\int d^4y\, G_{\rm R}^{\mu\nu|\rho\sigma}(x;y) h_{\rho\sigma}(y) + \frac{1}{8}\int d^4y \int d^4z\, G_{\rm R}^{\mu\nu|\rho\sigma|\tau\zeta}(x;y,z)\, h_{\rho\sigma}(y)\, h_{\tau\zeta}(z) + \ldots \\
&\equiv \langle T^{\mu\nu}_{(0)} \rangle + \langle T^{\mu\nu}_{(1)} \rangle + \langle T^{\mu\nu}_{(2)} \rangle + \cdots \ ,
\end{aligned} \tag{5}$$

where $\langle T^{\mu\nu} \rangle$ corresponds to $h = 0$ and $G_R^{\mu\nu|\cdots}$ are retarded n-point correlators. The fluid response to perturbations is derived via the conservation law $\nabla_\mu T^{\mu\nu} = 0$ and the conformal condition $T^\nu{}_\nu = 0$.

At zero order in derivatives, the energy momentum tensor reads

$$T^{\mu\nu}_{(0)} = (\epsilon + P)\, u^\mu\, u^\nu + P\, \bar{g}^{\mu\nu} \ , \tag{6}$$

where u^μ is the fluid four-velocity, ϵ denotes its energy density, P the pressure and $\bar{g}_{\mu\nu}$ is the unperturbed metric on the 4D boundary where the fluid lives. The response of the two-point function of the associated perturbed metric can then be written as the first order term $\langle T^{(1)}_{\mu\nu} \rangle$ in the expansion (5), $\langle T^{\mu\nu}(x) \rangle \sim \int dy\, G_R^{\mu\nu|\rho\sigma}(x;y)\, h_{\rho\sigma}(y)$, where the retarded Green function $G_R^{\mu\nu|\rho\sigma} = \langle T^{\mu\nu}(x)\, T^{\rho\sigma}(y) \rangle$ is employed [12]. Hence, from the gravity point of view of the correspondence, the above perturbation should arise from metric fluctuations $h_{\mu\nu}$ of the appropriate black brane metric.

The first-order expansion of the hydrodynamical stress-energy tensor, also known as the constitutive equation, includes dissipative terms, as well as shear and bulk viscosities. In curved space-times, the first order contribution is given by [29]

$$T^{\mu\nu}_{(1)} = -P^{\mu\rho}P^{\nu\sigma}\left[\left(\nabla_{(\rho}u_{\sigma)} - \frac{2}{3}\bar{g}_{\rho\sigma}\nabla_\tau u^\tau\right)\eta + \zeta\,\bar{g}_{\rho\sigma}\nabla_\tau u^\tau\right] \ , \tag{7}$$

where η is the shear viscosity, ζ the bulk viscosity, and ∇_μ represents the covariant derivative in the space-time metric $\bar{g}_{\mu\nu}$. In addition, $P^{\mu\nu} = \bar{g}^{\mu\nu} + u^\mu u^\nu$ is the projection tensor along spatial directions. The contribution to the shear viscosity η comes only from the component $h_{xy} = h_{xy}(t)$ of $h_{\mu\nu}$, and $h_{\mu\nu} = 0$, otherwise [1]. Such fluctuations around thermal equilibrium are small, making to assert the uniform fluid temperature $\mathcal{T}(x^\mu) = \mathcal{T}_0$, at rest in the chosen frame, $u^\mu = (1, u^i = 0)$.

The shear viscosity of the dual theory can be computed from gravity [13], by the Kubo formula, relating the viscosity to equilibrium correlation functions.

$$\delta \langle T^{xy} \rangle = -G_R^{xy|xy}(q) \, h_{xy} \ , \tag{8}$$

where $G_R^{xy|xy}(q) = -i \int d^4x \, e^{-i \, q_\mu \, x^\mu} \, \theta(x^0) \, \langle T^{xy}(x^\mu) \, T^{xy}(0) \rangle$. The Green-Kubo formula reads,

$$\eta = - \lim_{\omega \to 0} \frac{\Im \, G_R^{xy|xy}(\omega, 0)}{\omega} \ , \tag{9}$$

where $\Im$ denotes the imaginary part.

