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Surfactant effect on interaction of rising bubble and particle in a liquid subjected to vibrations

D.V. Lyubimov[1], L.S. Klimenko[2], T.P. Lyubimova[1,2], L.O. Filippov[3]

[1]Perm State University, Perm, Russia
[2]Institute of Continuous Media Mechanics, Perm, Russia
[3]Universite de Lorraine, Nancy, France

Email : lubimova@psu.ru

Abstract. The paper investigates the surfactant effect on the interaction of solid particle and gas bubble in a liquid subjected to vibrations. Surfactant transport between the bubble surface and the surrounding liquid is limited by the adsorption-desorption process. The particle is subjected to the Stokes, Basset and buoyancy forces, and average force related to the inhomogeneity of the pulsational field. The problem is solved numerically. It is found that in the presence of surfactant, the impact of vibrations on the particle-bubble collision is weaker than in the absence of surfactant. Thus, higher vibrations intensity is needed for increasing the collision efficiency, together with a higher consumption in energy.

1. Introduction

The effect of surfactant on particle-bubble interactions is the subject of interest in many studies (see, for example, [1], [2]). As well as direct measurements of the particle-bubble interaction [3], there are several theoretical investigations together with their comparison with experimental data [4]. The majority of these studies are focused on particle-bubble attachment, although for fine particles the most important sub-process is collision. The interest to this problem is justified by its applications to many technological and physical processes, especially flotation.

In flotation theory, the bubble–particle interaction can hardly be modeled without taking into account for the hydrodynamic approach. However, existing theories of the flotation process are either of empirical nature [5], [6] or do not account for the action of the external fields [7], [8].

A theoretical model for bubble-particle collision in a liquid under external vibrations was developed in [9]. The ambient liquid was assumed to be incompressible and submitted to gravitational forces. Analysis of the different components of the pulsational field scattered on the bubble allows the estimation of their contributions to the radiation force acting on the particle. Trajectories of the particle motion near a pulsating bubble were calculated. The evaluation of the efficiency of the flotation process exposed to an ultrasound action is then discussed.

In this study, the previously developed model is extended by taking into account the presence of surfactants. Analyzing the different components of the pulsational field scattered on the bubble surface in presence of surfactants, it is possible to calculate the average vibrational force acting on the particle. These results are compared with the cases where the surfactants are absent.

2. Problem statement

We study the behavior of a single solid particle near a gas bubble (Fig. 1). The system is subjected to the gravity field and to a vibrational field. The vibrations are considered as harmonic with amplitude b and frequency ω. We assume that the dissipative processes caused by the viscosity can be neglected as the investigated frequency is high. Thus, the viscosity of the fluid around the bubble and the particles is null (case of inviscid flow). Moreover the amplitude of the vibrations b is considered to be small compared to the bubble radius, this allows to linearize the governing equations. Additionally, we assume that the applied vibrations are such that the liquid can be considered as incompressible. To increase the collision efficiency between the bubble and the particle surface, a soluble surfactant is added. The transport of surfactant between the bubble surface and the surrounding liquid is limited by an adsorption-desorption process.

