

# Heat and Mass Transfer Model in Freeze-Dried Medium

Sayahdin Alfat<sup>1</sup> and Acep Purqon<sup>2</sup>

<sup>1</sup> Graduate Computational Science, Faculty of Mathematics and Science, Institute of Technology Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia

<sup>2</sup> Physics of Earth and Complex Systems, Institute of Technology Bandung, Jl. Ganesha 10 Bandung 40132, Indonesia

E-mail: sayahdin.alfat@yahoo.com

**Abstract.** There are big problems in agriculture sector every year. One of the major problems is abundance of agricultural product during the peak of harvest season that is not matched by an increase in demand of agricultural product by consumers, this causes a wasted agricultural products. Alternative way was food preservation by freeze dried method. This method was already using heat transfer through conduction and convection to reduce water quality in the food. The main objective of this research was to design a model heat and mass transfer in freeze-dried medium. We had two steps in this research, the first step was design of medium as the heat injection site and the second was simulate heat and mass transfer of the product. During simulation process, we use physical property of some agriculture product. The result will show how temperature and moisture distribution every second. The method of research use finite element method (FEM) and will be illustrated in three dimensional.

## 1. Introduction

Today, Indonesia is one of the countries producing and exporting coffee beans in the world. In Indonesia itself, the coffee beans is a major commodity and the coffee itself is the largest agricultural commodity exchange earners after oil palm, rubber and cocoa [1]. However, this product have some problems, one problem encounters is that the low value of exports because still low quality [2]. A modern solution, home freeze drying may be the best preservation approach. This approach is an artificial drying to biology product and an approach that is often used for the drying process [3].

Basically, the technology of freeze drying was developed during World War II. This technology as a way to keep vaccines for the wounded from spoiling due to uncertain refrigeration while transported to war zones. The technology was refined and used extensively during the NASA space program, providing varieties of freeze dried food for astronauts. This technology has been applied with success to diverse biological material, such as meat, seafood products, coffee, tea, vegetable, fruits, milk products, juices, dairy products, cells and bacteria and is now standard practice in the production of penicillin, protein hydrolyzes, hormones, blood plasma and vitamin preparations. Especially for food, this technology have many advantages such as minimize the decline in food nutrition, there is not shrink on food, and also freeze drying does not reduce aroma and flavor of food. The model or approach use heat and mass transfer to reduce water content in the food.

Heat and mass transfer are complex and complicated process. This phenomenon are found in other process such as; roasting [7, 5], cooking [9] and drying [14, 6, 10]. Understanding the heat and mass transfer in the products will help to improve drying process parameters and quality. There are advantages of heat and mass transfer In food product. First, heat and mass transfer treatment can



reduce microbial growth [8]. Second, understanding the heat and mass transfer can be minimize consumption energy during treatment process [7].

Mathematical modeling is a useful method to describe heat and mass transport during drying food. Several theoretical model have been proposed in the study of convection food drying [10]. Luikov's couple systems equation is heat and mass transfer equation, but thermo-physical properties is constant [11, 13].

The discrete model is a part of numerical method and it have been widely used in solving real engineering problem. One of the discrete model is finite element method (FEM), it is most powerful numerical method to show mechanism of heat ad mass transfer. This is one numerical method and can show mechanism of heat and mass transfer, can show distribution heat and mass inside product and also can be used to complex domain [4].

The work was divided on two objectives namely main and specific objective. The main objective of this paper is to design a numerical method for heat and mass transfer as a part of coffee drying. For specific objective, we want to investigate heat and mass flow between surface and center of product. However, distribution profile of temperature and moisture content of the product will be explained.

This paper have six section. Section 2, we will explain some of the assumption used during the simulation process. The computational model and some boundary condition are also shown in this section. Section 3, we will give parameters of coffee bean and air. For section that explain the numerical method that is section 4. As for section 5, the numerical results of heat and mass transfer will be shown and it is discussed with physical analysis. Finally, section 6 is closed by the main conclusion and the possible future works as the extension in this research.

