

The theory of hadronic parity violation

Matthias R Schindler

Department of Physics and Astronomy, University of South Carolina, Columbia, SC 29208,
USA

E-mail: mschindl@mailbox.sc.edu

Abstract. Parity-violating interactions between nucleons are the manifestation of an interplay of strong and weak interactions between quarks in the nucleons. Compared to the dominant parity-conserving part, the parity-violating component of the nuclear force is typically suppressed by approximately 6 to 7 orders of magnitude or more. Due to the short range of the weak interactions, however, it provides a unique probe of the strong dynamics that confine quarks into nucleons. An ongoing experimental program is mapping out this weak component of the nuclear force in few-nucleon systems. I will discuss recent theoretical progress based on effective field theory methods to analyze and interpret hadronic parity violation in few-nucleon systems, with a particular focus on two- and three-nucleon systems.

1. Introduction

The forces between nucleons are the manifestation of interactions between the quarks confined in these nucleons. While nucleon interactions are dominated by strong and electromagnetic effects, the confined quarks are also subject to the weak interactions. This leads to a parity-violating (PV) component in the interactions between nucleons. At the low energies typical of few-nucleon experiments, the PV part is expected to be suppressed by 6 to 7 orders of magnitude compared to the parity-conserving (PC) component. Effects due to the highly suppressed PV interaction can be isolated by considering observables that would vanish if parity was conserved. Typically, these are pseudoscalar observables involving a spin and a momentum vector, such as longitudinal and angular asymmetries and induced polarizations. The weak interactions are well understood at the quark level; however, when viewed at the hadronic level they are intertwined with the nonperturbative strong effects that confine the quarks in hadrons. Because of their short range, weak interactions are sensitive to quark-quark correlations at very short distances. PV nucleon interactions can therefore be viewed as a unique probe of nonperturbative strong effects.

PV effects can be enhanced by several orders of magnitude in some complex nuclei, e.g., by the presence of closely-spaced energy levels with opposite parity (see, e.g., Ref. [1]). While this is of great advantage experimentally, the theoretical description of such complex systems in terms of nucleon-nucleon interactions is difficult and can lead to uncontrolled theoretical errors. With the development of high-intensity neutron and photon sources and improved control over systematics, the measurement of PV effects in few-nucleon systems with $A \leq 5$ has become feasible, and an experimental program is underway to map out the PV component of the nucleon interactions. In these light systems, the theoretical description in terms of two- and three-nucleon systems is much more feasible. For recent reviews, see, e.g., Refs. [2, 3].

Traditionally, PV nucleon-nucleon interactions have been most commonly described in terms



of meson-exchange potentials with one PC and one PV meson-nucleon vertex. In particular, the formulation of Ref. [4] (often referred to as the DDH potential) has been used very frequently. More recently, effective field theory (EFT) methods, which have proven very successful in the PC sector, have been applied to PV observables. EFTs are model independent, treat PC and PV interactions as well as the coupling to external currents within a unified framework, and provide a method to estimate theoretical errors. The EFT analysis is performed at the hadronic level and does not directly relate the PV effects to interactions between standard model degrees of freedom. However, it provides a set of well-defined quantities with which any calculation performed at the quark level, such as potentially lattice QCD calculations, must be consistent.

2. Parity violation in pionless EFT

Many of the experiments in few-nucleon systems are performed at very low energies, e.g., using cold neutrons. At these energies, an EFT in which nucleons are the only dynamical degrees of freedom can be employed. In this so-called pionless EFT (EFT(π)) pions and all other degrees of freedom are integrated out and interactions between nucleons are described by contact terms with an increasing number of derivatives. All short-distance details of the interactions are encoded in the low-energy constants (LECs) of the theory that accompany each operator in the Lagrangian. The power counting of the theory provides a systematic method to determine observables order-by-order in terms of a small expansion parameter given by a ratio of typical scales involved in the reaction. For reviews of the highly successful application of EFT(π) in the PC sector, see, e.g., Refs. [5, 6, 7, 8].

