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Time Dependent $\hat{q}$ from AdS/CFT

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Abstract. We present the first ever AdS/CFT calculation of $\hat{q}$ for a light quark jet as a function of position or, equivalently, time. Our result does not suffer from the gamma factor blow up of the usual time-independent AdS/CFT heavy quark setup and is qualitatively similar to, but about a factor of 2/3 larger than, the light flavor result from Liu, Rajagopal, and Wiedemann. Our findings can be immediately implemented into any $\hat{q}$-based energy loss model.

Our $\hat{q}$ derivation relies on our calculation of the average distance squared, $s^2(t)$, travelled by the endpoint of a string falling in an AdS_3-Schwarzschild spacetime. The early time behavior is ballistic, $s^2(t) \sim t^2$, but the late time behavior is the usual diffusive Brownian motion, $s^2(t) \sim t$. These late time dynamics are universal and depend only on the near-horizon physics, which allows us to generalize our results to arbitrary dimensions and thus make contact with the physics explored by RHIC and LHC.

Additionally, we find that AdS/CFT predicts angular ordering for radiation in a medium, just as in vacuum, and in contradistinction to weak-coupling. Finally, our results also imply, sensibly, that AdS/CFT predicts a smooth interpolation between the angular correlations of open heavy flavor and light flavor observables.

1. Introduction

The remarkable success of nearly inviscid relativistic hydrodynamics in high multiplicity pp, pA, and AA collisions at RHIC and LHC [1–4] suggests the medium created in these collisions is strongly coupled and thus best described using the techniques of AdS/CFT [5]. A quantitative understanding of high momentum particles interacting with the quark-gluon plasma produced in these colliders provides a crucial test of this understanding of the dynamics of many-body quantum chromodynamics (QCD) at temperatures $T \gtrsim T_c$, where T_c is the temperature at which a phase transition from normal nuclear matter occurs [6–8].

Previous work [9–11] has demonstrated that AdS/CFT [12, 13] can provide a qualitatively correct description of the suppression pattern of decay products associated with open heavy flavor production at RHIC and LHC. Crucially, one must include the momentum fluctuations due to the thermal nature of the QGP medium [14–16]. Additional work has shown that, after a renormalization procedure, AdS/CFT also provides a qualitatively similar suppression for light flavor jets when compared to data [17–19].

It is natural then to ask: what is the impact of momentum fluctuations on the propagation of high momentum heavy quarks and, equivalently, light quarks?

We report in this proceedings the first attempt to answer this question.

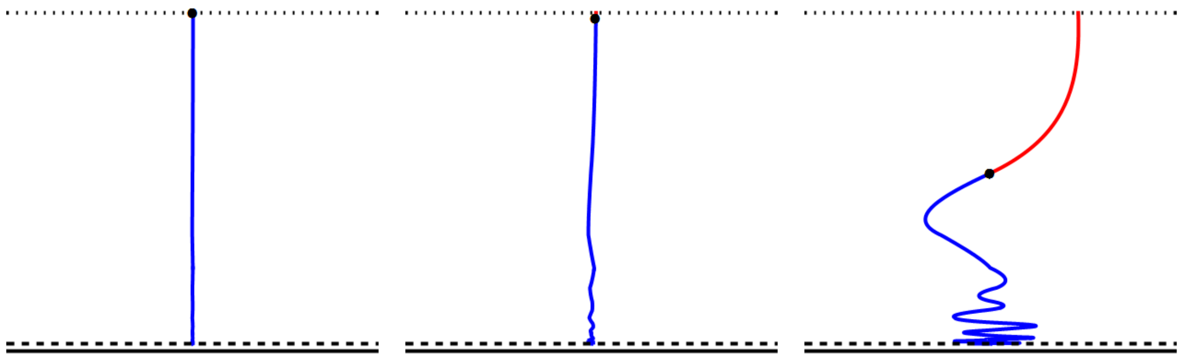

Figure 1: Snapshots in time (with time increasing from left to right) of a falling limp noodle (blue) equilibrating with a black hole (whose horizon is given by the dashed black line). The free endpoint of the falling limp noodle can be interpreted as an observer travelling down the stretched string of de Boer et al. [20] (red curve) at the local speed of light.