The CFM black string associated to the CFM I solution are related to black branes in AdS [1, 4], corresponding to a late time behaviour of their dual fluid. CFM black branes in AdS can be also obtained as a perturbative approach of CFM black holes on the brane [1, 4]. The 5D metric reads

$$g_{AB} \, dx^A \, dx^B = -N(r) \, dt^2 + B(r) \, dr^2 + \frac{r^2}{\ell^2} \, d\vec{x}^2 \ , \tag{10}$$

where $d\Omega^2 = \ell^{-2} \, d\vec{x}^2$, the length ℓ is related to the AdS curvature [20] and the metric coefficients N and B equal the CFM expression (4). The entropy of these black strings can be computed by using the Hawking-Bekenstein formula, $S = \frac{A}{4 \, G_5} = \frac{R^3 \, V_3}{4 \, G_5 \, \ell^3}$, where R is the event horizon radius and $V_3 = \ell \int \ell^{-2} \, d\vec{x}^2$. Moreover, the volume density of entropy $s = S/V_3$ must be finite, which requires that $\beta < 5/4$ in Eq. (4). Introducing the variable $u = R/r$, and defining $\bar{M} = G \, M$, yields

$$B(u) = \frac{1 - \frac{3 \, \bar{M}}{2 \, R} \, u}{\left(1 - \frac{2 \, \bar{M}}{R} \, u \right) \left[1 - \frac{\bar{M}}{2 \, R} \, u \, (4\beta - 1) \right]} \tag{11}$$

and the metric finally reads

$$g_{AB} \, dx^A \, dx^B = -N(u) \, dt^2 + B(u) \, \frac{R^2}{u^4} \, du^2 + \frac{R^2}{u^2} \, d\Omega^2 \equiv g_{uu} \, du^2 + g_{\mu\nu} \, dx^\mu \, dx^\nu \ . \tag{12}$$

The value for β was derived in [1] for the fixed black string temperature, from the KSS result (1). The aim here is to analyse how the KSS behaves as the temperature increases. The background metric (12) can be perturbed as $g_{AB} \mapsto g_{AB} + h_{AB}$ [13, 15]. As mentioned before, the contribution to the shear viscosity comes from a single component of the metric fluctuations. Denoting this term by $\phi = \phi(t, u, \vec{x})$, the wave equation reads, after taking $\phi \simeq e^{i \, \omega \, t} \, \Phi(u)$ [15]

$$\partial_u \left(\sqrt{-g} \, g^{uu} \partial_u \Phi \right) - \sqrt{-g} \, g^{tt} \, \omega^2 \, \Phi \approx 0 \ , \tag{13}$$

which can be specialised for both CFM I metric, provided by Eqs. (4), that is

$$\frac{d^2\Phi}{du^2} + \frac{V}{u} \frac{d\Phi}{du} + \left(1 - \frac{2 \, \bar{M}}{R} \, u \right) \omega^2 \, \Phi = 0 \ , \tag{14}$$

where V is given by Eqs. (29) in Ref. [1]. The retarded Green function [13, 14], $G_R(\omega, \vec{0}; \beta) = -\sqrt{-g}\, g^{uu}\, \Phi^* \frac{d\Phi}{du}\big|_{u \to 0}$, for both cases in Eqs. (29, 30) in Ref. [1] and then derive the shear viscosity from the Green-Kubo formula (9). The resulting propagator yields

$$\frac{\eta}{s} = -\frac{1}{s} \lim_{\omega \to 0} \frac{\Im\, G_R(\omega, k = 0; \beta)}{\omega} = \frac{1}{4\,\pi}. \tag{15}$$

where we remarked that both s and η do depend on β. Eq. (15) was obtained in the linear hydrodynamics regime, corresponding to the first two terms in Eq. (5), that is Eqs. (6) and (7). Including the second-order regime of a dual hydrodynamic prescription [10, 19, 30, 28], the parametrised post-Newtonian parameter β bound can be refined.

