

# Simulation of Fluid Flow and Heat Transfer in Porous Medium Using Lattice Boltzmann Method

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**Abstract.** Fluid flow and heat transfer in porous medium are an interesting phenomena to study. One kind example of porous medium is geothermal reservoir. By understanding the fluid flow and heat transfer in porous medium, it help us to understand the phenomena in geothermal reservoir, such as thermal change because of injection process. Thermal change in the reservoir is the most important physical property to known since it has correlation with performance of the reservoir, such as the electrical energy produced by reservoir. In this simulation, we investigate the fluid flow and heat transfer in geothermal reservoir as a simple flow in porous medium canal using Lattice Boltzmann Method. In this simulation, we worked on 2 dimension with nine vectors velocity (D2Q9). To understand the fluid flow and heat transfer in reservoir, we varied the fluid temperature that inject into the reservoir and set the heat source constant at 410°C. The first variation we set the fluid temperature 45°C, second 102.5°C, and the last 307.5°C. Furthermore, we also set the parameter of reservoir such as porosity, density, and injected fluid velocity are constant. Our results show that for the first temperature variation distribution between experiment and simulation is 92.86% match. From second variation shows that there is one pick of thermal distribution and one of turbulence zone, and from the last variation show that there are two pick of thermal distribution and two of turbulence zone.

## 1. Introduction

Fluid flow and convection heat transfer of porous medium have been studied by many researchers in different field of science and engineering including geothermal engineering, civil engineering and mechanics, chemistry and petroleum engineering, hydrology, and nuclear especially in cooling management. In geothermal engineering, heat transfer and fluid flow are very important thing to understand, because it has correlation with electrical energy produced by the reservoir. The property of fluid flow and heat transfer in porous medium had been studied and modeled it with some numerical methods. The numerical method used to simulate is divided into three groups i.e. macroscopic modeling (FDM, NS, FEM), microscopic modelling (MD, DSMC), and meso-scopic modelling (LBM, LGA, SPH). The meso-scopic modeling is a relatively new method that could bridge between two approaches macro-scale and micro-scale. In the meso-scopic modelling (LBM) stated that the behavior of group particles into the behavior of a single particle. Therefore, we do not need to declare the behavior of each particle. The behavior of a group particle is stated by distribution function. The distribution function fulfilled for this case is Boltzmann Distribution.

LMB has several advantages, two of these are it does not need computer with high of RAM and it does not need to solve Poisson equation for each iteration, therefore the running process is more faster. In this study, we simplify the process of fluid flow and heat transfer in geothermal reservoir as a simple



flow in porous medium canal with varying of fluid temperature injected, then we compare the result of simulation with experiment results by Olimpia Banete (2014).

## 2. Lattice Boltzmann Method

### 2.1. Basic Concept of Lattice Boltzmann Method

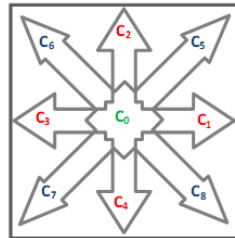
LBM is a kind of numerical method that can be used to simulate fluid flow. This method can bridge between two approaches macro-scale and micro-scale. In this method the behavior of group particles is stated into the behavior of a single particle. Therefore, we do not need to declare the behavior of each particle. The behavior of a group particle stated by distribution function. The distribution function fulfilled for this case is Boltzmann Distribution. The function is expressed by equation:

$$f_a(x + e_a \Delta t, t + \Delta t) = f_a(x, t) - \frac{\Delta t}{\tau} (f_a(x, t) - f_a^{eq}(x, t)) \quad (1)$$

where  $f_a(x, t)$  is Boltzmann distribution function to calculate density and velocity field,  $\tau$  is time relaxation that has correlated with kinematic viscosity  $\nu$ ,  $f_a(x + e_a \Delta t, t + \Delta t)$  is velocity distribution function at intervals time  $t + \Delta t$ , and  $f_a^{eq}(x, t)$  is distribution function in equilibrium state.

$$f_a^{eq}(x, t) = w_a \rho \left[ 1 + \frac{e_a \cdot u^{eq}}{c_s^2} + \frac{(e_a \cdot u^{eq})^2}{2c_s^4} - \frac{(u^{eq})^2}{2c_s^2} \right] \quad (2)$$

In this simulation, we used lattice model D2Q9, the figure of model as follow.



