

# A simple criterium for CP conservation in the most general 2HDM

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**Abstract.** We find a set of necessary and sufficient conditions for the CP conservation in the most general 2HDM in terms of observable quantities. This set contains two simple and relatively easily testable conditions instead of usually discussed more complex conditions.

## 1. Introduction

CP violation is one of the important yet not well understood aspect of the fundamental physics. Modern LHC data [1] allow to conclude that the observed particle  $h(125)$  is Higgs boson with spin-CP parity  $0^{++}$  only under assumption that this particle has a definite parity. Generally it can have no definite parity, what happens in many models. In this case mentioned data give no information about  $h(125)$  parity [2].

In the Standard Model the CP violation is described by the CKM matrix, but its origin remains unclear. The extension of SM with two Higgs doublets, called Two Higgs Doublet Model (2HDM), has been introduced in 1974 with the main aim for providing an extra source of CP violation [3]. Later, many variants of 2HDM were considered with different features, providing different physical realizations (see for review [6, 7]). Each of these variants is described by Lagrangian with many parameters. The choice of the set of these parameters, describing the same physical reality, (choice of basis in the parameter space) is ambiguous. In the non-minimal models, like 2HDM, the observed situation with properties of Higgs boson similar to those in the SM (*SM-like scenario* [4] or *alignment limit* [5]) can be described by different non-equivalent sets of parameters.

We discuss two type of problems which appear in the study of CP violation in these models.

- (A) Let we have a variant of 2HDM, constructed as a model for description of some set of phenomena.

*How to know whether CP symmetry is violated or not in this model without detailed calculations of each particular effect?* Here the basis independent recipe is desirable.

- (B) Consider 2HDM as an approximation in the description of Nature. How do we know whether CP violation is described by this approximation or the observed CP violation is an effect of some weaker interactions.

*How to check CP violation in the experiments with particles, which appear in 2HDM?*



## 2. 2HDM

The 2HDM describes a system of two spinless isospinor fields  $\phi_1, \phi_2$  with hypercharge  $Y = 1$ . The most general form of the 2HDM potential is as follows

$$V = -\frac{m_{11}^2}{2}(\phi_1^\dagger\phi_1) - \frac{m_{22}^2}{2}(\phi_2^\dagger\phi_2) - \left[ \frac{m_{12}^2}{2}(\phi_1^\dagger\phi_2) + \text{h.c.} \right] \\ + \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\ + \left[ \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{h.c.} \right]. \quad (1)$$

The potential parameters are restricted by the requirement that the potential should be positive at large quasiclassical values of  $\phi_i$  (*positivity constraints*). We assume also that these coefficients are not too big so that one can use estimates based on the lowest non-trivial approximation of the perturbation theory.

After EWSB the 2HDM contains 3 neutral Higgs bosons  $h_a \equiv h_{1,2,3}$  (in general with indefinite CP parity) and charged Higgs bosons  $H^\pm$  with masses  $M_a$  and  $M_\pm$ , respectively (*the numbering of  $h_a$  is independent on an order of masses  $M_a$* ).

### 2.1. Reparametrization freedom

2HDM describes system of two fields with identical quantum numbers. Therefore, its description in terms of original fields  $\phi_i$  or in terms of their linear superpositions  $\phi'_i$  are equivalent; this statement verbalizes a *reparameterization* (RPa) freedom of the model. The RPa group consists of RPa transformations  $\hat{\mathcal{F}}$  of the form:

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \hat{\mathcal{F}}_{gen}(\theta, \tau, \rho) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \hat{\mathcal{F}}_{gen} = e^{i\rho_0} \begin{pmatrix} \cos \theta e^{i\rho/2} & \sin \theta e^{i(\tau+\rho/2)} \\ -\sin \theta e^{-i(\tau+\rho/2)} & \cos \theta e^{-i\rho/2} \end{pmatrix}. \quad (2)$$

This transformation induces a transformation of the parameters of the Lagrangian in such a way that the new Lagrangian, written in terms of fields  $\phi'_i$ , describes the same physical content. We refer to these different choices as - the different RPa bases. A subgroup of the RPa group - the *rephasing* group RPh - describes a freedom in choice of the relative phase of fields  $\phi_i$ .