2. Method

Computing the thermal momentum spectrum for a light quark propagating with some velocity through a strongly coupled plasma is a very difficult problem. We thus first answer the question: what is the Brownian diffusion of the endpoint of a falling string (wobbly limp noodle) initially at rest in a strongly coupled plasma? See Fig. 1. Again, the problem is very difficult to solve in an AdS space in general dimension, so we will focus on AdS_3.

The outline of the solution is to 1) derive the classical solution; 2) quantize the perpendicular directions of motion on the string given the classical solution; 3) populate the quanta in these perpendicular directions according to Bose statistics, which is equivalent to the semi-classical approximation; and 4) compute the relevant correlators. For full details, one may read [21].

We will find it useful to interpolate between the heavy flavor result for the distance squared traversed by the string endpoint and that for a light quark. For a heavy quark, the free string endpoint stays at a fixed height in the direction orthogonal to the black brane horizon. For a light quark, the free string endpoint falls at the local speed of light. We will take a to be the fraction of the local speed of light at which we allow the free string endpoint to fall. Then we have that

$$s^2(t;a) = \langle :\big(\hat{X}_{\text{End}}(t;a)) - \hat{X}_{\text{End}}(0;a)\big)^2 : \rangle \tag{1}$$

$$= \frac{\beta^2}{4\pi^2\sqrt{\lambda}} \int_0^\infty \frac{d\omega}{\omega} \frac{1}{e^{\beta\omega}-1} |f_\omega(\sigma_f - at) - f_\omega(\sigma_f)e^{i\omega t}|^2. \tag{2}$$

3. Results

One is able to solve Eq. 2 analytically in several limits: for small virtualities at all times, for arbitrary virtualities at early times, and for arbitrary virtualities at late times.

3.1. Small Virtualities

For small virtualities we have that

$$s^2_{\text{small}}(t;a) = \frac{\beta^2}{\pi^2\sqrt{\lambda}} \int_0^\infty \frac{d\omega}{\omega} \frac{1}{e^{\beta\omega}-1} \left(1+\cos^2(a\omega t) - 2\cos(a\omega t)\cos(\omega t)\right)$$
$$= \frac{\beta^2}{4\pi^2\sqrt{\lambda}} \ln\left(\frac{2a\beta^3 \sinh^2\left(\frac{\pi(a+1)t}{\beta}\right)\sinh^2\left(\frac{\pi(a-1)t}{\beta}\right)\operatorname{csch}\left(\frac{2\pi a t}{\beta}\right)}{\pi^3\left(a^2-1\right)^2 t^3}\right). \quad (3)$$

Albeit fully analytic, the above formula is not the most transparent. One may expand Eq. 3 for early times, $t \ll 1/T$, to find

$$s^2_{\text{small}}(t;a) \xrightarrow{t\ll\beta} \frac{t^2}{6\sqrt{\lambda}} + \mathcal{O}((t/\beta)^4). \quad (4)$$

The $s^2 \sim t^2$ dependence is characteristic of ballistic motion.

One may also expand Eq. 3 for late times, $t \gg 1/T$, to find

$$s^2_{\text{small}}(t;a) \xrightarrow{\beta\ll t} \frac{\beta t}{\pi\sqrt{\lambda}}\left(1-\frac{a}{2}\right) + \frac{\beta^2}{4\pi^2\sqrt{\lambda}} \left\{ \begin{array}{ll} 4\ln\left(\frac{\beta}{2\pi t}\right), & \text{if } a=0 \\ \ln\left(\frac{a\beta^3}{4\pi^3(a^2-1)^2t^3}\right), & \text{if } 0<a<1 \\ \ln\left(\frac{\beta}{4\pi t}\right), & \text{if } a=1 \end{array} \right\} + \mathcal{O}(1). \quad (5)$$

The $s^2 \sim t$ dependence is characteristic of Brownian motion. Note the subdiffusive log corrections.

