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Calculations of superconducting parametric amplifiers performances

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Abstract. A superconducting parametric amplifier is an electromagnetic wave amplifier with high-quality characteristics such as a wide bandwidth, an extremely low noise, and a high dynamic range. In this paper, we report on the estimations of a YBCO superconducting parametric amplifier characteristic. The YBCO thin films were deposited on an MgO substrate by a pulsed laser deposition method. Based on the measured YBCO thin film parameters, theoretical calculations were implemented for evaluations of kinetic inductance nonlinearities and parametric gains. The nonlinearity of the YBCO thin film was estimated to be stronger than a single crystal NbTiN thin film. It is indicated that the YBCO parametric amplifier has a potential to be realized the amplifier with the high parametric gain. It is also expected that it could be operated in the range of the high frequency band, at the high temperature, and low applied current.

1. Introduction

In the fields of cryogenic electronics, there are some applications, such as superconducting qubit experiments for the realization of a quantum computer [1,2], a microwave kinetic inductance detectors (MKIDs) for radio astronomical observation [3,4], and so on. For such applications, a realization of a microwave amplifier with quantum noise limit, low power and broadband operations is desired [5]. However, in general, it is difficult to simultaneously satisfy low noise and broadband characteristics. For example, the HEMT amplifiers have a wideband characteristic, but have relatively high noise temperatures. The superconducting Josephson amplifiers have noise performances approaching the quantum limit, but the bandwidth is narrow [6]. On the other hand, a superconducting parametric amplifier was recently reported [7], and it has been shown that it has high-quality characteristics, such as a broad bandwidth, an extremely low-noise property, and a high dynamic range.

The parametric amplification process in the superconducting parametric amplifier uses a kinetic inductance (L_k), which has a nonlinear property. Since a total inductance (L_T) of the superconducting thin films is the sum of a magnetic inductance (L_m) and L_k, it is essential that L_k is dominant compared with L_m. L_k nonlinearly changes according to the applied current I as follows:

$$L_k(I) \approx L_k(0)\left[1 + \left(\frac{I}{I^*}\right)^2\right] \tag{1}$$

where I^* is same order as the critical current (I_C) of the superconductors. Its amplification process is a degenerate four-wave mixing, which is also utilized in an optical fiber parametric amplification process, but the phase matching method is different from that of the optical fiber. In the parametric amplifier, phase matching is realized by periodically changing the line impedance of a superconducting transmission line [7].

As the kinetic inductance nonlinearity per unit length is not so much, the length of the transmission line has to be long to obtain a high parametric gain. When a 2.2 m long coplanar waveguide (CPW) by using a polycrystalline NbTiN thin film was fabricated, the parametric gain was 15 dB in the microwave band [8]. We also have been studying the kinetic inductance nonlinearity of a single-crystal NbTiN and the polycrystalline NbTiN thin films to realize the superconducting parametric amplifier [9]. We have already reported that the nonlinearity of the single-crystal NbTiN is three times as much as that of the polycrystalline NbTiN. We are continuing the further study by using high temperature superconductors in order to operate it at the higher temperature. In this paper, we report on calculation results of the kinetic inductance nonlinearity for the YBCO thin films.

2. YBCO thin film fabrication and superconducting properties

The YBCO thin films were deposited on the MgO substrate by a pulsed laser deposition (PLD) method and treated by annealing in oxygen atmosphere. In the deposition process, the distance between the target and the substrate was 3.5 cm, a YAG laser energy density was 1.26 J/cm^2, a repetition frequency was 10 Hz, an oxygen pressure in the chamber was 400 mTorr, and a substrate temperature was 820 °C. The annealing treatment after the deposition was carried out for 60 minutes in the oxygen pressure of 475 Torr and the temperature of 430 °C. The YBCO thin film thickness was 180 nm. The thin film was processed into narrow lines with a line width of 6 μm and a line length of 1 mm by a photolithography and a dry etching. The superconducting critical temperature (T_C) and the critical current density (J_C) with a 6 μm wide YBCO line were measured by a four-terminal measurement. Figure 1(a) shows the temperature-resistivity characteristics of the YBCO thin film with bias currents of 0.1 to 15.0 mA, and T_C was obtained of 78 K for 0.1mA bias current. Next, we extracted the J_C values from the data of Fig. 1(a). Figure 1(b) shows the J_C characteristics of the YBCO thin film, and J_C at 77 K was obtained of 1.0×10^5 A/cm^2. The value of J_C is smaller than that of the general one. However, low J_C is suitable for parametric amplifier operations because of low drive current operation. From the next section, we will simulate the performance of the YBCO parametric amplifiers using the obtained parameters.