**Figure 1.** Lattice D2Q9 structure (Sukop, 2007)

For this model, we define  $e_a$  as :

$$e_a = \begin{cases} (0, 0) \rightarrow, a = 0 \\ \varphi_a c \left[ \cos \frac{(a-1)\pi}{4}, \sin \frac{(a-1)\pi}{4} \right] \rightarrow, a \neq 0 \end{cases} \quad \text{With } \varphi_a = \begin{cases} 1 \rightarrow, a = 1, 2, 3, 4 \\ \sqrt{2} \rightarrow, a = 5, 6, 7, 8 \end{cases} \quad (3)$$

Density and velocity define as

$$\rho = \sum_a f_a u_i = \frac{1}{\rho} \sum_a f_a c_{ai} u_i \quad (4)$$

### 2.2. LBM for heat transfer in pore medium

Niathiarasu et al. (1997) assumed if the Boussinesq in limit value are fulfilled and there is thermal equilibrium near fluid and solid, then the equations used to describe the behavior of incompressible fluid flow and convection proses in porous medium as follow:

$$\nabla \cdot \mathbf{u} = 0 \quad (5)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \left( \frac{\mathbf{u}}{\varepsilon} \right) = -\frac{1}{\rho f} \nabla(\varepsilon p) + \nu_e \nabla^2 \mathbf{u} + \mathbf{F} \quad (6)$$

$$\sigma \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot (\alpha_m \nabla T) \quad (7)$$

where coefficient  $\sigma$  is ratio of thermal capacity for solid and liquid,  $\mathbf{u}$  is discharge,  $p$  is pressure,  $\nu_e$  is effective viscosity,  $\mathbf{F}$  is total external force experienced by porous medium. Guo and Zhao (2005) defined the total of external force experienced by the medium as follow:

$$\mathbf{F} = -\frac{\varepsilon \nu}{K} \mathbf{u} - \frac{\varepsilon F_\varepsilon}{\sqrt{K}} |\mathbf{u}| \mathbf{u} + \varepsilon \mathbf{G} \quad (8)$$

where  $\nu$  is fluid viscosity,  $\mathbf{G}$  is gravitational force. The gravitational force  $\mathbf{G}$  is defined as follow :

$$\mathbf{G} = -g\beta(T - T_0) + \mathbf{a} \quad (9)$$

where  $g$  is gravitational acceleration,  $\beta$  is thermal expansion coefficient,  $T_0$  initial temperature,  $\mathbf{a}$  acceleration because of external force. Ergun (1952) developed a geometry function  $F$  and permeability  $K$  as a function of porosity based on their experiment. The function is defined as follow:

$$F_\varepsilon = \frac{1.75}{\sqrt{150\varepsilon^3}} \quad \text{and} \quad K = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2} \quad (10)$$

### 2.3. LBM for velocity field

Guo and Zhao (2002) developed the equation for fluid flow in porous medium defined as follow:

$$f_a(x + e_a \delta_t, t + \delta_t) - f_a(x, t) = -\frac{1}{\tau_v} [f_a(x, t) - f_a^{eq}(x, t)] + \delta_t F_a \quad (11)$$

Where  $\delta_t F_a$  is total force experienced by porous medium or total force experienced in the system. Guo and Zhao (2002) defined the distribution equation in equilibrium state for DnQb model as follow:

$$F_a = w_a \rho \left( 1 - \frac{1}{2\tau_v} \right) \left[ \frac{e_a \cdot \mathbf{F}}{c_s^2} + \frac{\mathbf{u} \cdot \mathbf{F} : (e_a e_a - c_s^2 \mathbf{I})}{\varepsilon c_s^4} \right] \quad (12)$$

Based on the equations above (Guo and Zhao),  $\mathbf{F}$  has non-linear correlation with  $\mathbf{u}$ . Guo and Zhao (2002) also defined the density and fluid velocity as follow:

$$\rho = \sum_a f_a \quad \text{and} \quad \mathbf{u} = \frac{\mathbf{v}}{c_0 + \sqrt{c_0^2 + c_1 |\mathbf{v}|}} \quad (13)$$

where  $\mathbf{v}$  is

$$\rho \mathbf{v} = \sum_a e_a f_a + \frac{\delta_t}{2} \varepsilon \rho \mathbf{G} \quad (14)$$