At the low energies of interest, PV interactions between nucleons can be described by transitions between S- and P-wave states [9, 10]. Incorporating isospin, the leading-order (LO) PV EFT(π) Lagrangian contains five independent operators [11, 12, 13]:

$$\begin{aligned} \mathcal{L}_{PV} = & - \left[g^{(3S_1-1P_1)} d_t^{i\dagger} \left(N^T \sigma_2 \tau_2 i \overleftrightarrow{\nabla}_i N \right) \right. \\ & + g_{(\Delta I=0)}^{(1S_0-3P_0)} d_s^{A\dagger} \left(N^T \sigma_2 \vec{\sigma} \cdot \tau_2 \tau_A i \overleftrightarrow{\nabla} N \right) \\ & + g_{(\Delta I=1)}^{(1S_0-3P_0)} \epsilon^{3AB} d_s^{A\dagger} \left(N^T \sigma_2 \vec{\sigma} \cdot \tau_2 \tau^B i \overleftrightarrow{\nabla} N \right) \\ & + g_{(\Delta I=2)}^{(1S_0-3P_0)} \mathcal{I}^{AB} d_s^{A\dagger} \left(N^T \sigma_2 \vec{\sigma} \cdot \tau_2 \tau^B i \overleftrightarrow{\nabla} N \right) \\ & \left. + g^{(3S_1-3P_1)} \epsilon^{ijk} d_t^{i\dagger} \left(N^T \sigma_2 \sigma^k \tau_2 \tau_3 i \overleftrightarrow{\nabla}^j N \right) \right] + \text{h.c.} \end{aligned} \quad (1)$$

Here, d_t and d_s are auxiliary fields representing two-nucleon states in the 3S_1 and 1S_0 channel, respectively [14], $a \overleftrightarrow{\nabla}_i b = a \mathcal{O} D_i b - (D_i a) \mathcal{O} b$, with \vec{D} a nucleon covariant derivative and \mathcal{O} some spin-isospin-operator, and the isospin matrix $\mathcal{I} = \text{diag}(1, 1, -2)$. The $g^{(X-Y)}$ are the PV LECs encoding the short-distance physics. They cannot be determined within the EFT itself, but are related to standard model parameters through nonperturbative QCD calculations. Given the complexity of such calculations, another approach is to fit the LECs to data. However, this requires the existence of a sufficient number of experimental results and the calculation of the corresponding observables in the EFT(π) framework.

3. Parity violation in two-nucleon systems

Because transition amplitudes can be determined analytically, two-nucleon systems present the simplest environments to determine PV observables theoretically, and a number of reactions

have been considered. Using the scattering of a longitudinally polarized nucleon beam on an unpolarized nucleon target, a longitudinal asymmetry A_L can be defined as

$$A_L^{NN} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad (2)$$

where σ_{\pm} represents the cross section for scattering of a nucleon beam with helicity ± 1 . Results of a measurement of A_L^{pp} in proton-proton scattering at an energy low-enough for EFT($\not{\pi}$) to be applicable are reported in Ref. [15]. Neglecting Coulomb effects in the proton-proton case and adjusting conventions to be consistent with Eq (1), the LO EFT($\not{\pi}$) results are given by [16, 17, 12]

$$A_L^{nn} = -\sqrt{\frac{32M}{\pi}} p \left(g_{(\Delta I=0)}^{(^1S_0-^3P_0)} - g_{(\Delta I=1)}^{(^1S_0-^3P_0)} + g_{(\Delta I=2)}^{(^1S_0-^3P_0)} \right), \quad (3)$$

$$A_L^{pp} = -\sqrt{\frac{32M}{\pi}} p \left(g_{(\Delta I=0)}^{(^1S_0-^3P_0)} + g_{(\Delta I=1)}^{(^1S_0-^3P_0)} + g_{(\Delta I=2)}^{(^1S_0-^3P_0)} \right), \quad (4)$$

$$\begin{aligned} A_L^{np} = & -\sqrt{\frac{32M}{\pi}} p \frac{\frac{d\sigma^{^1S_0}}{d\Omega}}{\frac{d\sigma^{^1S_0}}{d\Omega} + 3\frac{d\sigma^{^3S_1}}{d\Omega}} \left(g_{(\Delta I=0)}^{(^1S_0-^3P_0)} - 2g_{(\Delta I=2)}^{(^1S_0-^3P_0)} \right) \\ & - \sqrt{\frac{32M}{\pi}} p \frac{\frac{d\sigma^{^3S_1}}{d\Omega}}{\frac{d\sigma^{^1S_0}}{d\Omega} + 3\frac{d\sigma^{^3S_1}}{d\Omega}} \left(g^{(^3S_1-^1P_1)} + 2g^{(^3S_1-^3P_1)} \right), \end{aligned} \quad (5)$$

with p the momentum in the center-of-mass frame, M the nucleon mass, and $d\sigma/d\Omega$ the PC differential cross section in the corresponding channel. At the energy of the experiment of Ref. [15], Coulomb effects are shown to be of the order of approximately 3% and can therefore be safely neglected given the theoretical uncertainties of a LO calculation.

