

Simulation of Heat Transfer in Husk Furnace with Cone Geometry Based on Conical Coordinate System

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Abstract. Simulation of Heat Transfer in Husk Furnace with Cone Geometry Based on Conical Coordinates has been performed. This simulation aimed to study the heat distribution of temperature based on conduction and convection mechanism on conical coordinate system. Fluid dynamics inside the cone of husk furnace was obtained by solving the Navier - Stokes equations with laminar flow approach. The initial temperature in all parts of the cone is room temperature, except at the bottom of the cone is 700 °C. Through numerical calculation of heat conduction and convection equation by FDM method, we got that the velocity of fluid flow at the center cone is 13.69 m/s for 45 s, 11.90 m/s for 60 s, and 7.25 m/s for 120 s, with unfixed temperature condition in the cone.

1. Introduction

Husk furnace is a conical stove which uses fuel from rice husk. There are two component geometry in husk furnace that are cone part and cylinder part [1] (see Fig 1). Cylinder part function's used as heat source of a stove. While the cone function is the part of furnace that will ensure the heat is delivered well [2]. Heat distribution of temperature in the cone husk furnace can be determined theoretically by studying heat transfer based on conduction and convection process. Free convection flow in heat transfer can be solved by laminar model. The Graetz problem that we knew as the heat transfer problem of laminar fluid flow in ducts [3]. The Graetz problem could be studied with axial diffusion in circular tube, using a semi- infinite domain formulation with a specified inlet condition [4]. While the effects of axial diffusion in a infinite domain formed by an insulated preparation region followed by an isothermal wall was analyzed [5]. Noor [1] studied the heat transfer husk furnace in cylinder coordinate with laminar approach. Srinivasa [6] studied unsteady free convection flow and heat transfer from an isothermal truncated cone. Cone had been made by cylinder coordinate [7]. Except that to describe a cone can be modeled by conical coordinate system [8]. Fluid dynamics inside the cone husk furnace can be obtained by solving the Navier-Stokes equations to determines how convection works [9]. The pressure gradient of the fluid inside the cone can be derived from temperature gradient or fluid density gradient [1]. The aim of this research is to study the heat distribution of cone's temperature husk furnace



based on conduction and convection mechanism in conical coordinate system with laminar fluid flow approach.

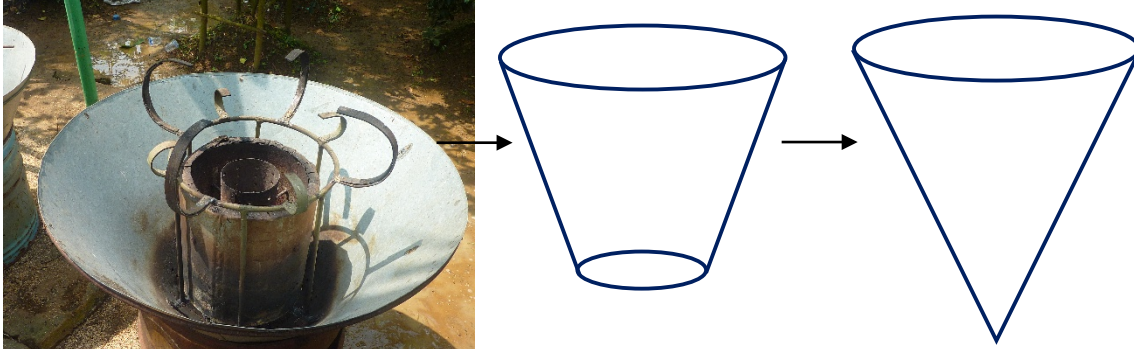


Figure 1. Husk Furnace and it's considered geometrical shape for the correspondingly numerical modeling

2. Result and Discussion

2.1 Mathematical Modelling of Heat Transfer in Conical Coordinate

Heat transfer on the cone husk furnace is associated with the phenomenon of conduction and convection [10]. The base governing equation for the heat transfer can be expressed as

$$\rho C_p \frac{\partial T}{\partial t} + \vec{U} \nabla T = k \nabla^2 T \quad (1)$$

With \vec{U} is fluid flow velocity, ρ is air density, C_p is constant of material specific heat, k is constant of conductivity thermal matter, α is constant of diffusivity thermal matter, and T is temperature on conical [8].