Transformation (2) is parameterized by angles  $\theta, \rho, \tau$  and  $\rho_0$ . The parameter  $\rho_0$  describes an overall phase transformation of the fields and can be ignored since it does not affect the parameters of the potential. The parameter  $\rho$  describes the RPh symmetry transformation of system.

The  $U(1)_{EM}$  symmetry preserving ground state of this system is given by a global minimum of the potential and reads

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix}, \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi}/\sqrt{2} \end{pmatrix}, \quad (3)$$

with a standard parameterization  $\tan \beta = v_2/v_1$ , or  $v_1 = v \cos \beta$ ,  $v_2 = v \sin \beta$ .

### 2.2. The Higgs basis

We use below the RPa basis with  $v_2 = 0$  (the Higgs, or Georgi, basis [8]), in which the 2HDM potential can be written in the form [9]

$$V_{HB} = M_\pm^2 (\Phi_2^\dagger \Phi_2) + \frac{\Lambda_1}{2} (\Phi_1^\dagger \Phi_1 - \frac{v^2}{2})^2 + \frac{\Lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \Lambda_3 (\Phi_1^\dagger \Phi_1 - \frac{v^2}{2}) (\Phi_2^\dagger \Phi_2) \\ + \Lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \left[ \frac{\Lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \Lambda_6 (\Phi_1^\dagger \Phi_1 - \frac{v^2}{2}) (\Phi_1^\dagger \Phi_2) + \Lambda_7 (\Phi_2^\dagger \Phi_2) (\Phi_1^\dagger \Phi_2) + \text{h.c.} \right]. \quad (4)$$

For this basis we use capital letters to denote fields and parameters of potential,  $\Phi_i$  and  $\Lambda_j$  respectively.

### 3. Relative couplings

In the discussion below we use the relative couplings for each neutral Higgs boson<sup>1</sup>  $h_a$  (in 2HDM  $a = 1, 2, 3$ ):

$$\chi_a^P = \frac{g_a^P}{g_{\text{SM}}^P}, \quad \chi_a^\pm = \frac{g(H^\pm H^- h_a)}{2M_\pm^2/v}, \quad \chi_a^{H^+W^-} = \frac{g(H^+W^- h_a)}{M_W/v}. \quad (5)$$

The quantities  $\chi_a^P$  (where  $P = V(W, Z)$ ,  $q = t, b, \dots$ ,  $\ell = \tau, \dots$ ) are the ratios of the couplings of  $h_a$  with the fundamental particles  $P$  to the corresponding couplings for the would be SM Higgs boson with mass  $M_a$ . The other relative couplings describe interaction of  $h_a$  with the charged Higgs boson  $H^\pm$ . Couplings  $\chi_a^V$  and  $\chi_a^\pm$  are real due to Hermiticity of Lagrangian, and are directly measurable. Couplings  $\chi_a^{H^+W^-}$ ,  $\chi_a^q$  and  $\chi_a^\ell$  are generally complex.

There are useful sum rules among these couplings, namely

$$(a) \sum_a (\chi_a^V)^2 = 1, \quad (b) (\chi_a^V)^2 + |\chi_a^{H^+W^-}|^2 = 1. \quad (6)$$

Both real and imaginary parts of Yukawa couplings  $\chi_a^q$  and  $\chi_a^\ell$  can be measured in principle, using distributions of Higgs bosons decay products in  $h_a \rightarrow \bar{q}q$ ,  $h_a \rightarrow \bar{\ell}\ell$ . The absolute value of the coupling  $\chi_a^{H^+W^-}$  is fixed by the sum rule (6 b) and is well measurable.