For a perpendicularly polarized neutron beam passing through an unpolarized target, PV interactions cause a rotation of the neutron spin. For a proton target with target density ρ , the rotation angle per unit length to next-to-leading (NLO) order is given by [18]

$$\frac{1}{\rho} \frac{d\phi_{PV}^{np}}{dl} = 4\sqrt{2\pi M} \left(\frac{2g^{(^3S_1-^3P_1)} + g^{(^3S_1-^1P_1)}}{\gamma_t} \frac{Z_t + 1}{2} + \frac{g_{(\Delta I=0)}^{(^1S_0-^3P_0)} - 2g_{(\Delta I=2)}^{(^1S_0-^3P_0)}}{\gamma_s} \frac{Z_s + 1}{2} \right), \quad (6)$$

where $Z_{t/s} = (1 - \gamma_{t/s} r_{t/s})^{-1}$, and $\gamma_{t/s}$ ($r_{t/s}$) are the poles in the NN S-wave scattering amplitude (effective ranges) in the $^3S_1/^1S_0$ channel. An order-of-magnitude estimate of the size of the rotation angle per unit length for a typical hydrogen target is in the range 10^{-7} rad/m to 10^{-6} rad/m.

Deuteron photodisintegration and radiative neutron capture on protons provide several PV observables. Of particular experimental interest is the angular asymmetry A_γ in the capture of polarized neutrons ($\vec{n}p \rightarrow d\gamma$), which has been the goal of a measurement at the Spallation Neutron Source at Oak Ridge National Laboratory [19]. The LO EFT($\not{\pi}$) result is

$$A_\gamma = \frac{4}{3} \sqrt{\frac{2}{\pi}} \frac{M^{\frac{3}{2}}}{\kappa_1 (1 - \gamma_t a_s)} g^{(^3S_1-^3P_1)}, \quad (7)$$

where κ_1 is the nucleon isovector anomalous magnetic moment and a_s the 1S_0 scattering length.

With ongoing developments in high-intensity photon sources, there is renewed interest in the measurement of the asymmetry A_γ^L in deuteron photodisintegration with polarized photons ($\vec{\gamma}d \rightarrow np$),

$$A_\gamma^L = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}, \quad (8)$$

where σ_{\pm} is the breakup cross section for a photon beam with helicity ± 1 . For exactly reversed kinematics, this asymmetry is identical to the induced photon polarization P_{γ} in unpolarized neutron capture ($np \rightarrow d\vec{\gamma}$). It provides independent and complementary information to A_{γ} , as can be seen from the LO result for P_{γ} [13, 20],¹

$$P_{\gamma} = -2\sqrt{\frac{2}{\pi}} \frac{M^{\frac{3}{2}}}{\kappa_1(1 - \gamma_t a_s)} \left[\left(1 - \frac{2}{3} \gamma_t a_s \right) g^{(3S_1-1P_1)} + \frac{\gamma_t a_s}{3} \left(g_{(\Delta I=0)}^{(1S_0-3P_0)} - 2g_{(\Delta I=2)}^{(1S_0-3P_0)} \right) \right]. \quad (9)$$

In particular, P_{γ} , or equivalently A_{γ}^L , is one of a very limited number of observables sensitive to the isotensor coupling $g_{(\Delta I=2)}^{(1S_0-3P_0)}$. Reference [21] determined the energy dependence of A_{γ}^L to NLO as well as a very simplistic figure of merit to determine at which energy to best perform the corresponding experiment. Because the PV LECs are currently not determined, Ref. [21] used different model estimates and found that the figure of merit is maximized for photon energies close to threshold, $2.259\text{MeV} < k < 2.264\text{MeV}$. However, these results only represent order-of-magnitude estimates because of the uncertainty in determining the LECs.