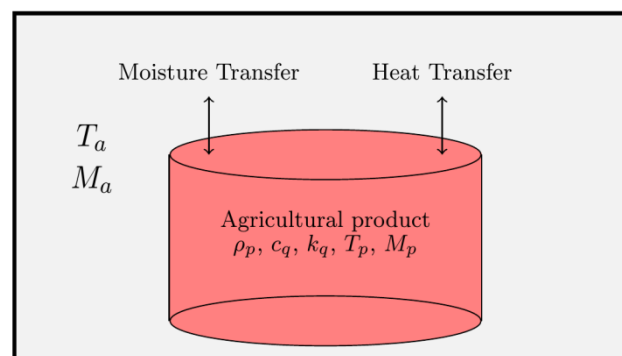
## 2. The Macroscopic governing equation

### 2.1. General distribution model

On freeze-drying medium, drying is usually described as a couple process of heat and mass transfer. In this study, the Luikov's couple system that simplified by Husain et, all [11] was used to simulate heat and mass transfer. Initial temperature and moisture content ( $T_0$ ,  $M_0$ ) of the agricultural product is uniform on the product (see fig 1).

In the model which will be presented here, the following consideration such as:

- There is no heat generation inside the product;
- The gas phase is water vapour, which behaves as an ideal gas;
- Thermo-physical properties of product are dependent on moisture;
- Heat transfer into the product through the entire surface by convection;
- Moisture content of product flowing out through the entire surface by diffusion;
- No chemical reaction takes place during drying, i.e. thermal and chemical properties of material, air and moisture are constant within the range of temperatures considered;
- Uniform distribution of air through the dryer;
- Gravity and shrinkage are negligible.



**Figure 1.** Schematic of physical model

## 2.2. Mathematical model

To simulate heat and mass transfer in freeze-dried medium, we use Luikov's equation that has been simplified by Husain, et, all [11]. As for the heat and mass transfer equation that are used as follows:

$$\rho_p c_t \partial_t T = \nabla \cdot (\kappa_t \nabla T) + L \rho_p \frac{\partial M}{\partial t} \quad x \in \Omega \quad (1)$$

$$\partial_t M = \nabla \cdot (D \nabla M) \quad x \in \Omega \quad (2)$$

where  $T$  is temperature of coffee (K),  $\rho_p$  is coffee bean density ( $\text{kg/m}^3$ ),  $c_t$  is the specific heat of coffee bean (J/kgK),  $\kappa_t$  is the thermal conductivity of coffee bean (W/ms),  $L$  is the latent heat of vaporization of water (J/kg),  $M$  is moisture content of coffee bean (%) and  $D$  is diffusion coefficient ( $\text{m/s}^2$ ).

## 2.3. Boundary condition and initial condition

According assumption in subsection of general distribution model. So, the boundary conditions associated with this system of equations were written is a generalized form as:

$$\kappa_t \left( \frac{\partial T}{\partial n} \right) + h_t (T - T_\infty) = 0 \quad \text{on } \Gamma_N \quad (3)$$

$$D \left( \frac{\partial M}{\partial n} \right) + h_m (M - M_\infty) = 0 \quad \text{on } \Gamma_N \quad (4)$$

$$D \left( \frac{\partial M}{\partial n} \right) = 0 \quad \text{on } \Gamma_D \quad (5)$$

$$\kappa_t \left( \frac{\partial T}{\partial n} \right) = 0 \quad \text{on } \Gamma_D \quad (6)$$

where  $n$  is the normal to the surface;  $T_\infty$  and  $M_\infty$  are the temperature and moisture of the air, respectively;  $h_t$  and  $h_m$  are the convection heat transfer coefficients ( $\text{W/m}^2\text{K}$ ) and the surface mass transfer coefficient ( $\text{kg/m}^2\text{s}$ ), respectively; and  $\Gamma_N$  and  $\Gamma_D$  are Neumann boundary and Dirichlet boundary, respectively.