3.2. Arbitrary Virtualities

Unfortunately one cannot solve analytically the arbitrary virtualities result for arbitrary times. One can, however, find the early and late time expansions. For $t \ll 1/T$

$$s^2(t;a)\Big|_{t\ll\beta} = \frac{4t^2}{\sqrt{\lambda}} \int_0^\infty d\nu \frac{\nu}{e^{2\pi\nu}-1} \frac{1+\nu^2}{1+\nu^2\tilde{r}_0^2}$$
$$= \frac{t^2}{6\tilde{r}_0^4\sqrt{\lambda}} \left(\tilde{r}_0^2 - 6(\tilde{r}_0^2-1)\left(-2\gamma_E - \pi\cot\left(\frac{\pi}{\tilde{r}_0}\right) + H_{-1/\tilde{r}_0} + H_{1/\tilde{r}_0} + 2\ln(\tilde{r}_0)\right)\right), \quad (6)$$

where γ_E is the Euler-Mascheroni constant and $H_{\mp 1/\tilde{r}_0}$ are harmonic numbers. Again, one finds ballistic $s^2 \sim t^2$ behavior.

For $t \gg 1/T$ one finds

$$s^2(t;a)\Big|_{t\gg\beta} = s^2_{\text{small}}(t;a), \quad (7)$$

where $s^2_{\text{small}}(t;a)$ is exactly the expression of Eq. 3.

Critically, notice that the late time dynamics are *universal* in the sense that 1) there is only one scale that determines the difference between early and late times, $1/T$, (in particular, there is no dependence on a) and 2) the late time behavior is independent of the initial string length (virtuality of the quark). Thus the late time behavior is set completely by the near horizon dynamics of the string.

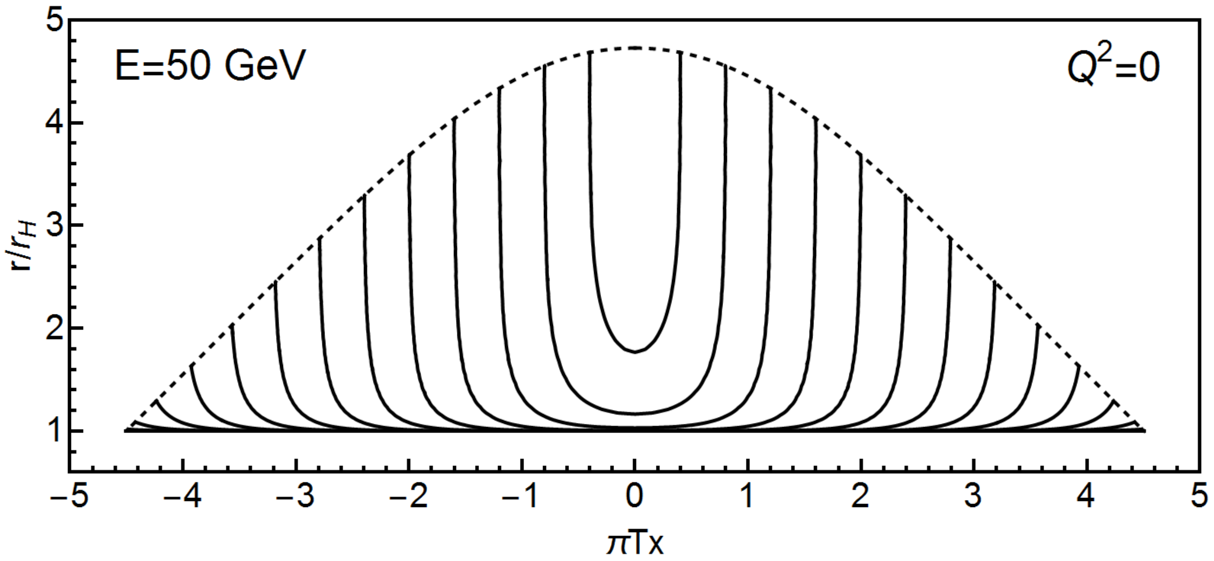

Figure 2: Snapshots in time of a typical numerical falling string profile (black curve) of Chesler et al. [22] representing a hard light quark–anti-quark pair production event in the dual field theory. The falling string nucleates at $x = 0$ as a point and evolves to an extended object with the endpoints falling towards the stretched horizon at $r \sim r_H$ along trajectories (dashed black curve) approximated by null geodesics. Initially the transverse fluctuations are characterized by $a = 0$ while close to the stretched horizon $a = 1$.