$c_0$  and  $c_1$  are defined as

$$c_0 = \frac{1}{2} \left( 1 + \varepsilon \frac{\delta_t}{2} \frac{F_\varepsilon}{\sqrt{K}} \right) \quad \text{and} \quad c_1 = \varepsilon \frac{\delta_t}{2} \frac{F_\varepsilon}{\sqrt{K}} \quad (15)$$

## 2.4. Temperature Field Equation

He et al 1998 found the distribution function for equilibrium state. The equation is defined

$$g^{eq} = \frac{\rho e}{(2\pi RT)^{D/2}} \exp\left(-\frac{e^2}{2RT}\right) \left[ \frac{e^2}{DRT} + \left(\frac{e^2}{DRT} - \frac{2}{D}\right) \frac{(e.u)}{RT} + \frac{(e.u)^2}{2(RT)^2} - \frac{u^2}{2RT} \right] +$$

$$\frac{\rho e}{(2\pi RT)^{D/2}} \exp\left(-\frac{e^2}{2RT}\right) \left[ \left(\frac{e^2}{DRT} - \frac{D+4}{D}\right) \frac{(e.u)^2}{2(RT)^2} - \left(\frac{e^2}{DRT} - \frac{D+2}{D}\right) \frac{u^2}{2RT} \right] \quad (16)$$

For D2Q9 model, Seta et al 2006 used equation 17 until 19 to describe the distribution of density in equilibrium state which is the value of the function is discrete.

$$g_0^{eq} = -\frac{2\rho e u^2}{3c^2} \quad (a=0) \quad (17)$$

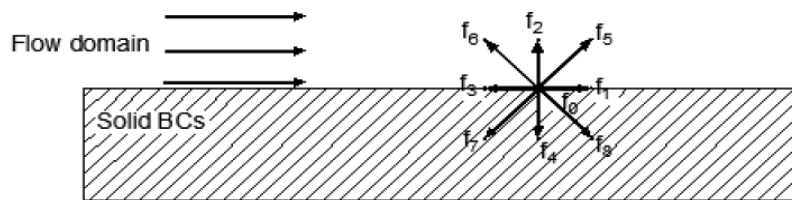
$$g_{1-4}^{eq} = \frac{\rho e}{9} \left[ \frac{3}{2} + \frac{3}{2} \frac{e_a . u}{c^2} + \frac{9}{2} \frac{(e_a . u)^2}{c^4} - \frac{3}{2} \frac{u^2}{c^2} \right] \quad (a=1,2,3,4) \quad (18)$$

$$g_{5-8}^{eq} = \frac{\rho e}{36} \left[ 3 + \frac{6e_a . u}{c^2} + \frac{9}{2} \frac{(e_a . u)^2}{c^4} - \frac{3}{2} \frac{u^2}{c^2} \right] \quad (a=5,6,7,8,9) \quad (19)$$

Internal energy has correlation with temperature,  $e = 3RT/2$  where R is ideal gas constant (R=8.314 J/mol-K).

## 2.5. Boundary Condition

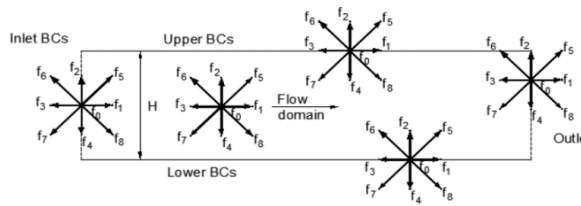
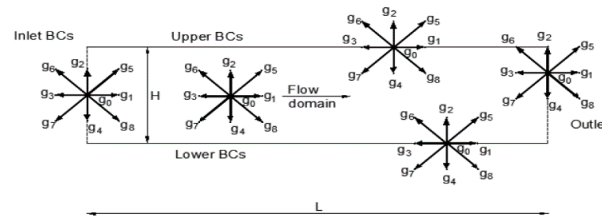
In this simulation, we used many kind of boundary condition, such as bounce back boundary condition and inlet velocity boundary condition. The particles bounce when those hit the wall in back boundary condition. This is a simple boundary condition to make a solid formation in the system.