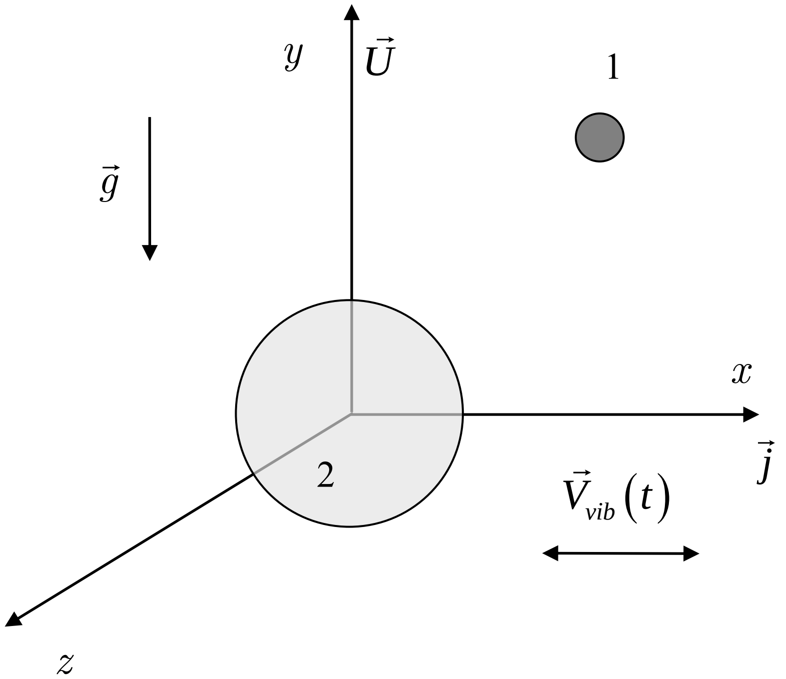

Figure 1 Problem configuration, 1 – solid particle, 2 – gas bubble

For studying the behavior of the system, it is convenient to decompose the motion into a fast (pulsational) and slow (average) components. Let us begin with the bubble pulsational motion under vibrations.

3. Pulsational velocity of ambient liquid

We assume that the particle radius is much smaller than the bubble radius. It allows ignoring the effect of the particles on the pulsational flow scattered on the bubble. Under the effect of the vibration of frequency ω, the total liquid flow around the bubble is composed of a *vibration field* in the absence of the bubble and a *pulsational field* scattered by the bubble and associated with its radial (or *monopole* [10]) and translational motion

$$\boldsymbol{V} = \boldsymbol{v}_{\mathrm{p}} + \boldsymbol{v}_{\mathrm{m}} + \boldsymbol{v}_{\mathrm{tr}}\,, \tag{1}$$

where the uniform pulsating field of amplitude V_{vib} can be written as

$$\boldsymbol{v}_{\mathrm{p}} = \mathrm{Re}(\boldsymbol{V}_{\mathrm{vib}}\, \mathrm{e}^{\mathrm{i}\omega t}) \tag{2}$$

Monopole bubble motion. We define the liquid flow $\boldsymbol{v}_{\mathrm{m}}$ associated with the monopole (it does not depend on the angle), the bubble motion induced by the external pressure part is independent of the coordinates. The pressure at a long distance from the bubble can be represented in the form

$$p = \mathrm{Re}\left(q\mathrm{e}^{\mathrm{i}\omega t}\right) \tag{3}$$

with q is the pressure amplitude, Re is the real part, t is the time, and i is the imaginary unit. As liquid is assumed incompressible, only the radial component must be account for

$$v_{\mathrm{m}} = \mathrm{Re}\left(\frac{c_1}{r^2}\mathrm{e}^{\mathrm{i}\omega t}\right) \tag{4}$$

where c_1 is an unknown constant, and r is the spherical coordinate. Then, according to the linearized Euler equation ($\rho\,\partial v_m/\partial t = -\partial p/\partial r$), the pressure in the surrounding liquid is given by

$$p = \mathrm{Re}\left\{\left(i\omega\rho\frac{c_1}{r} + q\right)\mathrm{e}^{\mathrm{i}\omega t}\right\} \tag{5}$$

where ρ is the fluid density. On the bubble surface a kinematic condition and a normal stresses balance condition in the presence of surfactants are imposed:

$$\frac{\partial\zeta}{\partial t} = v_{\mathrm{m}},$$
$$p - p' - \frac{2}{R_{\mathrm{b}}}\gamma'\Gamma = 2\frac{\gamma}{R_{\mathrm{b}}}\zeta, \tag{6}$$

where $p' = -3\rho_{\mathrm{g}}c^2\zeta/R_{\mathrm{b}}$ is the gas pressure inside the bubble where ρ_g is the gas density and c is the speed of sound, R_{b} is the bubble radius in equilibrium, γ is the surface tension coefficient and its derivative $\gamma' = \partial\gamma/\partial\Gamma$, where Γ is the surface concentration in surfactant, and ζ is the bubble surface deviation (i.e. the position of the bubble surface at time t determined from it position when the bubble is at equilibrium).

We assume that the bubble remains spherical and that the thickness of the film formed by a surfactant is sufficiently small; these assumptions allow writing the equation of mass conservation for the surfactant as [11], [12]

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t} + 2\Gamma_0\frac{v_{\mathrm{m}}}{R_{\mathrm{b}}} = 0 \tag{7}$$

where Γ_0 is the surface concentration of the surfactant at equilibrium, and Γ at time t.