3.3. Arbitrary Dimensions

Since the late time dynamics is determined solely by the near horizon physics, we may extend our derivation of the late time dynamics to arbitrary numbers of spatial dimensions. From the metric for AdS$_d$ space and repeating the computation that led to Eq. 3 in AdS$_3$, we find that

$$s^2_{\text{small}}(t;a,d) = \frac{1}{\sqrt{\lambda}}\left(\frac{(d-1)\beta}{4\pi}\right)^2 \ln\left(\frac{2a\beta^3\sinh^2\left(\frac{\pi(a+1)t}{\beta}\right)\sinh^2\left(\frac{\pi(a-1)t}{\beta}\right)\operatorname{csch}\left(\frac{2\pi a t}{\beta}\right)}{\pi^3\left(a^2-1\right)^2 t^3}\right), \qquad (8)$$

which reduces to our previous result for $d = 3$, Eq. 3.

For processes that exhibit Brownian $s^2 \sim t$ motion, one may define the diffusion coefficient,

$$s^2(a,d) \equiv 2D(a,d)t. \qquad (9)$$

Expanding Eq. 8 in powers of β/t, we can again extract the diffusion coefficient by comparing to Eq. 9:

$$D(a,d) := \frac{(d-1)^2\beta}{8\pi\sqrt{\lambda}}\left(1-\frac{a}{2}\right). \qquad (10)$$

Using this very different approach to computing the diffusion coefficient, we exactly reproduce the known low velocity heavy quark, $a = 0$, $d = 5$ results of [14, 15].

3.4. $\hat{q}(t)$

Let's now make a non-trivial conjecture that we can apply the above results to compute the dynamical $\hat{q}(t)$ for the first time in AdS/CFT. Recall that $\hat{q}$ is the average transverse momentum