For initial condition ( $t = 0$ ), temperature ( $T_0$ ) and moisture content ( $M_0$ ) of agriculture product are uniform:

$$T(x, y, z, 0) = T_0(x, y, z) \quad (x, y, z) \in \Omega \quad (7)$$

$$M(x, y, z, 0) = M_0(x, y, z) \quad (x, y, z) \in \Omega \quad (8)$$

On the model, one of the assumptions used were air is considered as an ideal gas; if the air pressure decreases, air density will decrease according to the ideal gas law:

$$\rho_a = \frac{p_a}{RT_a} \quad (9)$$

where  $\rho_a$  is the air density ( $\text{kg/m}^3$ ),  $p_a$  is the air pressure (Pa) and  $R$  is the air constant ( $8.314 \text{ J/Kmol}$ ).

In heat and mass simulation, heat and mass transfer coefficient for freeze dried medium were calculate using Nusselt number. The Chilton-Colburn analogy holds [5]:

$$Nu = 0.664 Re^{0.5} Pr^{0.33} \quad (10)$$

with

$$Re = \frac{\rho_a v_a L}{\mu}, Pr = \frac{c_a \mu_a}{\kappa_a}$$

$$Sc = \frac{\mu_a}{\rho_a D}, Le = \frac{v_a}{D}$$

### 3. Thermo-physical properties of coffee bean

In this section, we will show thermo-physical properties of the agricultural product that is used on simulation. Here, the kind of the agricultural products is coffee bean, then the parameters that are used entirely a physical parameter coffee.

**Table 1.** Thermo-physical properties of coffee bean.

Thermo-physical properties	Value	Ref
Density, $\rho_p$ ( $m^2/s$ )	$78.845M + 1723$	[7]
The thermal conductivity, $\kappa_t$ ( $W/mK$ )	$0.0116M + 0.062$	[7]
The thermal expansion coefficients, $\alpha$ ( $1/K$ )	$5 \times 10^{-9}M + 6 \times 10^{-8}$	[7]
The convection heat transfer coefficients, $h_t$ ( $W/m^2 K$ )	$\frac{3.26\rho_a c_a v_a}{Pr^{2/3}} Re^{-0.65}$	[10]
The surface mass transfer coefficient, $h_m$ ( $kg/m^2 s$ )	$\frac{h_t Pr^{2/3}}{\rho_a c_a Sc^{2/3}}$	[10]
The specific heat, $c_t$ ( $J/kgK$ )	$\frac{\kappa_t}{\rho_p \alpha}$	[10]
The latent heat of vaporization of water, $L$ ( $J/kg$ )	$2.3 \times 10^6$	[7]

**Table 2.** Thermo-physical properties of air .

Thermo-physical properties	Value	Ref
The kinematic viscosity, $\nu$ ( $m^2/s$ )	$16.97 \times 10^{-6}$	[16]
The air velocity, $v_a$ ( $m/s$ )	1	-
The specific heat, $c_a$ ( $J/kgK$ )	$1.005 \times 10^3$	[16]
The thermal conductivity, $k_a$ ( $W/mK$ )	0.0271	[16]
The dynamic viscosity, $\mu$ ( $Ns/m^2$ )	$\nu \mu$	[16]
The diffusion coefficient, $D$ ( $m^2/s$ )	$2.17 \times 10^{-5} \left( \frac{101.325}{P_a} \right) \left( \frac{T_\infty}{273.16} \right)^{1.88}$	[5]

### 4. Numerical methods

In simulation heat and mass transfer of coffee bean, we use finite element method with continuous piecewise linear (P1-elements) function to solve the governing equation (1 - 2). As for the software is already in use is FreeFEM++, which was created especially for the finite element method.



**Figure 2.** Grid profile of computational domain use GMSH software

To solve heat and mass transfer equation [1-2], we have to do a semi discretization in time. Let  $T^k, M^k, \rho_p^k, c_t^k, \kappa_t^k, D^k$  be approximation of  $T, M, \rho_p, c_t, \kappa_t, D$ . we set  $\Delta t > 0$  as a constant time step at

$t_k = k\Delta t$  ( $k = 0, 1, 2, \dots$ ). The numerical solution of heat and mass transfer equation ( $T^k, M^k$ ) are recomputed from ( $T^{k-1}, M^{k-1}$ ) with the implicit scheme:

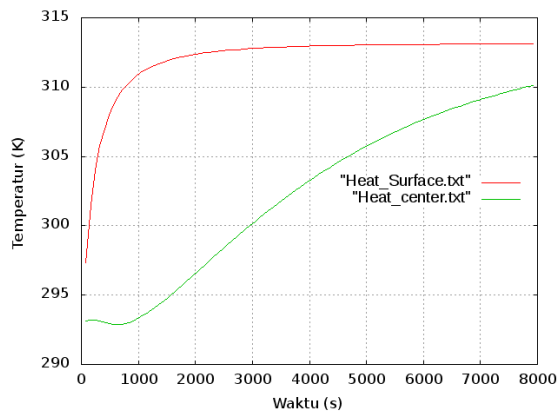
$$\rho^k c_t^k \left( \frac{T^k - T^{k-1}}{\tau_1} \right) = \nabla \cdot (\kappa_t^k \nabla T^k) + L \rho^k \left( \frac{M^k - M^{k-1}}{\tau_1} \right) \quad (11)$$

$$\rho^k c_t^k \left( \frac{T^k - T^{k-1}}{\tau_1} \right) = \nabla \cdot (\kappa_t^k \nabla T^k) + L \rho^k \left( \frac{M^k - M^{k-1}}{\tau_1} \right) \quad (12)$$

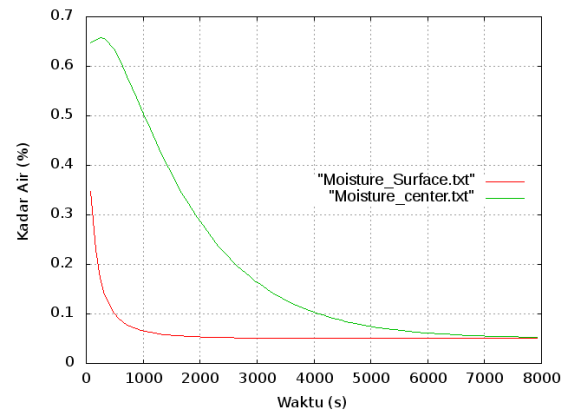
$$\kappa_t^k \frac{\partial T^k}{\partial n} = h_t (T_\infty - T^k) \quad \text{on } \Gamma_N \quad (13)$$

$$D^k \frac{\partial M^k}{\partial n} = h_m (M_\infty - M^k) \quad \text{on } \Gamma_N \quad (14)$$

$$\kappa_t^k \frac{\partial T^k}{\partial n} = 0, D^k \frac{\partial M^k}{\partial n} = 0 \quad \text{on } \Gamma_D \quad (15)$$



**Figure 3.** Predicted heat transfer between center and surface of coffee bean; pressure was kept at 20 kPa.



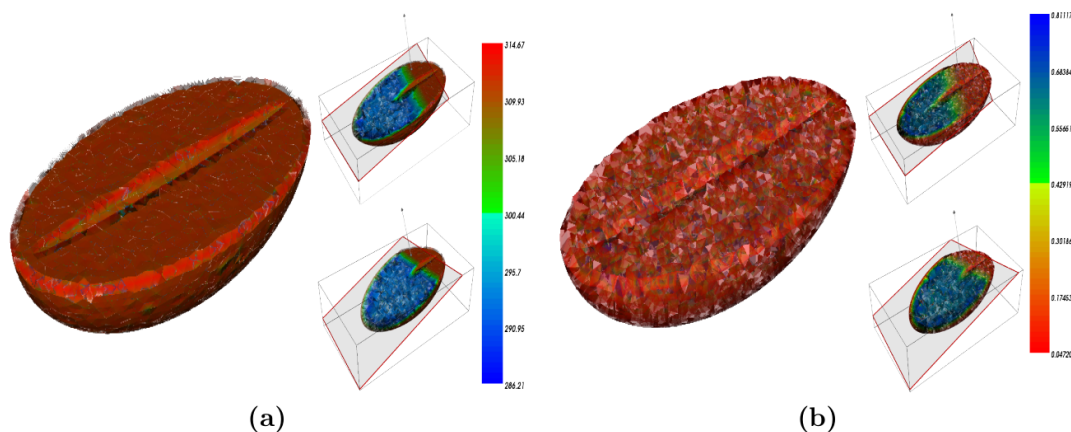
**Figure 4.** Predicted mass transfer between center and surface of coffee bean; pressure was kept at 20 kPa.