The unitarity of the rotation matrix describing transition from components of fields  $\phi_i$  to the physical Higgs fields  $h_a$ , allows to obtain the following relations for couplings  $\chi_a^{H^+W^-}$  (the factor  $e^{i\rho}$  represents the rephasing freedom in the Higgs basis):

$$\begin{aligned} \chi_1^{H^+W^-} &\equiv \left( \chi_1^{H^-W^+} \right)^* = -e^{i\rho} \frac{\chi_1^V \chi_2^V - i\chi_3^V}{\sqrt{1 - (\chi_2^V)^2}}, \\ \chi_2^{H^+W^-} &\equiv \left( \chi_2^{H^-W^+} \right)^* = e^{i\rho} \sqrt{1 - (\chi_2^V)^2}, \\ \chi_3^{H^+W^-} &\equiv \left( \chi_3^{H^-W^+} \right)^* = -e^{i\rho} \frac{\chi_2^V \chi_3^V + i\chi_1^V}{\sqrt{1 - (\chi_2^V)^2}}. \end{aligned} \quad (7)$$

Note, that here we discuss couplings which appear in the Lagrangian. Radiative corrections (RC) change these couplings, however in most cases these corrections are small and therefore the corresponding observable differ weakly from those presented in the Lagrangian. In this sense we treat the latter ones as being measurable. Therefore the identities (6), (7) are valid with accuracy to RC.

For the interaction of Higgs bosons with the gauge bosons  $V = W, Z$  one should distinguish interactions of different structure, that are vectorial  $g_a^V h_a V_\mu V^\mu$ , tensor  $g_{aT} h_a V_{\mu\nu} V^{\mu\nu}$  and axial-tensor  $\tilde{g}_{aT} h_a \tilde{V}_{\mu\nu} V^{\mu\nu}$  with corresponding coupling constants  $g_a^V$ ,  $g_{aT}^V$  and  $\tilde{g}_{aT}^V$ , respectively (where  $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ ,  $\tilde{V}_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} V^{\alpha\beta}$ ). These interactions can be separated in the experiment from each other by the study of angular correlations in the decays like  $h_a \rightarrow ZZ \rightarrow e^+e^- \mu^+ \mu^-$  [1]. **In the paper we concentrate on vectorial couplings appearing in Lagrangian** (and modified weakly by RC).

In the 2HDM tensor and axial-tensor interactions appear only due to RC (mainly from  $t$ -loops). In the most of cases they give very small contributions to the observed decays rates of  $h_a$  and hardly observable.

<sup>1</sup> We omit the adjective "relative" below.

The axial-tensor interaction can imitate CP violation. This interaction was studied in ref. [11] for CP conserved variant of 2HDM, in which one of neutral Higgs bosons is pseudoscalar  $A$ . It was found that the corresponding decay  $A \rightarrow e^+e^-\mu^+\mu^-$ , etc. can be observable in very narrow range of parameters only ( $M_A \approx 2m_t$ , low  $\tan\beta$ ). If it will be realized, the study of angular correlation of leptons allows to distinguish CP odd nature of initial state (as it is done currently at LHC in the study of CP properties of the Higgs boson [1]) without violation of CP symmetry.

#### 4. A minimal complete set of observables in 2HDM

In ref. [9] a *minimal complete set of directly measurable quantities (observables)* defining the 2HDM was found ( $a = 1, 2, 3$ ), namely :

- (1) *v.e.v. of Higgs field*  $v = 246 \text{ GeV}$ , (2-5) *masses of Higgs bosons*  $M_a, M_{\pm}$ ,
- (6-7) *2 (out of 3) couplings*  $\chi_a^V$ , (8-10) *3 couplings*  $\chi_a^{\pm}$ ,
- (11) *quartic coupling*  $g(H^+H^-H^+H^-)$ .

In the most general 2HDM, these 11 observables are independent of each other. In particular variants of 2HDM additional relations between these parameters may appear.

The parameters of potential in the Higgs basis are expressed through these observables and free parameter  $\rho$  (it appears in  $\Lambda_{5,6,7}$  via couplings  $\chi_a^{H^+W^-}$  given in eq. (7)):

$$\begin{aligned}\Lambda_1 &= \sum_a (\chi_a^V)^2 M_a^2 / v^2; \quad \Lambda_5 = \sum_a (\chi_a^{H^-W^+})^2 M_a^2 / v^2; \\ \Lambda_4 &= (\sum_a M_a^2 - M_{\pm}^2) / v^2 - \Lambda_1; \quad \Lambda_3 = 2 (M_{\pm}^2 / v^2) \sum_a \chi_a^V \chi_a^{\pm}; \\ \Lambda_6 &= \sum_a \chi_a^V \chi_a^{H^-W^+} M_a^2 / v^2; \quad \Lambda_7 = 2 (M_{\pm}^2 / v^2) \sum_a \chi_a^{H^-W^+} \chi_a^{\pm}; \\ \Lambda_2 &= 2g(H^+H^-H^+H^-).\end{aligned}\tag{9}$$