**Figure 2.** Bounce Back boundary Condition (Muhammed,2011)

By used inlet velocity boundary condition this boundary condition, we could take some assumption that there is no velocity in y-direction. The velocity in x-direction equal to 0 when the particle hit the wall. After the streaming process, there are unknown distribution function  $f_1, f_5, f_8$  and also the density. To get the distribution function we used non-equilibrium equation (4). Temperature distribution function for lattice model D2Q9 defined

$$T = \sum_{g=0}^8 g_a = g_0 + g_1 + g_2 + g_3 + g_4 + g_5 + g_6 + g_7 + g_8 \quad (20)$$

**Figure 3.** velocity Boundary Condition**Figure 4.** Temperature Boundary Condition

### 3. Model and Simulation Algorithm

The simulations were performed in this study divided in to two categories, the first categories is single phase fluid flow simulation for poissellie flow and heat convection, and second is simulation of fluid flow and heat transfer in porous medium.

#### 3.1. Single Phase Fluid Flow

In the first category, we designed the simulation which the fluid flows in a pipe with both ends open to each other, otherwise it is assumed that there is no friction between the fluid and the pipe and there is no interaction between fluid particles. The fluid in the system is non-viscous and incompressible fluid. In this simulation we used Lattice Boltzmann Method D2Q9 to described the fluid flow in the cylinder. We used some assumption i.e. the interaction between particles only collision, there is no external force, and flow occurs just because of the differences pressure between inlet and outlet. We used bounce back boundary condition on the cylinder wall and assumed no slip. While in inlet and outlet of the cylinder we used periodic boundary conditions, therefore the fluid can flow continuously. The dimension of the models we used 400x100 lattice unit. The Reynolds number is 100, density is 1, and the inlet pressure is 0.005. The time relaxation is defined by this formula  $\tau = 3\nu / 2$  with  $\nu = 2v_{maks} / \text{Re}$ . From this simulation we plot the velocity profile, then we compare the result to the theory.

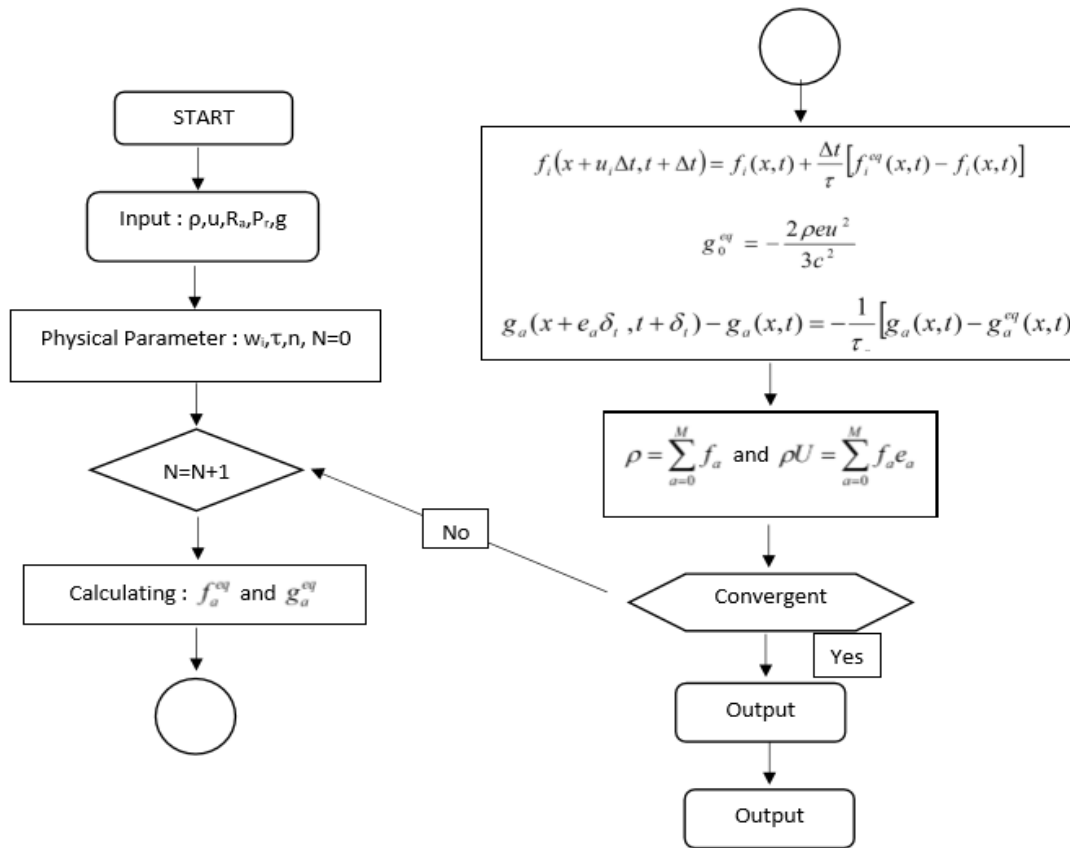
#### 3.2. Natural Convection in the Water