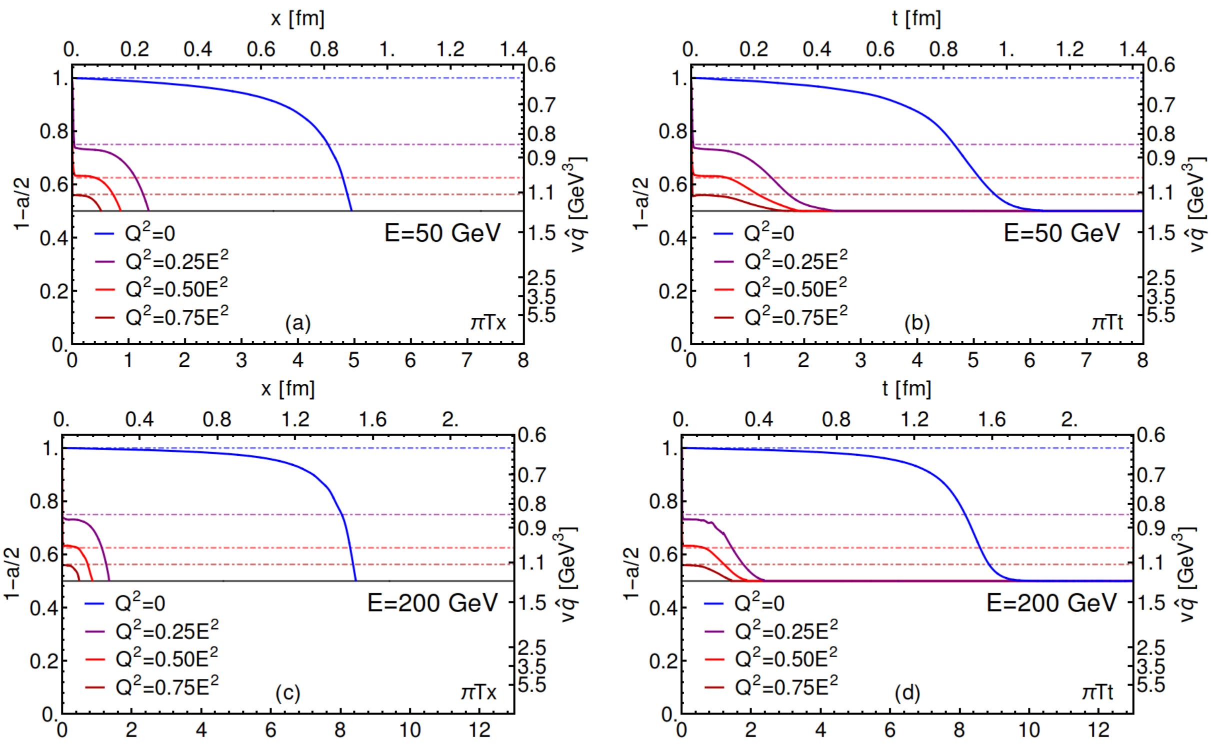

Figure 3: $1 - a/2 \propto 1/(v \times \hat{q})$ plotted as a function of x (left column) and t (right column) for an $E = 50$ GeV quark (top row) and $E = 200$ GeV quark (bottom row) for various virtualities in a $T = 350$ MeV plasma with $\lambda = 5.5$.

squared transferred from the plasma to the probe quark per unit distance travelled by the probe quark [7, 23]. $\hat{q}$ is an important transport coefficient, which can be used to determine the rate of radiative energy loss of a probe quark in pQCD [7, 23].

From the above definition, one may relate $\hat{q}$ to the diffusion coefficient [14],

$$\begin{aligned}\hat{q} &= \frac{\langle p_T^2 \rangle}{\lambda_{mfp}} = \frac{2\kappa_T}{v} = \frac{4T^2}{vD} \\ &= \frac{32\pi\sqrt{\lambda}T^3}{(d-1)^2(1-a/2)v},\end{aligned} \tag{11}$$

where in the second line we used Eq. 10 applicable for times large compared to $1/T$.

We thus find that for heavy quarks ($a = 0$) in the usual 4 spacetime dimensions, we have for a high momentum heavy quark moving at approximately constant velocity v but for which we do not input energy to maintain a constant velocity,

$$\hat{q}_{MH} = \frac{2\pi\sqrt{\lambda}T^3}{v}, \tag{12}$$

which should be compared to the previous calculation in which the heavy quark is explicitly kept at a constant velocity [14],