The term on the right side of Equation (1) corresponds to the heat conduction. The Laplacian on the equation in conical coordinate is written as :

$$\begin{aligned} \nabla^2 T = & \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \\ & + \frac{1}{r^2(\theta^2 - \lambda^2)} \left\{ (\theta^2 - b^2)(c^2 - \theta^2) \frac{\partial^2 T}{\partial \theta^2} - \theta[2\theta^2 - (b^2 + c^2)] \frac{\partial T}{\partial \theta} \right. \\ & + (b^2 - \lambda^2)(c^2 - \lambda^2) \frac{\partial^2 T}{\partial \lambda^2} \\ & \left. + \lambda[2\lambda^2 - (b^2 + c^2)] \frac{\partial T}{\partial \lambda} \right\} \end{aligned} \quad (2)$$

While, the second term on left side of the equation corresponds to the heat convection and in conical coordinate the term can be written as:

$$\begin{aligned} \vec{U} \nabla T = & \vec{U}_r \frac{\partial T}{\partial r} + \frac{1}{r(\theta^2 - \lambda^2)^{\frac{1}{2}}} \left\{ \vec{U}_\theta (\theta^2 - b^2)^{\frac{1}{2}} (c^2 - \theta^2)^{\frac{1}{2}} \frac{\partial T}{\partial \theta} \right. \\ & \left. + \vec{U}_\lambda (b^2 - \lambda^2)^{\frac{1}{2}} (c^2 - \lambda^2)^{\frac{1}{2}} \frac{\partial T}{\partial \lambda} \right\} \end{aligned} \quad (3)$$

With $\overline{U_r}$ is fluid flow velocity for r radius direction and $\overline{U_\theta}$ is fluid flow velocity for θ direction. $\overline{U_\lambda}$ is fluid flow velocity for λ degrees direction. Cause of fluid flow velocity that used is r radius direction than the others direction assumed is 0, so from equation (3) can simplified be :

$$\nabla T = \overline{U_r} \frac{\partial T}{\partial r} \quad (4)$$

From equation (2) and (4), we combine it to equation (1), so the equation be :

$$\begin{aligned} \frac{dT}{dt} = \alpha \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right. \\ \left. + \frac{1}{r^2(\theta^2 - \lambda^2)} \left\{ (\theta^2 - b^2)(c^2 - \theta^2) \frac{\partial^2 T}{\partial \theta^2} - \theta[2\theta^2 - (b^2 + c^2)] \frac{\partial T}{\partial \theta} \right. \right. \\ \left. \left. + (b^2 - \lambda^2)(c^2 - \lambda^2) \frac{\partial^2 T}{\partial \lambda^2} + \lambda[2\lambda^2 - (b^2 + c^2)] \frac{\partial T}{\partial \lambda} \right\} \right) - \frac{1}{\rho C_p} \left(\overline{U_r} \frac{\partial T}{\partial r} \right) \end{aligned} \quad (5)$$

2.2 Fluid dynamics

Fluid dynamics inside the cone husk furnace was obtained by solving the Navier-Stokes equations with laminar flow approach. Analytical solution of the equation for laminar flow case is given by [10]:

$$\begin{aligned} U_r(r) \\ = \frac{\beta}{4\mu} (a^2 \\ - r^2) \end{aligned} \quad (6)$$

Where boundary condition at the wall is :

$$U_r(0) = \text{finite} ; r = 0$$

$$U_r(a) = 0 ; r = a$$

With **a** is constant which have same value with **r**.

2.3 Numerical Calculation of Heat Conduction and Convection equation in Conical

Numerical calculation of heat conduction and convection equation in conical coordinates can be solved by Finite Difference Method (FDM) based on forward and center formulas [11]. Finite difference method is one of numerical technique to solved differential equation problem [1]. The initial temperature in all parts of the cone is at room temperature, except the bottom of the cone is at 700 °C. There are two times simulation that used is 45 and 60 second with unfixed cone blanket temperature case. For unfixed temperature assumed that the cone walls in no contact with low heat conductivity medium.