The surface deviation can be obtained from Eq. (6) by substitution Eq. (4) and Eq. (5) and taking into account Eq. (7). Indeed, from Eq. (7) $\omega\Gamma = 2i\Gamma_0 c_1 R_b^{-3}$, thus the boundary conditions Eq. (6) reads

$$i\omega\zeta = c_1 R_b^{-2},$$
$$\frac{i\omega\rho}{R_b}c_1 + q + 3\zeta\frac{\rho_{\mathrm{g}}c^2}{R_b} - \frac{4i\gamma'\Gamma}{\omega R_b^4}c_1 = 2\frac{\gamma}{R_{\mathrm{b}}}\zeta,$$

which gives the amplitude of surface deviation as

$$\zeta = \frac{q}{\omega^2\rho R_{\mathrm{b}} - c^2\frac{3\rho_{\mathrm{g}}}{R_{\mathrm{b}}} + \frac{2\gamma}{R_{\mathrm{b}}^2} - \frac{4\Gamma_0}{R_{\mathrm{b}}^2}\gamma'} \tag{8}$$

Taking into account that the natural frequency of the bubble $\omega_0^{\,2} = \left(3\rho_{\mathrm{g}}c^2R_{\mathrm{b}} - 2\gamma\right)/\rho R_{\mathrm{b}}^{\,3}$ and $\gamma' < 0$, the surface deviation can be rewritten in the following form

$$\zeta = \frac{qR_\mathrm{b}^2}{\rho R_\mathrm{b}^3\left(\omega^2-\omega_0^2\right)+4\Gamma_0\left|\gamma'\right|} \tag{9}$$

Thus, the velocity and pressure in the surrounding liquid caused by a monopole bubble motion is

$$\mathrm{v_m} = \mathrm{Re}\left(\frac{\mathrm{i}\omega\zeta R_\mathrm{b}{}^2}{r^2}\mathrm{e}^{\mathrm{i}\omega t}\right) = \mathrm{Re}\left(\frac{\mathrm{i}\omega qR_\mathrm{b}^4}{\rho R_\mathrm{b}^3\left(\omega^2-\omega_0^2\right)+4\Gamma_0\left|\gamma'\right|}\frac{1}{r^2}\mathrm{e}^{\mathrm{i}\omega t}\right) \tag{10}$$

$$p = \mathrm{Re}\left(\frac{-\omega^2 qR_\mathrm{b}^4\rho}{\rho R_\mathrm{b}^3\left(\omega^2-\omega_0^2\right)+4\Gamma_0\left|\gamma'\right|}\frac{1}{r}\mathrm{e}^{\mathrm{i}\omega t}+q\,\mathrm{e}^{\mathrm{i}\omega t}\right) \tag{11}$$

The velocity and pressure depend on the unknown liquid pressure q far from the bubble, the far-field pressure is found from the linearized Euler equation in the form

$$q\mathrm{e}^{i\omega t} = -\mathrm{i}\omega\rho\left(\boldsymbol{V}_\mathrm{vib}\cdot\boldsymbol{r}_\mathrm{b}\right)$$

where r_b is the radius-vector of the bubble center which is counted from the pressure nodes, and $\boldsymbol{V}_{vib}$ the fluid velocity due to the far field oscillating pressure. The coordinates of the nodes are determined by the type of generation of oscillations in the liquid.

Finally, the velocity of the surrounding liquid due to the monopole bubble motion is obtained as

$$\boldsymbol{v}_\mathrm{m} = \mathrm{Re}\left(\frac{\omega^2\rho R_\mathrm{b}^4}{\rho R_\mathrm{b}^3\left(\omega^2-\omega_0^2\right)+4\Gamma_0\left|\gamma'\right|}\left(\boldsymbol{V}_\mathrm{vib}\cdot\boldsymbol{r}_\mathrm{b}\right)\frac{\boldsymbol{r}}{r^3}\mathrm{e}^{\mathrm{i}\omega t}\right) \tag{12}$$