The second category is simulation of natural convection in the water. In this simulation, we used two distribution function, there are distribution function for liquid particle and distribution function for temperature, and we also used two kind of lattice model; there are D2Q9 for particle and D2Q5 for temperature. Both of distribution function we stacked together in the system. We assumed that the interaction between particles on the system is only collision, no external forces except gravity, and flow occurs because of the differences density between cold and hoot water. We used three kind of boundary conditions; there are no-slip boundary condition, periodic boundary condition, and streaming boundary condition. In this simulation we composed the model into dimension 200x100 lattice unit, the Rayleigh number 20000. In this condition, heat source comes from the bottom wall of the system. We compared the result of this simulation with another simulation used FDM that has been done by Septian Setyoko to see the pattern of distribution temperature.

#### 3.3. Fluid Flow and Herat Transfer in Porous Medium

In this simulation, firstly we simulate the fluid flow and heat transfer in porous medium with heat source comes from left wall, then we simulate fluid flow and heat transfer with various fluid temperature injected into the system and we set the heat source (comes from bottom wall) at constant temperature. From this simulation we plot the profile of temperature distribution.

### 3.4. Simulation Algorithm

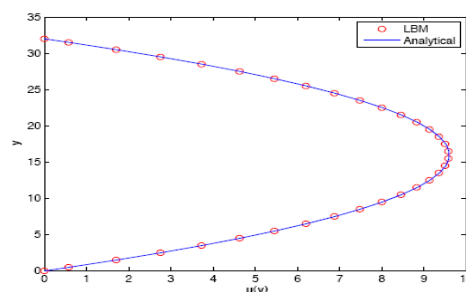


**Figure 5.** Simulation Algorithm

## 4. Results and Discussion

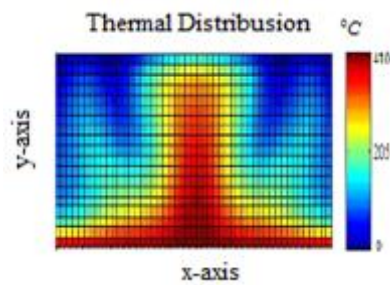
### 4.1. Single Phase Fluid Flow

From the simulations for the Poiseuille flow, we got the result as follows.

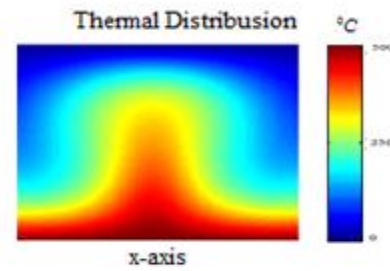


**Figure 6.** Velocity profile for poiseuille flow in the pipe (analytic and simulation) .

In Figure 5 we known that from simulation and the analytic result are the same. The fluid velocity profile has the shape of hyperbolic functions where the highest rate velocity located in the center of the pipe. Further validation process by comparing the visualization result from the simulation using LBM and Finite Difference method that has been done by Septian Setyoko (2012). The pattern of the temperature distribution results can be seen in the picture below.