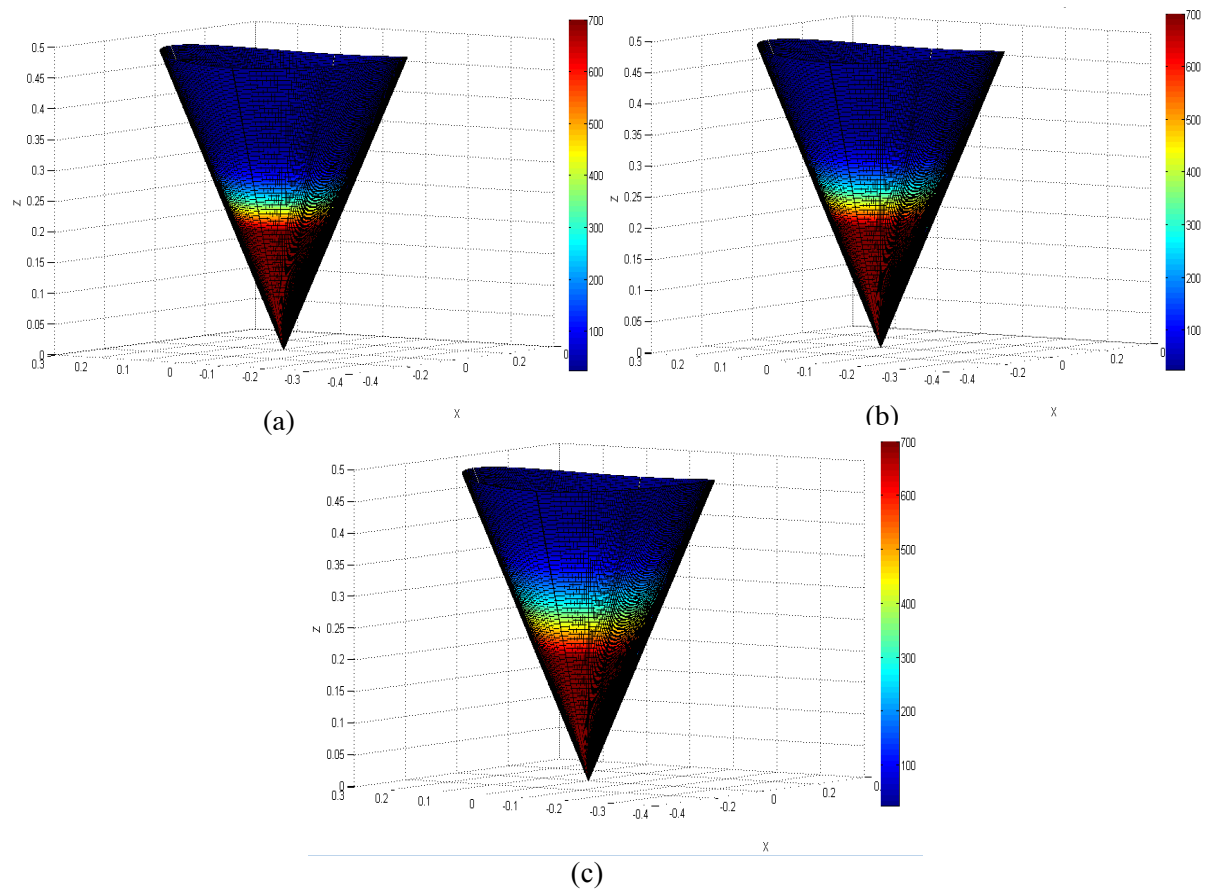
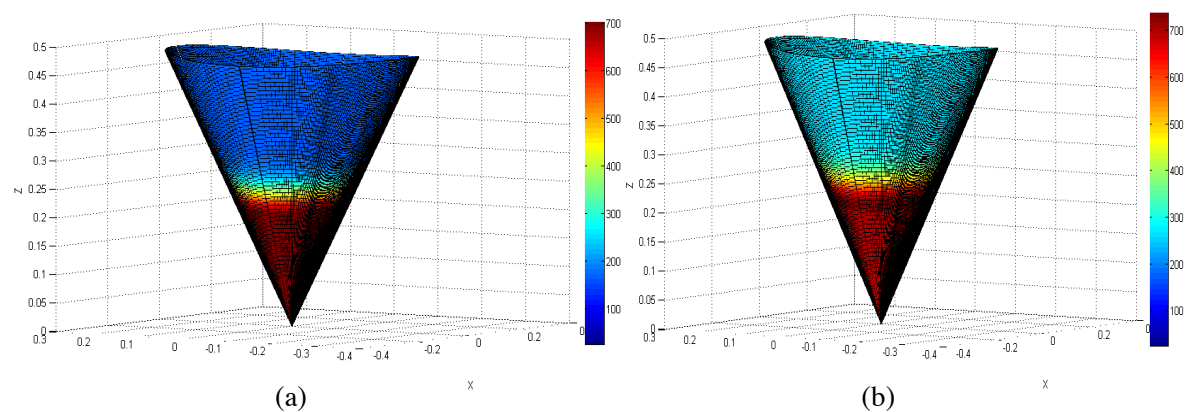