Translational bubble motion. Let us proceed to the determination of the liquid flow induced by the translational oscillations of the bubble. The Euler and the continuity equations for incompressible liquid in a reference frame moving with the bubble center read

$$\frac{\partial\boldsymbol{v}_\mathrm{tr}}{\partial t} = -\frac{\nabla p}{\rho}-\frac{\mathrm{d}\boldsymbol{U}_\mathrm{b}}{\mathrm{d}t}, \tag{13}$$

$$\mathrm{div}\boldsymbol{v} = 0, \tag{14}$$

where $\boldsymbol{U}_\mathrm{b}(t)$ is the velocity of the bubble center relative to the laboratory reference frame.

Far from the bubble the velocity is given by $\boldsymbol{v} = b\omega\cos(\omega t)\boldsymbol{j}-\boldsymbol{U}_\mathrm{b}$, where $\boldsymbol{j}$ is a unit vector in the vibration direction.

The kinematic condition and the condition of stress balance at the bubble surface under the assumption of negligible thickness of the surfactant film, are (see, for example[11], [12]):

$$\begin{gathered}\frac{\partial\zeta}{\partial t} = \boldsymbol{v}_\mathrm{tr}\cdot\frac{\boldsymbol{r}}{r},\\ p-\frac{2}{R_\mathrm{b}}\gamma'\Gamma = \frac{\gamma}{R_\mathrm{b}^2}\left(\nabla_\vartheta^2+2\right)\zeta,\end{gathered} \tag{15}$$

where $\nabla_\vartheta^2 = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial}{\partial\vartheta}\right)$ (we assume that the velocity field has axial symmetry), and ϑ is the angle counted from the vibration direction. To have the problem closed, the mass conservation equation for the surfactant is needed

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}t}+\Gamma_0\frac{1}{R_\mathrm{b}}\left(\frac{\partial v_\vartheta}{\partial\vartheta}+v_\vartheta\cot\vartheta\right)+2\Gamma_0\frac{v_r}{R_\mathrm{b}} = 0 \tag{16}$$

It is convenient to express the velocity in terms of flow potential φ so that the continuity equation has the form of a Laplace's equation whose solution can be written as [13]

$$\varphi = \mathrm{Re}\left\{\left(b\omega r+\frac{c_2}{r^2}\right)\cos\vartheta\,\mathrm{e}^{\mathrm{i}\omega t}-\boldsymbol{U}_\mathrm{b}\cdot\boldsymbol{r}\right\}, \tag{17}$$

$$p=\mathrm{Re}\left\{-\mathrm{i}\rho\omega\left(b\omega r+\frac{c_2}{r^2}\right)\cos\vartheta\,\mathrm{e}^{\mathrm{i}\omega t}\right\}.$$

where c_2 is an unknown constant, and $\boldsymbol{U}_b$ is the velocity of the bubble center.

From the kinematic condition, the surface deviation can be thus written as $\zeta=\mathrm{Re}\left\{c_3\cos\vartheta\,\mathrm{e}^{\mathrm{i}\omega t}\right\}$. However as the bubble is immobile in the reference frame attached to the bubble center, the translational oscillations of the bubble as a whole is null so $c_3=0$. Thus, the bubble surface deviation vanishes. It follows that the ambient liquid velocity due to the bubble translation motion takes the form

$$\boldsymbol{v}_{\mathrm{tr}}=\frac{\rho b\omega^3R_{\mathrm{b}}^3}{\rho R_{\mathrm{b}}^3\omega^2+12\Gamma_0|\gamma'|}\mathrm{Re}\left\{\left(-2+2\frac{R_{\mathrm{b}}{}^3}{r^3}\right)\cos\vartheta\,\mathrm{e}^{\mathrm{i}\omega t}\boldsymbol{e}_r+\left(2+\frac{R_{\mathrm{b}}{}^3}{r^3}\right)\sin\vartheta\,\mathrm{e}^{\mathrm{i}\omega t}\boldsymbol{e}_\vartheta\right\}. \tag{18}$$