**Figure 7a.** Temperature distribution using FDM

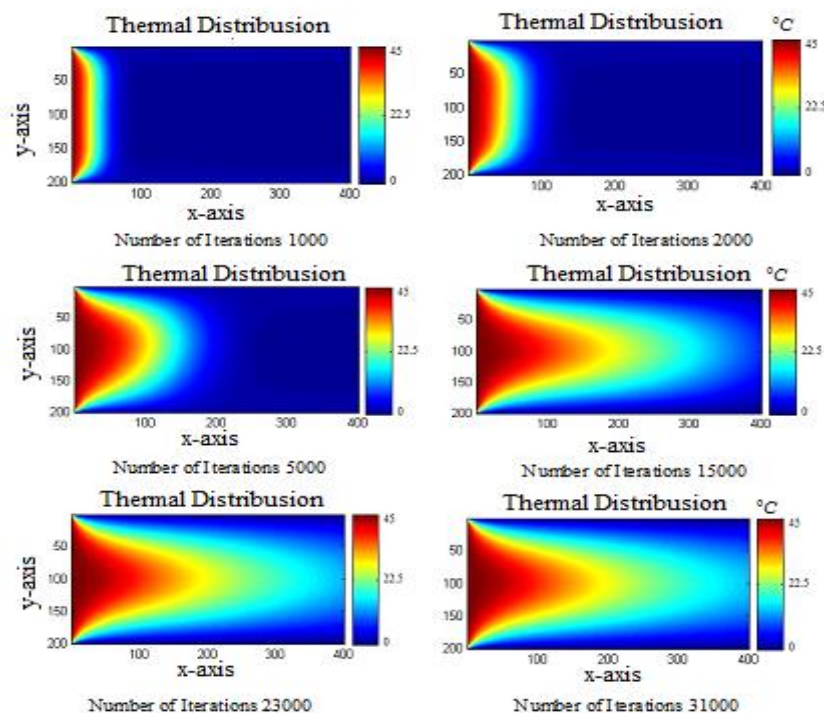


**Figure 7b.** Temperature distribution using LBM

From the simulation, using FDM and LBM we got similar profile of temperature distribution. The big difference is likely due to differences in physical parameters used in the simulation. Even though we can use the LBM to approach heat convection process in a porous medium.

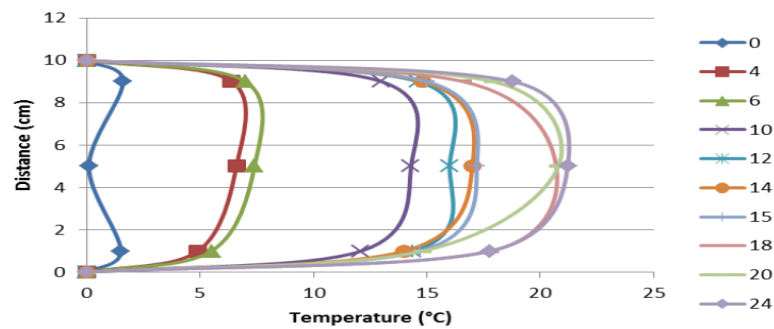
#### 4.2. Fluid Flow and Herat Transfer in Porous Medium

After the simulations with analytical validation was done, then we simulate heat transfer in a porous medium when the heat source comes from left wall and also simulate heat transfer in a porous medium with variation temperature fluid injected where the source comes from the bottom. Firstly we simulate fluid flow and heat transfer where the heat source comes from the left of the wall. The temperature on the left wall we set at  $45^{\circ}\text{C}$ , while the upper and lower walls we set and we maintained into  $0^{\circ}\text{C}$ .



**Figure 8.** Temperature distribution profile in porous medium as convection proces used LBM with temperature fluid flow  $45^{\circ}\text{C}$  and upper bottom walls temperature  $0^{\circ}\text{C}$

