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Gravitational waves

Andrzej Trautman

Faculty of Physics, University of Warsaw

E-mail: `Andrzej.Trautman@fuw.edu.pl`

Abstract. Historical remarks on early theoretical work on the subject. Very early on, Einstein introduced the notion of gravitational waves, but later became convinced that they did not exist as a physical phenomenon. Exact solutions of Einstein's equations representing waves were found by a number of authors, contributing to their final acceptance as part of physics.

The recent observation of gravitational waves emitted by a system of two orbiting black holes [1] increased the interest in the phenomenon of gravitational radiation. This short note recalls some of the early theoretical work on this subject.

The first two papers on gravitational waves were by Albert Einstein [2] [3]. There was a small computational error in the first one.

In the linear approximation, the metric tensor is written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and the following coordinate conditions, analogous to the Lorentz condition, have been used by Einstein:

$$(h^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}\eta_{\rho\sigma}h^{\rho\sigma})_{,\nu} = 0.$$

They are the linear form of the harmonic coordinates, later introduced by V.A. Fock,

$$(\sqrt{|g|}g^{\mu\nu})_{,\nu} = 0 \qquad \text{or} \qquad \Box x^\mu = 0.$$

The linearized Einstein field equations reduce to

$$\Box h^{\mu\nu} = \text{const.}\, T^{\mu\nu}.$$

By solving them, Einstein found plane gravitational waves and radiation from a bounded source – all of this was valid in the linear, weak field approximation to the field equations.

In the 1920s there appeared a paper [4] by the mathematician H. W. Brinkmann on exact plane gravitational waves (that name was not used by the author). At that time the paper was not noticed by physicists.

In the 1930s, probably under the influence of work on cosmology, Einstein recognizes the importance of using full – rather than linearized – equations. With Nathan Rosen he attempted to find exact plane waves. They obtained solutions of $R_{\mu\nu} = 0$ exhibiting singularities and, in 1936, they sent to the *Physical Review* a manuscript entitled *Do gravitational waves exist?*, concluding that they do not.

The referee – thanks to Daniel Kennefick [5] we now know that was H.P. Robertson – pointed out that the singularities may be due to the choice of coordinates and suggested revisions.

Einstein wrote to the editor of *Physical Review*: "We sent you the paper for publication and had not authorized you to show it to specialists before it is printed... I prefer to publish the

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paper elsewhere." Indeed, there is a paper [6] by Einstein and Rosen "On gravitational waves", published in *Journal of the Franklin Institute*, **223** (1937) 43–54, where they accepted Robertson suggestion and interpreted the solution as representing cylindrical waves.

Leopold Infeld, a Polish physicist, was with Einstein in Princeton in 1936-38. With Banesh Hofmann they then created a new approximate method (EIH) of finding the equations of motion of gravitating bodies. Infeld did not participate in the Einstein–Rosen collaboration, but under Einstein's influence, became convinced that gravitational waves did not exist. After the war, he returned to Poland and created the Institute of Theoretical Physics in Warsaw. Jerzy Plebański was his first collaborator here; in 1954 he gave a course of theoretical physics at Politechnika Warszawska and invited Andrzej Trautman (AT), one of the students, to join Infeld's group as a Ph. D. student. He suggested gravitational waves as the subject, warning that Infeld did not believe in their existence. In 1957, using Sommerfeld's outgoing radiation conditions, adapted to the case of gravitation, AT showed that an isolated system could radiate energy and that the curvature tensor was then of the form

$$N/r + O(r^{-2})$$

where N is a tensor of the same, simple form, as it appears for plane waves.

When AT has shown his results to Felix Pirani, who was then visiting Warsaw, Felix invited him to King's College, London, to give lectures (1958). Ivor Robinson (IR) and AT met there for the first time.

In 1958, there appeared a paper [7] by H. Bondi, F. Pirani and I. Robinson giving a new account of plane gravitational waves, free of singularities, making an explicit use of the 5-dimensional group of symmetries of those waves.

Robinson introduced the notion of congruences of (foliations by) null geodesics without shear; later they were characterized (by IR and AT) by a null vector field k in space-time M such that the flow generated by k preserves the conformal geometry of the screen bundle

$$\mathcal{K}^{\perp}/\mathcal{K} \to M, \qquad \mathcal{K}_p = \mathbb{R}k_p.$$

Many new gravitational fields were found using the requirement that they admit such a shear-free flow. The most important among them was the field of a general, rotating, black hole discovered by Roy Kerr (1963) [8].

IR and AT found [9] a new class of solutions of Einstein equations, containing gravitational waves with spherical wave fronts, carrying information in the sense of depending on arbitrary functions of $u = t - r$. They are given in coordinates (x, y, u, r) with the function $p(x, y, u)$ as

$$2\,\mathrm{d}r\,\mathrm{d}u + (K - 2p^{-1}\frac{\partial p}{\partial u}r - \frac{2m}{r})\,\mathrm{d}u^2 - r^2p^{-2}(\mathrm{d}x^2 + \mathrm{d}y^2),$$

where

$$K = \Delta \log p, \qquad \Delta = p^2(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}),$$

is the Gaussian curvature of the surface $r = 1,\ u = \text{const.}$.

The field equations $R_{\mu\nu} = 0$ reduce to

$$12m\frac{\partial}{\partial u}\log p + \Delta\Delta \log p = 0.$$

If $m = 0$, then the dependence of p on u is unrestricted.

There is 'peeling off' of the curvature exhibiting its Petrov types,

$$\text{Riemann tensor} = \frac{N}{r} + \frac{III}{r^2} + \frac{D}{r^3}.$$

The Schwarzschild metric is among those solutions; in this case the Riemann tensor is D/r^3.
There is a clear electromagnetic analog of the N/r solution: plane e.m. reflected from a perfectly conducting paraboloid of revolution.

A discrete aside. Spinors that appear throughout physics have a 'discrete aspect'. The Pythagorean equation $x^2+y^2=z^2$ can be interpreted as a condition for the vector $(x,y,z)\in\mathbb{Z}^3$ to be *null* w.r.t. a scalar product of signature $(2,1)$. Euclid's solution

$$x=p^2-q^2,\quad y=2pq,\quad z=p^2+q^2,$$

written as

$$2\begin{pmatrix}p\\ q\end{pmatrix}\begin{pmatrix}p & q\end{pmatrix}=\begin{pmatrix}x+z & y\\ y & z-x\end{pmatrix}$$

reads now as the equality

$$(\text{spinor})^2=\text{null vector}.$$

There is a homomorphism of groups
$\rho:\mathrm{SL}_2(\mathbb{Z})\to\mathrm{SO}_{2,1}$ given by

$$\rho(A)X=AXA^t,\quad X=\begin{pmatrix}x+z & y\\ y & z-x\end{pmatrix},$$

where now $(x,y,z)\in\mathbb{R}^3$ and $A^t=$ transpose of the matrix $A\in\mathrm{SL}_2(\mathbb{Z})$.

One shows that the group $\mathrm{SL}_2(\mathbb{Z})$ acts transitively on the set of integer-valued spinors with relatively prime components. The group $\rho(\mathrm{SL}_2(\mathbb{Z}))$ contains matrices with half-integer elements, e.g.

$$\rho\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ -1 & 1 & 1\\ -\frac{1}{2} & 1 & \frac{3}{2}\end{pmatrix}.$$

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