## 5. Numerical results and discussion

The effects of drying temperature and pressure on the freeze drying medium are shown in Figs. 3 and 4, in which drying curves (moisture ratio vs. time) and temperature evolution in the sample centre under different drying conditions are plotted.

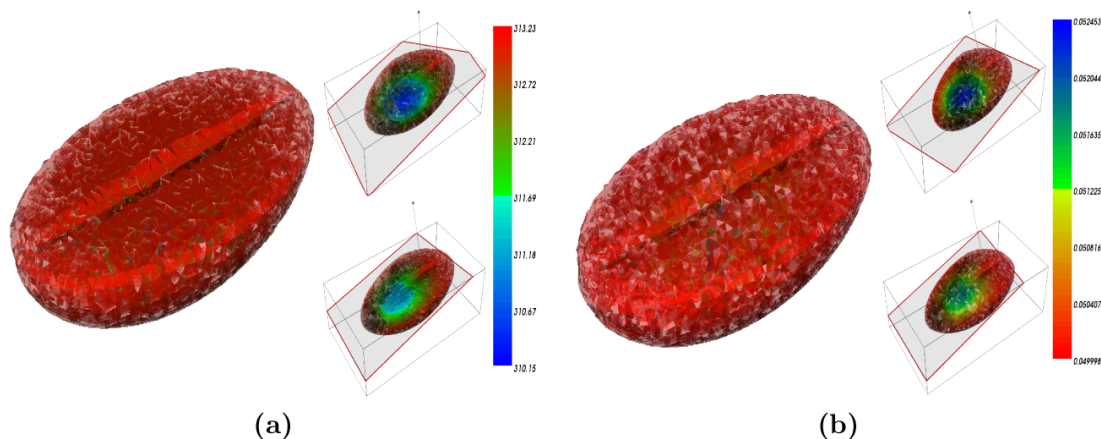
According figure 3 - 4, show that there are significant different temperature and moisture content between surface and centre of coffee. In figure 3, the rate of temperature very fast in the surface of coffee. The fastest rate of temperature increase occurs between time interval 0 – 1000 s. On this state, the temperature is 311 K. In centre of coffee bean, no significant changes of temperature. The evolution temperature significantly increase in the centre of product after 1000 s. However, temperature increase slowly in the surface of product. Temperature increase should be offset by a decrease in the moisture content of the coffee bean. If we see picture 4, this show similarities. the drying rate of coffee surface is very fast. At  $t = 2000$  s, moisture content is 0.05 % on surface. But in the same time, moisture content is 0.29 % on centre.

Physically, Drying phenomena occurs heat flow or transfer on coffee bean during time  $t$ . Beside that, thermo-physical parameters and pressure factor take effect on drying process. In this simulation of heat and mass transfer, time interval is 8000 s and air pressure is 20 kPa. The use of low air pressure can cause drying process more quickly.



**Figure 5.** Snapshot of distribution (a) temperature and (b) moisture content at time  $t = 40$  s.

Qualitatively, the graph output is satisfies to illustrate drying process of coffee bean. However, display distribution profile of heat and mass transfer on drying process is better. On figure 5 – 6 shown distribution profile of temperature and moisture content inside coffee bean at time  $t$ . Distribution profile of temperature and moisture content at  $t = 40$  s (see fig. 5a), temperature is 313 K in surface of coffee and 293 K in centre of coffee. In the same time, there is moisture content decrease, but the rate of decline was not significant. Moisture content in surface and centre of coffee is 0.04 % and 0.6 %, respectively. Moisture content distribution is almost uniform inside coffee bean at  $t = 8000$  s, it is like that shown in the figure 6. As well as to the temperature distribution is almost same but it is not maximum in the centre of product. If we see figure 5 - 6, show that geometry of coffee bean is not smooth, this is because the number of triangle or grid slightly and also type of element is continuous piecewise linear



**Figure 6.** Snapshot of distribution (a) temperature and (b) moisture content at time  $t = 8000$  s.

## 6. Conclusion and future work

The finite element method (FEM) has been implemented to solve heat and mass transfer model in freeze dried medium. The method already successful to simulate drying phenomena. The method is also very easy to simulate phenomena on complex domain case. From this result also, we get that increase of temperature in the surface is very fast than in the centre of coffee. For drying process, in the surface is very vast that in the centre of coffee.