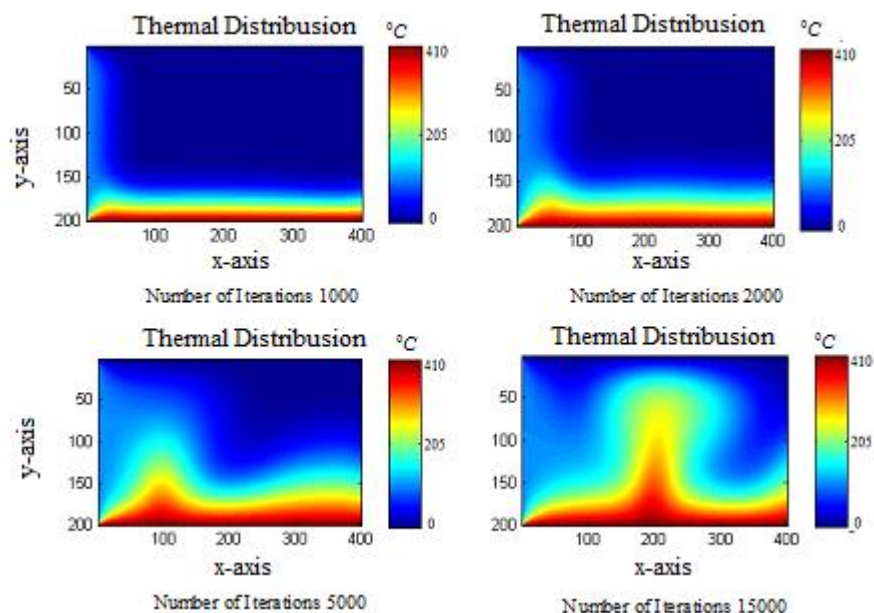




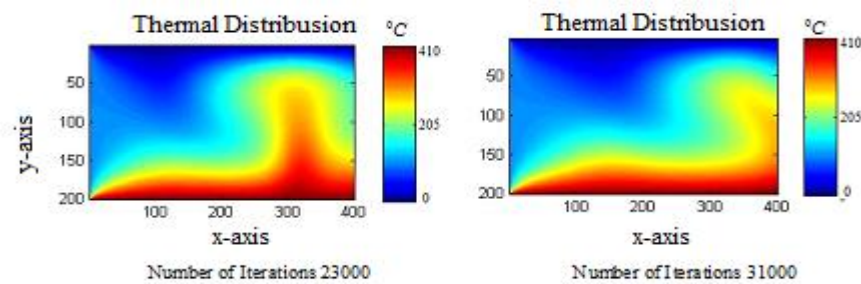
**Figure 9.** Experiment result of thermal temperature distribution profile in porous medium as convection process by Olimpia Banete.

From the simulations and experiments conducted by Olimpia Banete, we have the similar profile. The profile has a parabolic shape with a peak on the right. If we compare it, the peak has a value almost the same,  $22.5^{\circ}\text{C}$  for the experiments and  $21^{\circ}\text{C}$  for the simulation, but there is a striking difference around the walls. In the simulation, temperature distribution around the wall rise rapidly, while the experimental relatively slow to rise.

Secondly, we simulate fluid flow and heat transfer where the heat source comes from the bottom wall, in this case, there is a hot fluid injection from the left wall. We set the temperature at bottom wall constant at  $410^{\circ}\text{C}$  and injection fluid temperature at  $102.5^{\circ}\text{C}$  with the rate injection  $0.02\text{m/s}$ . Their rate of injection in the left wall intended to approximate the behavior of the temperature distribution at the time of the geothermal reservoir injection process. This process is very important since it has relationship to the energy that would be generated by the reservoir. From the simulation, temperature distribution profile is obtained as follows.

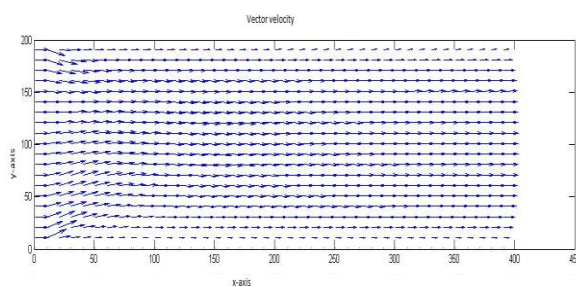




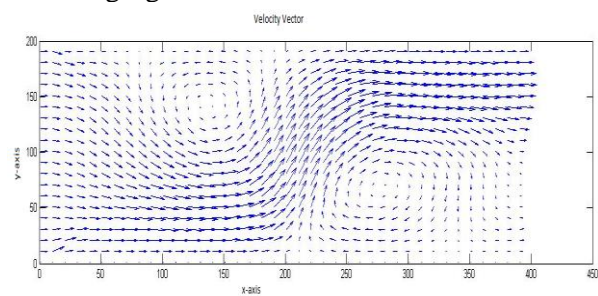


**Figure 10.** temperature distribution profile with heat source 410  $^{\circ}\text{C}$  and injection fluid temperature 102.5  $^{\circ}\text{C}$ . y-axis and x-axis is dimension of system .