The parameters of the potential (1) with the defined values of  $\tan\beta$  and  $\xi$  (3) are obtained from parameters (9) with the aid of transformation (2) in the following form

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \hat{\mathcal{F}}_{HB} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix}, \quad \hat{\mathcal{F}}_{HB} = \hat{\mathcal{F}}_{gen}(\theta = -\beta, \tau = \xi, -\rho).\tag{10}$$

#### 5. Conditions for a CP conservation

In the models like 2HDM neutral scalar particles coincide with their antiparticles. Therefore, in such models one can discuss the P-parity violation, but not C-parity. When in addition we consider fermions, this P violation is transformed to the CP violation (see e.g. [6, 7]).

##### 5.1. Basic facts

The CP symmetry is conserved in the 2HDM and similar models containing spinless bosons if <sup>2</sup>

- *Each observable physical neutral spinless boson has definite P-parity* (11a)

- *There are no P-violating interactions between these bosons.* (11b)

In the 2HDM we denote neutral spinless P-even bosons as  $h_1, h_2$  (they are called often as  $h$  and  $H$ ) and P-odd boson as  $h_3$  (it is called often as  $A$ ). The condition (11b) means an absence of the interactions<sup>3</sup> ( $i, j, k = 1, 2$ )

$$h_i h_j h_3, \quad h_3 h_3 h_3, \quad h_i h_j h_k h_3, \quad h_i h_3 h_3 h_3.\tag{12}$$

<sup>2</sup> Note that CP violation, obliged by fermions and (or) vector bosons, appears in the interactions of spinless particles via radiative correction.

<sup>3</sup> Similar conditions were discussed in [12].

It is well known that in the 2HDM CP conservation holds if

*There exists the RPa basis in which:*

$$\bullet \text{ all parameters of potential are real,} \quad (13a)$$

$$\bullet \text{ relative phase (3) } \xi = 0. \quad (13b)$$

In is worth mentioning, that the condition (13a) forbids an explicit CP violation while conditions (13a) and (13b) together forbid a spontaneous CP violation.

The statement (11) only describes CP-conservation, but does not provide a criterion for CP conservation in the considered model. The description (13) is RPa basis-dependent. Below we discuss both (A) and (B) set-ups of our problem, presented in the Introduction.

Many authors consider solution of the problem (A) as a necessary step in solving the problem (B). Our approach is different – we look for a solution of the problem (B), with a solution of problem (A) appearing as a by-product.

### 5.2. Method of the CP-odd basis-independent invariants

Many author found the basis independent criterium for CP violation conservation in terms of parameters of the Higgs potential – this corresponds to the problem (A) stated in the Introduction. For this goal they constructed the RPa-invariant CP-odd combinations of parameters of the potential and as a condition for CP conservation they demand vanishing of all these invariants. For 2HDM three such invariants  $Im\mathcal{J}_{1,2,3}$  were found in ref. [10], the procedure for construction of such invariants for multi-Higgs models is developed in ref. [13].

To solve the corresponding problem (B) the invariants [10] for 2HDM were expressed via measurable quantities in refs. [14, 15]. In the terms of quantities (8), the corresponding conditions for the CP conservation read as

$$\begin{aligned} Im\mathcal{J}_1 &= \sum_{i,j,k} \varepsilon_{ijk} \frac{2M_i^2 M_{\pm}^2}{v^4} \chi_i^V \chi_k^V \chi_j^{\pm} = 0, & Im\mathcal{J}_2 &= 2\chi_1^V \chi_2^V \chi_3^V \sum_{i,j,k} \varepsilon_{ijk} \frac{M_i^4 M_k^2}{v^6} = 0, \\ Im\mathcal{J}_{30} &= 4 \sum_{i,j,k} \varepsilon_{ijk} \mathcal{T}_i \chi_j^V \chi_k^{\pm} = 0, & \text{where } \mathcal{T}_i &= \frac{(M_{\pm}^2 \chi_i^{\pm} + M_i^2 \chi_i^V) M_i^2 M_{\pm}^2}{v^6}. \end{aligned} \quad (14)$$