Figure 2. Heat distribution in the cone by conduction for unfixed blacket temperature at time 45 second (a) 60 second (b) and 120 second (c)

Figure 2 show the simulation of heat distribution of cone furnace caused by conduction process for 45 second and 60 second. It is appears that the rate of heat transfer in the (a) is lower than (b) unfixed temperature case. In this case, heat transfer in the cone is unsteady state condition. The temperature within the system does vary with time. While in unfixed temperature case, there is a contribution from the wall heat conduction with higher conductivity than fluid, so the temperature of cone edge is higher than its center.



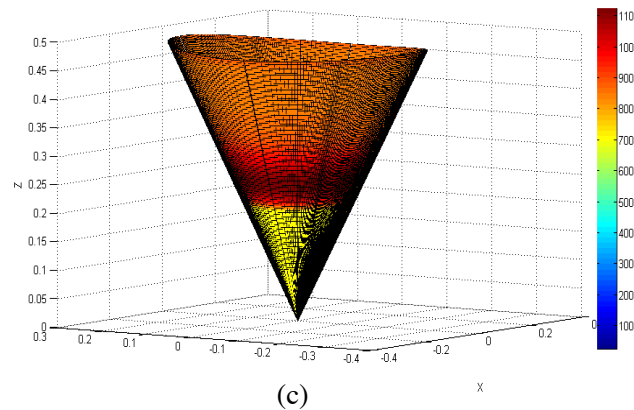


Figure 3. Heat distribution in the cone by convection for unfixed blanket temperature at time 45 second (a) 60 second (b) and 120 second (c)

Figure 3 show the simulation of heat distribution of cone furnace caused by conduction and convection process simultaneously for 45 second, 60 second, and 120 second. By comparing Fig 2 and Fig 3, it can be concluded that the heat transfer in the cone husk furnace is dominated by convection mechanism. On the mechanism of convection, the heat transfers through the movement of air particles due to the pressure gradient caused by the temperature gradient. While in conduction, there is only vibration between atoms of air due to a temperature gradient.