Notice that the velocities and coordinates rules for converting these quantity from the local reference frame attached to the bubble center into the fix laboratory reference frame are:

$$\boldsymbol{V}'=\boldsymbol{V}+\boldsymbol{U}_{\mathrm{b}},\ \boldsymbol{r}'=\boldsymbol{r}+\boldsymbol{R}, \tag{19}$$

where the velocity $\boldsymbol{U}_b$ of the bubble center reads

$$\boldsymbol{U}_{\mathrm{b}}=3b\omega\frac{\rho R_{\mathrm{b}}^3\omega^2-4\Gamma_0|\gamma'|}{\rho R_{\mathrm{b}}^3\omega^2+12\Gamma_0|\gamma'|}\boldsymbol{j}\cos\omega t, \tag{20}$$

The coordinate are found by integrating Eq.(20) over time t (assuming $|\gamma'|$ and R_b constant)

$$\boldsymbol{R}=3b\frac{\rho R_{\mathrm{b}}^3\omega^2-4\Gamma_0|\gamma'|}{\rho R_{\mathrm{b}}^3\omega^2+12\Gamma_0|\gamma'|}\boldsymbol{j}\sin\omega t, \tag{21}$$

Thus, the velocity in the laboratory reference frame can be obtained from the converting rules Eq.(19)

$$\boldsymbol{V}'(\boldsymbol{r}')=\boldsymbol{V}(\boldsymbol{r}'-\boldsymbol{R})+U_{\mathrm{b}}\boldsymbol{j}=\boldsymbol{V}(\boldsymbol{r}')+U_{\mathrm{b}}\boldsymbol{j}-\nabla\boldsymbol{V}(\boldsymbol{r}')\cdot\boldsymbol{j}R \tag{22}$$

Neglecting the nonlinear terms ($\nabla\boldsymbol{V}(\boldsymbol{r}')\cdot\boldsymbol{j}R$ which is small under the small amplitude of vibrations assumptions, and omitting the primes in the liquid translation velocity in the laboratory reference frame, it comes:

$$\boldsymbol{V}_{tr}=\frac{\rho b\omega^3R_{\mathrm{b}}^3}{\rho R_{\mathrm{b}}^3\omega^2+12\Gamma_0|\gamma'|}\left(3\frac{R_{\mathrm{b}}{}^3}{r^5}(\boldsymbol{j}\cdot\boldsymbol{r})\boldsymbol{r}+\left(1-\frac{R_{\mathrm{b}}{}^3}{r^3}\right)\boldsymbol{j}\right)\cos\omega t. \tag{23}$$

Finally, the liquid pulsational velocity around the gas bubble under external vibrations has the following form

$$\boldsymbol{V}=\frac{\rho\omega^2R_{\mathrm{b}}^4}{\rho R_{\mathrm{b}}^3\left(\omega^2-\omega_0^2\right)+4\Gamma_0|\gamma'|}\left(\boldsymbol{V}_{\mathrm{vib}}\cdot\boldsymbol{r}_{\mathrm{b}}\right)\frac{\boldsymbol{r}}{r^3}+\frac{\rho\omega^2R_{\mathrm{b}}^3}{\rho R_{\mathrm{b}}^3\omega^2+12\Gamma_0|\gamma'|}\left((\boldsymbol{r}\cdot\boldsymbol{V}_{\mathrm{vib}})\frac{3R^3}{r^4}\frac{\boldsymbol{r}}{r}+\boldsymbol{V}_{\mathrm{vib}}-\boldsymbol{V}_{\mathrm{vib}}\frac{R^3}{r^3}\right), \tag{24}$$

which is in a good agreement with the results obtained in the absence of surfactant [7].

4. Trajectories of particle

Let us consider the average particle motion. The particle is subjected to gravity $\boldsymbol{g}$, buoyancy, Stockes, Basset forces and average force caused by the vibrations. The equation of the particle motion reads

$$\left(\rho_{\mathrm{s}}+\frac{1}{2}\rho\right)\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}=\boldsymbol{F}_{\mathrm{vib}}-\frac{9}{2R_{\mathrm{p}}{}^2}\eta\left(\frac{d\boldsymbol{r}}{dt}-\boldsymbol{U}\right)-\frac{9\rho}{2R_{\mathrm{p}}}\sqrt{\frac{\nu}{\pi}}\int_0^t\frac{\mathrm{d}}{\mathrm{d}t_1}\left(\frac{d\boldsymbol{r}}{dt_1}-\boldsymbol{U}\right)\frac{\mathrm{d}t_1}{\sqrt{t-t_1}}+(\rho_{\mathrm{s}}-\rho)\boldsymbol{g}$$