Note that in this approach there are four CP-odd invariants but one should check vanishing only of two of them (see e.g. [7]). Since the choice of these two invariants is not fixed from beginning, the presented set contains three conditions, instead of necessary two. In our opinion the equations (14) are too complicated and their experimental verification, discussed in [14, 15, 16], requires very complex procedure.

### 5.3. A direct criterium for CP conservation

In the direct method we approach the mentioned above problem (B) directly. We start with a description of CP conservation (11) and use only observables, which by definition are basis independent.

In Refs. [17], [18] we proposed conditions for CP conservation, based only on one condition<sup>4</sup> (11a), without checking up of condition (11b), in the form

$$\prod_a \chi_a^V = 0, \quad \prod_a \chi_a^{\pm} = 0, \quad \left| \prod_a \chi_a^f \right| = \prod_a \left| \chi_a^f \right|. \quad (15)$$

<sup>4</sup> Particular version of such approach was used in [19].

Below we simplify these conditions to prove that the set of new conditions is a *necessary and a sufficient* one.

### • The direct criterium

In the 2HDM for the CP conserved case all  $h_a$  should have definite P-parity. In particular, one of them is P-odd, while two others are P-even (11a). Therefore, the **necessary condition** for a CP conservation is an existence of one neutral Higgs boson (we denote it  $h_3$ ), which doesn't couple to the CP-even states  $VV$  and  $H^+H^-$ . So it reads:

$$\boxed{\text{There exists a neutral Higgs boson } h_3 \text{ for which vectorial coupling } g_3^V \equiv g(h_3 VV) = 0 \text{ and } g_3^\pm \equiv g(h_3 H^+ H^-) = 0.} \quad (16)$$

Now, one has to check condition (11b). In order to do this, we substitute Eq-s. (16), (7) into (9), choosing  $\rho = 0$ . Choice  $\rho = 0$  together with constraint  $\chi_3^V = 0$  in (7) makes both  $\chi_{1,2}^{H^+W^-}$  real while  $\chi_3^{H^+W^-}$  is imaginary. Next, we insert these  $\chi_a^{H^+W^-}$  into (9). The parameter  $\Lambda_5$  contains real negative quantity  $(\chi_3^{H^+W^-})^2$  while in  $\Lambda_6$  and  $\Lambda_7$  this imaginary  $\chi_3^{H^+W^-}$  is multiplied by  $\chi_3^V$  or  $\chi_3^\pm$ , which are equal to 0 (16). Hence, with this choice all parameters of potential  $\Lambda_a$  in the Higgs basis are real. Therefore, in view of a statement (13), the CP-symmetry of model is not violated. In particular, the CP violating interactions (12) don't appear. (Besides, it is easy to check that conditions (16) ensure compliance of conditions (14) and first two conditions (15).) Therefore we conclude that **the conditions (16) are necessary and sufficient** for establishing CP conservation in the 2HDM (problem B).

Note, that the eq-s (9) after substitution equation (16) together with (10) can be treated as a solution of the problem (A).

### • A discussion of the direct criterium

Certainly, for the experimental verification one can use different conditions instead of one or both conditions (16). In particular, the condition  $g(h_3 VV) = 0$  means that the scalar  $h_3$  has no P-even admixture. This leads immediately to the identities  $g(h_3 h_k Z) = 0$  ( $k = 1, 2$ ). This conclusion allows to replace checking up of the condition  $g(h_3 VV) = 0$  by checking of one of conditions  $g(h_3 h_k Z) = 0$ , i. e. non-observation of corresponding decays, as it was proposed<sup>5</sup> in [19]. Besides, the conditions for CP conservation, written via the CP-odd invariants (14) are fulfilled if some masses  $M_a$  are degenerated and some special relations among observable couplings  $\chi_a^V$  and  $\chi_a^\pm$  took place. This combination can be treated as an alternative form of our necessary and sufficient condition in this particular case.