From figure above, in 1000 number of iteration, the temperature does not rise significantly from the bottom and sides. In 5000 number of iteration, there is a peak of temperature distribution due to the injection process and the heating continuously from the bottom walls. The results show that the peak temperature distribution getting higher and shifting toward to the right. The high peak temperature distribution affects the pattern of fluid flow in the system. At low temperature, fluid flows in a laminar pattern, while at the high temperatures the fluids flow would be turning into turbulent pattern. it could be seen from the velocity vector of fluid flow in the following figure.

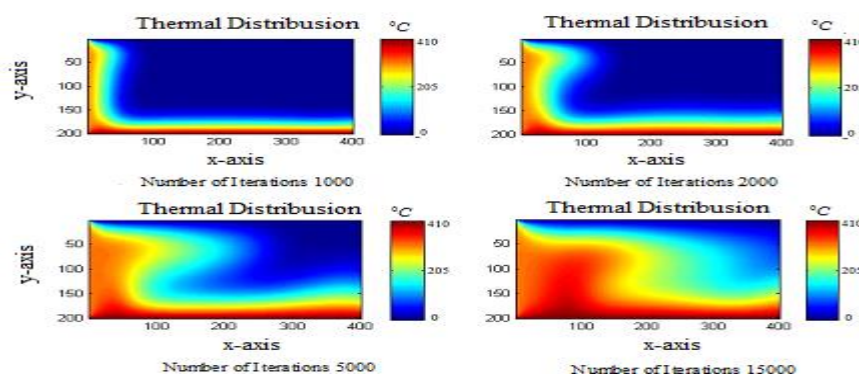


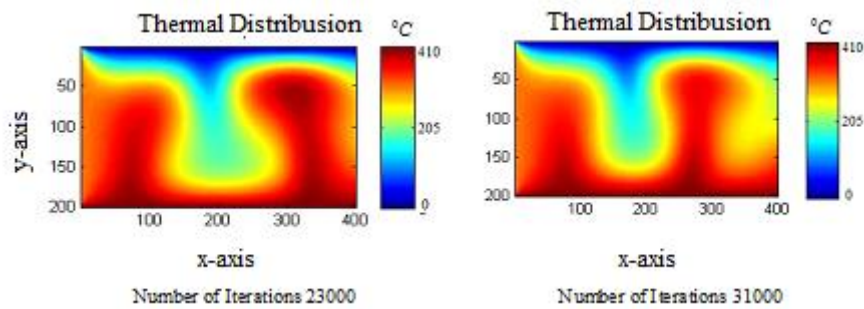
**Figure 11a.** Vector velocity in low temperature



**Figure 11b.** Vector velocity in low temperature

For the third simulation we set the bottom temperature into 410  $^{\circ}\text{C}$  and injection fluid temperature 307.5  $^{\circ}\text{C}$  with the rate of injection 0.02m/s. The simulation process obtained the temperature distribution profile as show in figure 11. From the simulation, the temperature increases rapidly and the profile almost different from the previews due to the differences fluid temperature injection with the previews simulation is significant, around 200 degrees. At 15000 number of iteration, there are two peak temperature. The first peak occurs due to a combination temperature from heat source and fluid temperature injection, while the second peak temperature comes from warming the fluid that has a higher temperature than the initial temperature.





**Figure 12.** Temperature distribution profile with heat source  $410^{\circ}\text{C}$  and injection fluid temperature  $102.5^{\circ}\text{C}$ .

## 5. Conclusion

From this simulation, we could conclude that the model composed by LBM can be used to solve fluid flow and heat transfer in a porous medium problems. The first simulation simulate fluid flow in a pipe, from the simulation that there are similarities with the theory of fluid flow profile for Poiseuille flow. The second simulation simulate natural water convection which is the heat source comes from the bottom of the wall. The results using LBM have the same profile with the Finite Difference Method, it can be concluded that LBM could also be used to simulate the heat flow in the fluid. In the simulation of fluid flow and heat transfer in a porous medium provides the results of the temperature distribution profile similar to the experiment, with the rate reaching 92.8%, it show in the temperature distribution profile.

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