4. Parity violation in three-nucleon systems

In systems with three or more nucleons, three-nucleon (3N) forces have to be taken into account. The EFT power counting rules based on dimensional analysis indicate that 3N interactions are suppressed relative to NN interactions. However, in the PC sector of the theory a cutoff dependence is present in the LO solutions of the scattering amplitude for neutron-deuteron scattering in the spin doublet channel [22]. This unphysical dependence on the cutoff can be removed by the introduction of a 3N contact term at LO. The accompanying LEC can be related to a three-body observable, such as a nucleon-deuteron scattering length or a three-body binding energy. Given the difficulty in obtaining a sufficient number of experimental results to pin down the PV two-nucleon LECs, an analogous enhancement of a 3N interaction in the PV sector would present a significant obstacle in describing hadronic parity violation in few-nucleon systems.

In Ref. [23] the ultraviolet behavior of the PV neutron-deuteron scattering amplitude for S-P wave transitions is analyzed. It is shown that no divergent contribution appears at LO, while at NLO the spin-isospin structures of the allowed PV 3N operators are different from those of possibly divergent terms. Therefore, up to and including NLO, no PV 3N operators are required, and up to an accuracy of roughly 10% two-nucleon interactions should be sufficient to analyze parity violation in few-nucleon systems.

Considering a deuteron target, the spin rotation angle of a polarized neutron beam to NLO is $\text{EFT}(\not{\pi})$ is given by [18] (also see Ref. [24] for a LO calculation)

$$\frac{1}{\rho} \frac{d\phi_{\text{PV}}^{nd}}{dl} = \left([16 \pm 1.6] g^{(3S_1-1P_1)} + [34 \pm 3.4] g^{(3S_1-3P_1)} + [4.6 \pm 1.0] (3g_{(\Delta I=0)}^{(1S_0-3P_0)} - 2g_{(\Delta I=1)}^{(1S_0-3P_0)}) \right) \frac{\text{rad}}{\text{MeV}^{\frac{1}{2}}}. \quad (10)$$

Theoretical uncertainties are estimated using three different methods, and the reported errors are the most conservative of these three estimates. An order of magnitude estimate of the rotation angle per unit length is again in the range of 10^{-7} rad/m to 10^{-6} rad/m .

5. Parity violation in chiral EFT

The range of applicability of $\text{EFT}(\not{\pi})$ is restricted to very low energies and to few-nucleon systems. To analyze processes involving higher energies and/or more complex nuclei, the

¹ Ref. [20] includes a formally higher-order contribution.

theoretical description has to include pions as dynamical degrees of freedom. Effective field theory methods can also be applied in this case. The LO PV pion-nucleon Lagrangian was constructed in Refs. [25, 26]. These results can be used to derive a PV nucleon-nucleon potential based on chiral symmetry [16, 27]. In addition to a one-pion-exchange contribution at LO, at NLO contact terms analogous to those in EFT(π) and two-pion-exchange contributions appear. In addition to two-nucleon observables, chiral EFT has been used to determine the longitudinal asymmetry in the charge-exchange reaction $\vec{n} \ ^3\text{He} \rightarrow p \ ^3\text{H}$ [27, 28].

6. Parity-violating interactions in the large- N_c expansion

Given the challenges in obtaining experimental results to determine the PV LECs, additional theoretical constraints on the couplings can be very valuable. QCD in the limit of the number of colors N_c becoming large [29, 30] provides one method of arriving at such constraints. In order to apply the large- N_c analysis to two-nucleon interactions, the NN potential is defined by [31]

$$V(\mathbf{p}_-, \mathbf{p}_+) = \langle (\mathbf{p}'_1, C), (\mathbf{p}'_2, D) | \hat{H} | (\mathbf{p}_1, A), (\mathbf{p}_2, \beta, B) \rangle, \quad (11)$$

where A, B, C, D collectively denote nucleon spin and isospin quantum numbers. The momenta \mathbf{p}_\pm are related to the momenta of the incoming and outgoing nucleons through $\mathbf{p}_\pm \equiv \mathbf{p}' \pm \mathbf{p}$, where $\mathbf{p}' = \mathbf{p}'_1 - \mathbf{p}'_2$ and $\mathbf{p} = \mathbf{p}_1 - \mathbf{p}_2$. The Hartree Hamiltonian is given by [30, 31]

$$\hat{H} = N_c \sum_n \sum_{s,t} v_{stn} \left(\frac{\hat{S}}{N_c} \right)^s \left(\frac{\hat{I}}{N_c} \right)^t \left(\frac{\hat{G}}{N_c} \right)^{n-s-t}, \quad (12)$$