Moreover, $\sigma^{\alpha\beta}$ denotes the shear tensor and $\Omega^{\alpha\beta}$ is the vorticity tensor. The second order dissipative part in the expansion of the stress-energy tensor (5) reads

$$
\begin{aligned}
T^{\mu\nu}_{(2)} =\ & \eta\,\tau_\Pi \left[\langle g_{\rho\alpha} u^\alpha \nabla_\rho \sigma^{\mu\nu} \rangle_\perp + \frac{1}{2} \sigma^{\mu\nu} (\nabla_\rho u^\rho) \right] + \kappa (g_{\rho\zeta} - 2 u_\rho u_\zeta) \left(\frac{1}{2} P_\alpha{}^\mu P_\gamma{}^\nu R^{\rho(\alpha\gamma)\zeta} \right. \\
& \left. -\frac{1}{3} P_{\alpha\gamma} P^{\mu\nu} R^{\rho\alpha\gamma\zeta} \right) + \frac{1}{2} P_\alpha{}^\mu P_\gamma{}^\nu \left(\lambda_1 \sigma_\zeta^{(\alpha} \sigma^{\gamma)\zeta} + \lambda_2 \sigma_\zeta^{(\alpha} \Omega^{\gamma)\zeta} + \lambda_3 \Omega_\zeta^{(\alpha} \Omega^{\gamma)\zeta} \right) \\
& -\frac{1}{3} P_{\alpha\gamma} P_{\mu\nu} \left(\lambda_1 \sigma_\zeta^\alpha \sigma^{\gamma\zeta} + \lambda_2 \sigma_\zeta^\alpha \Omega^{\gamma\zeta} + \lambda_3 \Omega_\zeta^\alpha \Omega^{\gamma\zeta} \right) ,
\end{aligned} \tag{16}
$$

where $\langle\,\cdot\,\rangle_\perp$ denotes the traceless component that is orthogonal to the fluid velocity. The coefficients in (16) have different interpretations [28]. It is worth mentioning that the second-order hydrodynamic transport coefficients κ and λ_i have been calculated from the leading order AdS/CFT correlators [16, 28, 30]to , corresponding to the black brane horizon response properties. For instance, $\lambda_3 = -4 \lim_{\substack{k_1 \to 0 \\ k_2 \to 0}} \frac{\partial}{\partial k_1} \frac{\partial}{\partial k_2} \lim_{\substack{\omega_1 \to 0 \\ \omega_2 \to 0}} G^{xy|0x|0y}$. The obtained expressions are given by Eq. (37) in Ref. [1], irrespectively of the transport coefficients in the range in Refs. [13, 19, 31].

If one now takes, as usual in the literature, $h_{xy} = h_{xy}(t, z)$, Eq. (16) yields

$$T_{(1)xy} + T_{(2)xy} = -P\, h_{xy} - \eta\, \dot{h}_{xy} + \eta\,\tau_\Pi\, \ddot{h}_{xy} - \frac{\kappa}{2} \left[\ddot{h}_{xy} + \partial_z^2 h_{xy} \right] , \tag{17}$$

where dots denote derivatives with respect to the time coordinate. The retarded Green function reads

$$G_R^{xy|xy}(\omega, k) = P - i\,\eta\,\omega + \eta\,\tau_\Pi\,\omega^2 - \frac{\kappa}{2} \left[\omega^2 + k^2 \right] , \tag{18}$$

providing the analogue of the Kubo formula with second-order derivative terms. The kinetic parameters τ_Π and κ are the coefficients of the ω^2 and k^2 terms in the low-momentum expansion of the retarded propagator. With the above expressions, and repeating the previous analysis, one finds bounds – temperature dependent – for the parametrised post-Newtonian parameter β, shown in Fig. 1. All terms in Eq. (16) are included, with transport coefficients provided by Eq. (37) in Ref. [1]. The KSS bound is illustrated in Fig. 2, as a function of the mass and the temperature. Although the KSS bound implies higher values for β as the temperature increases, these values still match the experimental/observational bounds [23].