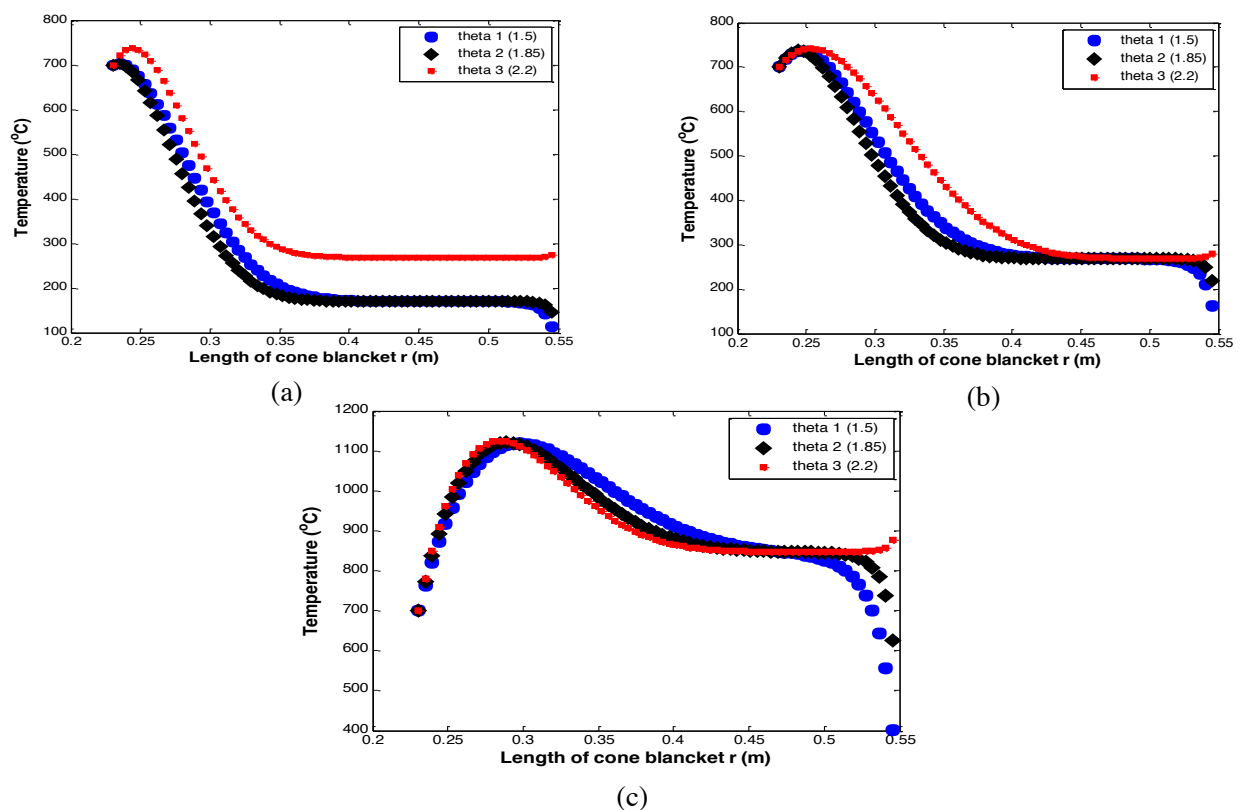


Figure 4. Graph of temperature vs length's cone blanket at time 45 second (a) 60 second (b) and 120 second (c)

Figure 4 show the graph temperature distribution by conduction and convection process simultaneously in the cone. There are three theta used, that is theta 1 (cone center), theta 2 (between center and cone edge) and theta 3 (cone edge) for 45 second, 60 second, and 120 second, respectively. For 45 second, temperature at the $r=0.35$ m in theta 1 is 201.94°C and 287.66°C in theta 3. For 60 second, temperature at the $r=0.35$ m in theta 1 is 337.70°C and 430.60°C in theta 3. there is a contribution from the wall heat conduction with higher conductivity than fluid, so the temperature of cone edge (theta 3) is higher than its center (theta 1).