where

$$\hat{S}^i = \hat{q}^\dagger \frac{\sigma^i}{2} \hat{q}, \quad \hat{I}^a = \hat{q}^\dagger \frac{\tau^a}{2} \hat{q}, \quad \hat{G}^{ia} = \hat{q}^\dagger \frac{\sigma^i \tau^a}{4} \hat{q}, \quad (13)$$

and \hat{q} denotes the light-quark doublet. The matrix elements of these operators scale as [32, 33]

$$\langle N' | \hat{S} | N \rangle \sim \langle N' | \hat{I} | N \rangle \sim 1, \quad \langle N' | \hat{G} | N \rangle \sim \langle N' | \mathbb{1} | N \rangle \sim N_c. \quad (14)$$

in the large- N_c limit. The coefficients v_{stn} contain factors of momentum, and momenta scale as $\mathbf{p}_- \sim 1$ and $\mathbf{p}_+ \sim N_c^{-1}$ [32, 33, 31]. When applied to PV interactions, the large- N_c analysis leads to a hierarchy of terms [34, 35]. The LO terms are given by

$$V_{N_c}^{\mathcal{P}} = N_c \left[U_{\mathcal{P}}^1(\mathbf{p}_-^2) \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \vec{\tau}_1 \cdot \vec{\tau}_2 + U_{\mathcal{P}}^2(\mathbf{p}_-^2) \mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) [\tau_1 \tau_2]_2^{zz} \right], \quad (15)$$

with $[\dots]_2$ the symmetric and traceless rank-two tensor, and the coefficient functions $U_{\mathcal{P}}^i(\mathbf{p}_-^2)$ scaling as $\mathcal{O}(1)$. There are four terms at NLO in the large- N_c expansion that are all isovector, while the six operators at next-to-next-to-leading order are isoscalar and isotensor like the LO terms.

These results have important implications when combined with the EFT(π) approach [35]. While in the EFT power counting all five LECs in Eq. (1) are expected to be of the same size, the large- N_c analysis implies that only two of these terms are dominant in a combined EFT(π) and large- N_c expansion. In addition, the large- N_c analysis implies a relation between the two isoscalar LECs that should hold up to corrections of order N_c^{-2} , or roughly 10%. The analysis of Ref. [35] shows that these results are not inconsistent with existing measurements.

The reduction in the number of dominant terms and the relation between the LECs in the large- N_c limit provide significant constraints on PV interactions. In particular the fact that the isotensor term is LO in the combined EFT and large- N_c analysis indicates the importance

of finding corresponding experimental constraints and to perform a lattice QCD calculation of this term. While in principle the existing measurement of the longitudinal asymmetry in $\vec{p}p$ scattering at 221 MeV [36, 37] could provide information on the isotensor coupling, this energy is far outside the range of applicability of EFT(π). At low energies, a measurement of the PV asymmetry in $\vec{\gamma}d \rightarrow np$ could put important experimental constraints on the isotensor component of the PV interaction.

7. Conclusions

Hadronic parity violation originates in an interplay of weak and nonperturbative strong interactions between quarks inside hadrons. It therefore provides a unique probe of our understanding of the standard model. While experimentally more accessible in complex nuclear systems, the recent focus has been on parity violation in few-nucleon systems, which can be more readily described in terms of two- and three-nucleon interactions. Effective field theories provide a model independent framework to systematically analyze and interpret experimental results and to provide theoretical error estimates. At LO in EFT(π), which is applicable at the very low energies relevant to many of the recent experiments, parity violation is described in terms of five S-P transition operators. A variety of observables has been calculated within this framework. In addition, the large- N_c expansion of QCD provides important theoretical constraints on the PV interactions. In a combined EFT(π) and large- N_c expansion, the number of LO terms reduces from five to two, and a relation between the two isoscalar LECs is predicted. These results provide strong motivation to determine the isotensor component of the PV interactions, both from a measurement of the longitudinal asymmetry in $\vec{\gamma}d \rightarrow np$ and from lattice QCD.

Acknowledgments

I thank H. W. Griebhammer, D. R. Phillips, R. P. Springer, and J. Vanasse for their collaboration and many interesting and stimulating discussions. This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award No. DE-SC0010300.

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