Refined expressions involving more transport coefficients, like in Ref. [32], could be used. However, such contributions in the viscosity-to-entropy ratio would amount at most to order 10^{-5} corrections in the bounds for β. These preliminary results including second-order terms are thus fully compatible with the observational bound $|\beta - 1| \lesssim 0.003$ provided by the perihelion shift [23]. Nevertheless, the main result here is that, since the parametrised post-Newtonian
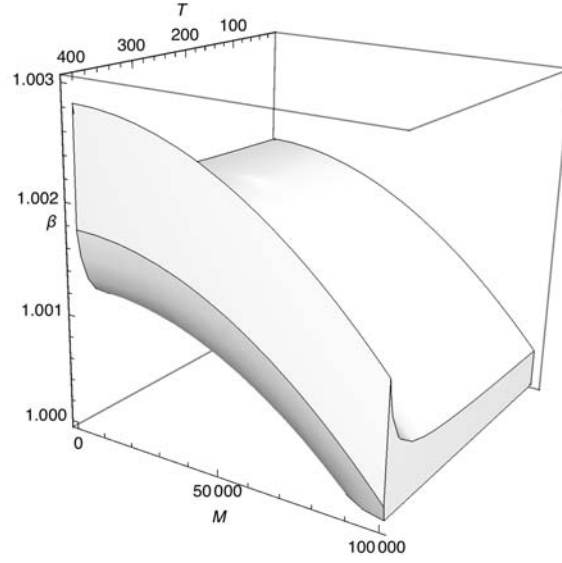
Figure 1. Allowed range for the parametrised post-Newtonian parameter β as a function of the mass M (in Solar mass units), for the CFM I case including second-order corrections.

parameter is bound to be close to the unity also in five dimensions, the CFM black branes must effectively reduce to their Schwarzschild counterparts. In fact, the CFM metric reduces to the Schwarzschild metric in the limit $\beta \to 1$. Here, we proved that the KSS result forces the CFM black branes to reduce to Schwarzschild black branes. For the boundary energy-momentum expansion (5), the first order approximation (7) provides $|\beta - 1| \lesssim 10^{-1}$, whereas the second order approximation (7) provides $|\beta - 1| \lesssim 3 \times 10^{-2}$. When the higher order term $\langle T^{\mu\nu}_{(3)} \rangle$ is included in Eq. (5), one finds $|\beta - 1| \lesssim 10^{-4}$, suggesting that $\beta \to 1$ when sufficiently high-order terms $\langle T^{\mu\nu}_{(n)} \rangle$ are taken into account in Eq. (5). This is in agreement with the conjecture of Ref. [33], that the Schwarzschild black brane [34, 35, 36] is the unique static black hole solution of vacuum gravity with a negative cosmological constant. In fact, Schwarzschild-AdS black branes can be recovered from CFM-AdS black branes, providing fundaments for the proof of a conjecture [33], that asserts that the Schwarzschild-AdS black hole is the unique, static, asymptotically AdS, black-hole solution of gravity with a negative cosmological constant.

4. Conclusions

The shear viscosity-to-entropy ratio has been obtained with respect to the parametrised post-Newtonian parameter of CFM-AdS black branes. The KKS result (1) yields the bounds on β, for different black brane masses. For fixed values of the black brane mass, β is shown to increase with the black brane temperature, still in agreement with observational bounds. The KSS bound yields the CFM black branes to converge to Schwarzschild black branes, when sufficiently high-order terms in the boundary energy-momentum tensor (5) are taken into account, in agreement with the conjectured unicity of the Schwarzschild black brane as a static black-hole solution of vacuum gravity with a negative cosmological constant [33].

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