$$\hat{q}_{Gubser} = \frac{2\pi\sqrt{\lambda}T^3}{v}\sqrt{\gamma}. \tag{13}$$

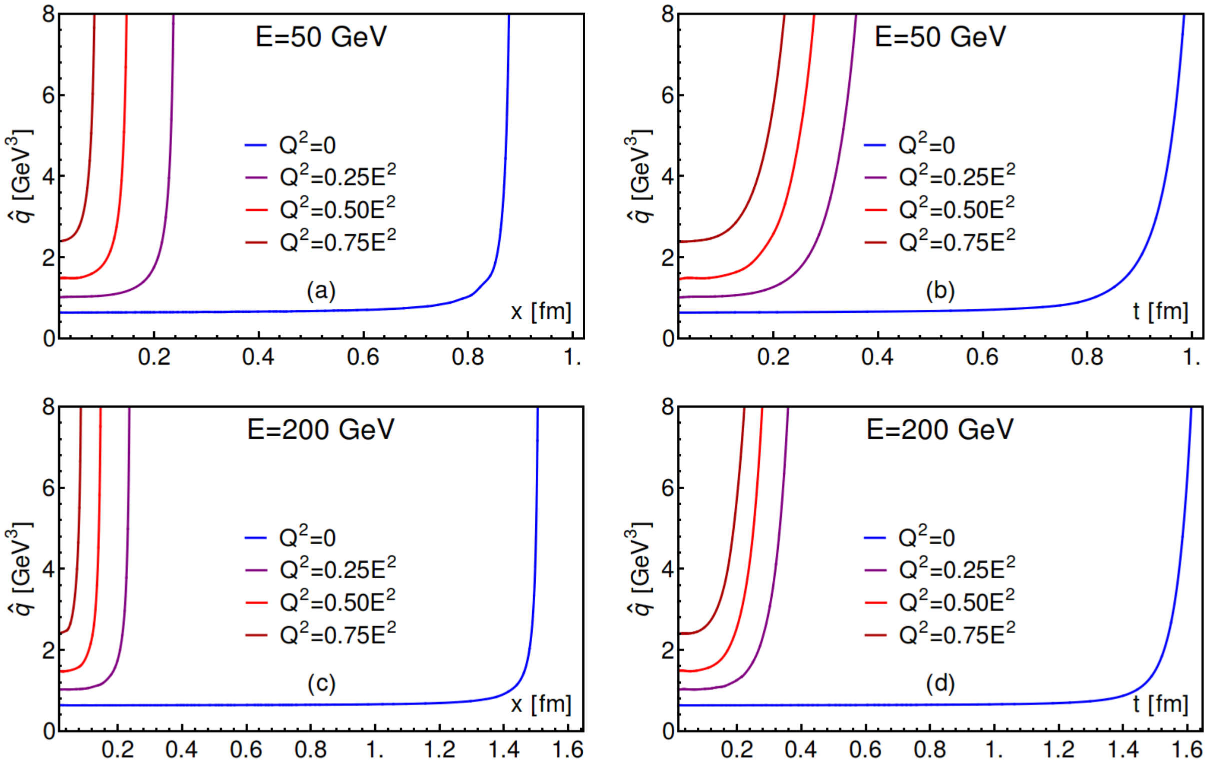

Figure 4: (Colour online) $\hat{q}$ plotted as a function of x (left column) and t (right column) for an $E = 50$ GeV quark (top row) and $E = 200$ GeV quark (bottom row) for various virtualities in a $T = 350$ MeV plasma with $\lambda = 5.5$.

For high momentum light quarks, we have that

$$\hat{q}_{MH} = \frac{2\pi\sqrt{\lambda}T^3}{(1-a/2)v} \simeq (2-4)\pi\sqrt{\lambda}T^3, \tag{14}$$

which should be compared to the LRW result [24]

$$\hat{q}_{LRW} = \frac{\pi^{3/2}\Gamma(3/4)}{\Gamma(5/4)}\sqrt{\lambda}T^3 \simeq 7.5\sqrt{\lambda}T^3. \tag{15}$$

We can, however, go even further for the light quark case by numerically solving the falling string equations of motion for the usual light flavor initial conditions. We show snapshots in time of the shape of the falling string in Fig. 2. From the numerical solution, we may extract explicitly $a(t)$, as shown in Fig. 3. One may thus determine $\hat{q}(t)$, which we show in Fig. 4. Our derivation for the diffusion coefficient in AdS$_5$ required the assumption of examining the probe system at times late compared to $1/T$. We nevertheless show $\hat{q}$ for all times for phenomenologically relevant values of energy and virtuality for purposes of illustration. At very late times, the endpoint of the falling Chesler string has essentially stopped moving in the horizontal direction, and $\hat{q}$ blows up trivially.

4. Conclusions

We have shown here the first results for $\hat{q}(t)$ for light and heavy quarks from AdS/CFT. For heavy quarks, we find, crucially, that the momentum diffusion picked up per unit distance does not grow with heavy quark velocity. This lack of growth with the Lorentz boost factor implies that the application of Langevin energy loss model to heavy quarks does not break down due to unphysically large momentum fluctuations and leads to a prediction of much greater suppression of heavy quarks at high momenta [25].

For light flavor, we found $\hat{q}(t)$ by mapping the rate of descent of the endpoint of a falling numerical open string in AdS$_5$/Schwarzschild space to the Brownian motion of a falling limp noodle as derived in [21]. We show the result for $\hat{q}(t)$ in Fig. 4.

One can see from Fig. 3 that a is a monotonically increasing function of time (or, equivalently, distance travelled by the string endpoint). Thus the quark experiences its greatest transverse fluctuations at the initial time, and the fluctuations decrease monotonically with time. This decrease in transverse fluctuations with time/distance is consistent with the pQCD result of angularly ordered gluon emissions from an off-shell parton in vacuum [26, 27] and in contradistinction to the anti-angular ordering observed in medium [28].

Finally, since the transverse momentum fluctuations smoothly interpolate between light flavor and heavy flavors through a, one naturally expects a smooth interpolation between the angular correlations of $q\bar{q}$ pairs at high-p_T.

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