where ρ_{s} is the particle density, ρ,η are density and dynamic viscosity of the surrounding liquid, and the $\boldsymbol{U}$ is the velocity field induced by the rising bubble, which according to [13] has the form

$$\boldsymbol{U} = \frac{uR_{\mathrm{b}}}{2r}\left[\frac{\boldsymbol{r}}{r^2}\left(\boldsymbol{k}\cdot\boldsymbol{r}\right)+\boldsymbol{k}\right]-u\boldsymbol{k}$$

where $u = gR_{\mathrm{b}}^2/3\nu$ is the bubble rising velocity, $\boldsymbol{k}$ is the direction of its motion.

For incompressible fluid, the average vibrational force acting on the particle from the oscillating liquid, is given in [14]. In the laboratory reference frame after averaging over the vibration period, the expression for the average vibrational force per unit volume was obtained in the form:

$$\boldsymbol{F}_{\mathrm{vib}} = \frac{3}{4}\rho\frac{\rho_{\mathrm{s}}-\rho}{\rho_{\mathrm{s}}+\frac{1}{2}\rho}\nabla\overline{V^2}\,.$$

Here $\boldsymbol{V}$ is the pulsational velocity of the liquid, described in the previous section (Eq. (24)).

The equation of particle motion was solved numerically using the Euler method. For each time step, the Basset force was computed from the previous time steps using a Caputo-type fractional derivative algorithm for solving partial differential equations [15]. The calculations show that in the presence of surfactant, the effect of vibrations on the particle-bubble interaction is lower than in the absence of surfactant which means that to increase the interaction efficiency, higher vibrations intensity is needed, and thus more energy.

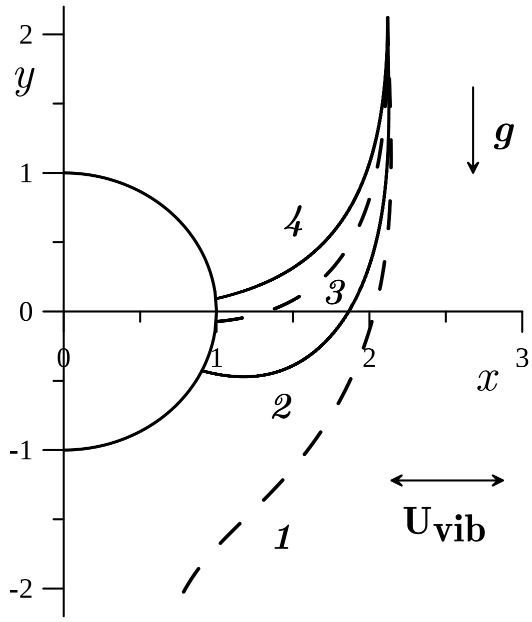

Figure 2 Trajectories of the particle motion, dot lines with surfactants, solid lines without surfactants, lines 1,2 – $\rho_s/\rho = 1.2$, lines 3, 4 – $\rho_s/\rho = 2$.

5. Conclusions

The solid particle behavior near a rising gas bubble in an incompressible liquid with surfactants was investigated under a gravity field and an external vibrational field. The surfactant transfer between the bubble surface and the surrounding liquid is limited by the adsorption-desorption process. The particle is subjected to the Stokes, Basset and buoyancy forces, and the average force due to the inhomogeneity of the pulsational field. The surfactants impact the pulsational bubble motion and decrease the bubble radius variations. It changes the radiation force value acting on the particle.

The problem is solved numerically using the Euler method; the trajectories of the particle motion have been calculated. In the presence of surfactant, it has been shown that vibrations decrease the particle-bubble collision efficiency compared to the case without surfactant; thus, to increase the collision efficiency a higher vibrations intensity is required, leading to larger energy consumptions.

It should be noted, that the surfactants are widely used in the flotation process and their effects on other flotation sub-processes (attachment-detachment) are important. In future we plan to extend this analysis to other sub-processes of bubble-particle interaction.

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