## Acknowledgements

I would like to acknowledge the financial support in the form of *Bantuan Kerja Mahasiswa* (BKM) from Institute of Technology Bandung (ITB) so that the research have done. I also express appreciations to the entire FreeFEM++ authors; Frédéric Hecht, Sylvian Auliac, Oliver Pironneau, Jacques Morice, Antoine Le Hyaric and Kohji Ohtsuka, for their help when first use this program.

## References

- [1] Anonim 2015 *Rencana Strategis Kementerian Pertanian Tahun 2015-2019* 24-25. (Jakarta: Kementerian Pertanian)
- [2] B. Dradjat, A. Agustian and A. Supriatna 2007 *Ekspor dan daya saing kopi biji Indonesia di pasar internasional: implikasi strategis bagi pengembangan biji kopi organis* (Pelita Perkebunan) p 159-179.
- [3] K. Haghighi, and L.J. Segerlind 1988 *Modeling simultaneous heat and mass transfer in an isotropic sphere-a finite element approach* American Society of Agricultural Engineers Vol. 31 p 629-637.
- [4] Sh. Rafiee, M. Kashaninejad, A.R. Keyhani and A. Jafari 2009 *Pistachio nut (ohadi variety) mass transfer simulation during process of drying using finite element method* J. Agriculture Science Technology Vol. 11 p 137-146.
- [5] F. Nadi, G.H. Rahimi, R. Younsi, T. Tavakoli, and Z. Hamidi Esfahani 2012 *Numerical Simulation of Vacuum Drying by Luikov's Equation* (Drying Technology Taylor & Francis) p 197-206
- [6] Allan L. Phillips 1965 *Drying coffee with solar-heated air* Solar Energy Vol. 9 Issue 4 p 213-216
- [7] Angelo Fabbri, et all 2011 *Numerical Modeling of Heat and Mass Transfer During Coffee Roasting Process* Journal of Food Engineering Vol. 205 p 264-269
- [8] Jafar F and Farid M *Analysis of heat and mass transfer in freeze-drying* Drying Technology Vol. 21 p 249-263
- [9] Haiqing Chen, et all 1999 *Modeling Coupled Heat and Mass Transfer for Convection Cooking of Chicken Patties* J. of Food Engineering Vol. 24 p 139-146
- [10] Hector J. Ciro-Velasquez, Luis C. Abud-Cano and Luis. R. Perez-Alegria 2010 *Numerical Simulation of Thin Layer Coffee Drying by Control Volumes* Dyna Vol. 163 p 270-278
- [11] Husain A, Sun C.C and Clayton J.T 1973 *Simultaneous Heat and Mass Diffusion in Biological Materials* Journal Agricultural Engineering Research Vol. 18 p 343-354
- [12] Luikov, A.V 1966 *Heat and mass transfer in capillary-porous bodies* (New York: Pregamon Press)
- [13] A. Kovács, E. Lakatos, G. Milics dan M. Neményi 2010 *Finite Element Modeling of Coupled Heat and Mass Transfer of a Single Maize Kernel Based on Water Potential Using COMSOL Multiphysics Simulation* The Proceedings of The COMSOL Conference
- [14] Miklos Nemenyi, et all 2000. *Investigation of Simultaneous Heat and Mass Transfer within The Maize Kernels During Drying* J. Computers and Electronics in Agriculture p 123-135.
- [15] Michael J. Moran and Howard N. Shapiro 2004 *Termodinamika Teknik, Edisi 4* (Jakarta: Erlangga) p 229
- [16] [http://www.engineeringtoolbox.com/air-properties-d\\_156.html](http://www.engineeringtoolbox.com/air-properties-d_156.html)