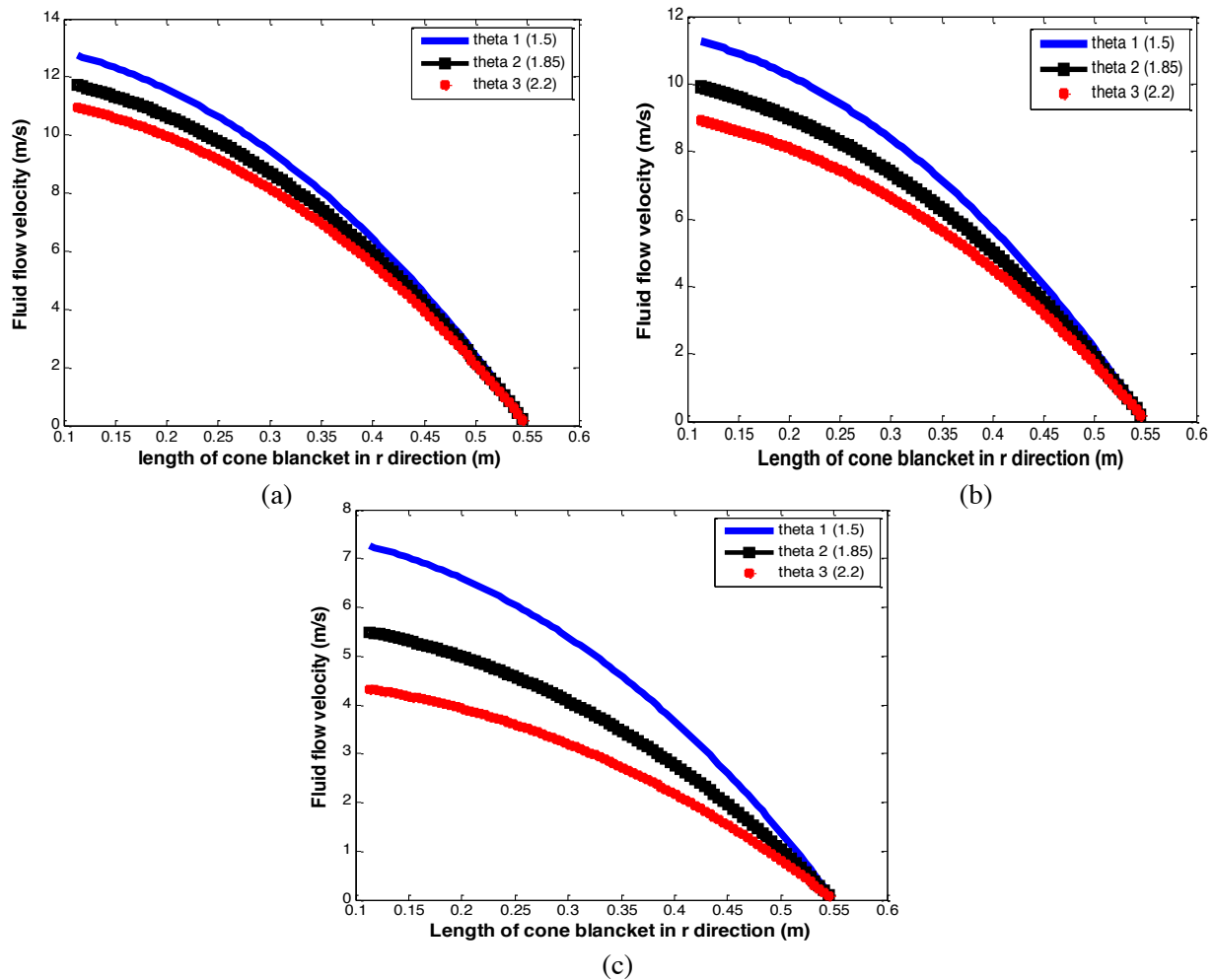


Figure 5. Graph of fluid flow velocity in r direction vs length's cone blanket at time 45 second (a) 60 second (b) and 120 second

Figure 5 above show the graph of fluid flow velocity in r direction at the time 45 and 60 second with unfixed temperature case. The fluid flow velocity at the center of the cone is bigger than fluid flow velocity at cone edge. The highest fluid flow velocity at $r=0.11$ m is 13.69 m/s, 11.90 m/s, and 7.25 m/s for 45, 60, and 120 second, respectively. The different result is due to the pressure gradient for 45 second is the biggest than 60 second and 120 for unfixed temperature case.

3. Conclusion

We have performed simulation of heat transfer on cone husk furnace with FDM (Finite Difference Method). Based on numerical calculation with FDM for heat conduction and convection, temperature gradient convection is bigger than conduction. On convection, the heat transfer through the movement

of air particles due to the pressure gradient caused by the temperature gradient. While in conduction, there is only vibration between atoms of air due to temperature gradient. The highest fluid flow velocity at $r=0.11\text{m}$ is 13.69 m/s and 11.90 m/s for 45 and 60 second, respectively. The different result is due to the pressure gradient for 45 second is bigger than 60 second unfixed temperature case. The velocity with maximum speed is at the center of the cone.

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