### • Consequence for the Yukawa interaction.

The Yukawa interaction can violate CP symmetry of the model only in the case of a P non-conservation for the scalars. The conditions (16) guarantee that  $h_a$  are pure state of CP parity ( $h_1$  and  $h_2$  are CP even,  $h_3$  is CP odd). In this case Yukawa interactions cannot generate CP violation, and the third condition from the equation (15) is fulfilled automatically.

## 6. Possibilities for a verification

The verification of the CP conservation requires an observation of all scalars of the model. In the realized in Nature SM-like scenario this looks difficult (see e.g. [18]). Moreover, one should

<sup>5</sup> Let us remind that such condition (including all its forms discussed in [19]) is insufficient to establish CP conservation in the considered model in the general case, since does not guarantee the fulfillment of the condition (11b), i. e. non-appearance of CP odd vertices (12).

check if some measurable quantities are equal to zero. In any case, these measurements cannot be performed with a high accuracy. From this point of view the proposal to change a direct criterium to a condition for a non-observation of decay  $h_3 \rightarrow h_1 Z$ , etc. given in [19] looks attractive. Nevertheless, one can not hope for a high accuracy in testing CP conservation in the 2HDM. Let us remind that the possible observation of weak enough decay  $h_3 \rightarrow ZZ \rightarrow \ell_1^+ \ell_1^- \ell_2^+ \ell_2^-$  doesn't contradict CP conservation. If this decay will be observed the correlations between momenta of leptons should be studied. The CP conservation for  $h_3$  will be confirmed if these correlations show axial-tensor nature of this interaction, i.e. pseudoscalar nature of  $h_3$ .

If in future the experiments will show, within the definite experimental uncertainties, the agreement with criteria (16) the important problem arises for the further studies, namely *whether one can expect that the more accurate measurements will show violation of CP in our 2HDM, i. e. violation of criteria (16), or the observed CP violation is in fact an effect beyond approximation given by 2HDM, and some new weaker interactions should be implemented in the description of Nature.*

This situation is similar to that in atomic physics. Atomic interaction (QED) conserves P-parity. Parity non-conservation appears at the smallest distances due to next level weak interaction. It is observed as a small effect in rare atomic transitions.

## 7. Conclusions

We present here a compact set of necessary and sufficient conditions for CP conservation in 2HDM (16), which are common for all mechanisms of CP violation. We prove that the verification of CP conservation in 2HDM requires to measure two simple and relatively easily testable observables instead of more complex conditions with CP-odd invariants (14) discussed by many authors [10], [14, 15, 16].

## Appendix. Necessary condition for CP conservation in Multi Higgs Doublet Model

The criterion of CP conservation in the Multi Higgs Doublet Models – nHDM are also of interest. Here, the method of CP-odd invariants allows to construct many equations, which can be used for obtaining conditions for CP conservation. Both complete set of these equations and their expressions via measurable quantities are absent up to now (see e. g. [13]).

The direct method used above allows to formulate for the nHDM simple necessary conditions for the CP conservation.

After EWSB the nHDM contains  $2n - 1$  neutral Higgs bosons  $h_a$ , generally with indefinite CP parity, and  $n - 1$  charged Higgs boson  $H_b^\pm$  with masses  $M_a$ ,  $M_{b\pm}$ , respectively. The couplings  $h_a VV$  obey first sum rule (6). In the case of CP conservation one can split spinless neutral particles  $h_a$  into two groups: P-even  $h_1, \dots, h_n$  and P-odd  $h_{n+1}, \dots, h_{2n-1}$ . Similarly to (16), the condition for a CP conservation in the nHDM can be written as

$$\boxed{\begin{aligned} &\text{There exist } n - 1 \text{ neutral Higgs bosons } h_c, \text{ for which } g(h_c VV) = 0, \\ &g(h_c H_b^+ H_b^-) = 0 \text{ (with } n + 1 \leq c \leq 2n - 1, 1 \leq b \leq n - 1). \end{aligned}} \quad (17)$$

These  $n(n - 1)$  conditions are *necessary* for CP conservation. We don't know now whether these conditions are *sufficient* or not.

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