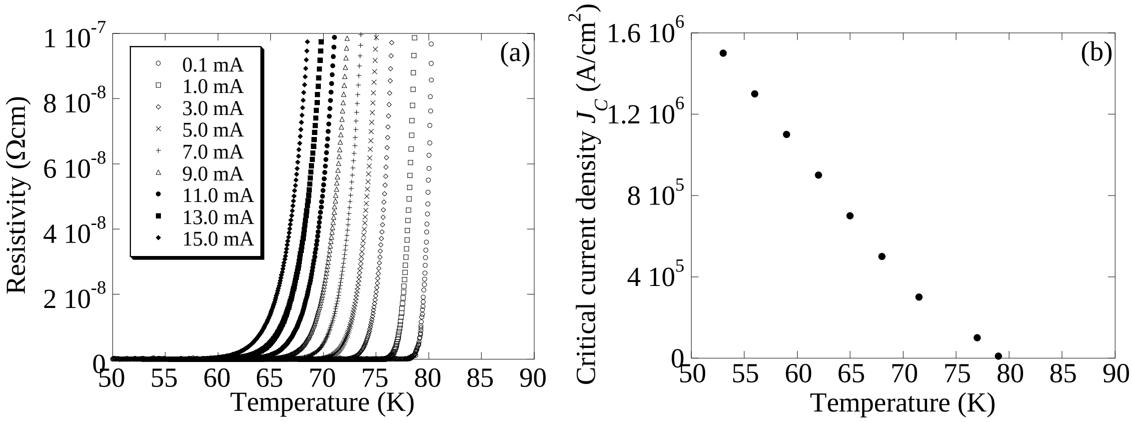

Figure 1. (a) Temperature-resistivity characteristics of the patterned YBCO thin film with 6 μm width. (b) Temperature-critical current density characteristics of the YBCO thin film.

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3. Superconducting parametric amplifier using YBCO thin film CPW

3.1. Kinetic inductance nonlinearity

In this section, we calculate the kinetic inductance nonlinearity in the YBCO thin film CPW. The L_T per unit length in superconducting CPW is expressed as

$$L_T = L_m + L_k. \tag{2}$$

When the film thickness d is thinner than the penetration depth λ ($d < 2\lambda$), both of the inductances are expressed by conformal mapping by TEM mode approximation [10].

$$L_m = \frac{\mu_0}{4}\frac{K(k')}{K(k)} \tag{3}$$

$$L_k = \mu_0 \frac{\lambda^2}{dw} g(s,w,d) \tag{4}$$

Where μ_0 is the permeability of vacuum, d is the film thickness, w is the width of signal line, s is the gap width between the signal line and the ground line, $K(k)$ is the complete elliptic integral of first kind and parameters k and k' is

$$k = \frac{w}{w + 2s}, \; k' = \sqrt{1 - k^2}. \tag{5}$$

Moreover, $g(s,w,d)$ is a geometrical factor of CPW given as:

$$\mathrm{g}(s,w,d) = \frac{1}{2k^2 K(k)^2}\left[-\ln\frac{d}{4w} - \frac{w}{w + 2s}\ln\frac{d}{4(w + 2s)} + \frac{2(w + s)}{w + 2s}\ln\frac{s}{w + s}\right] \tag{6}$$

From the above equations, the magnetic inductance depends only on the line pattern, whereas the kinetic inductance depends on both of the line pattern and the penetration depth. From Eq. (2), in order for the nonlinearity of the kinetic inductance to appear conspicuously in the superconducting transmission line, the kinetic inductance have to be dominant in the total inductance. Therefore, we examined the line pattern dependences of the kinetic inductance. Figure 2(a) and 2(b) show the film thickness and the line pattern dependences of the kinetic inductance, respectively. The kinetic inductance was normalized with the total inductance. From these results, the kinetic inductance becomes dominant by thinning the film thickness and narrowing the line width.

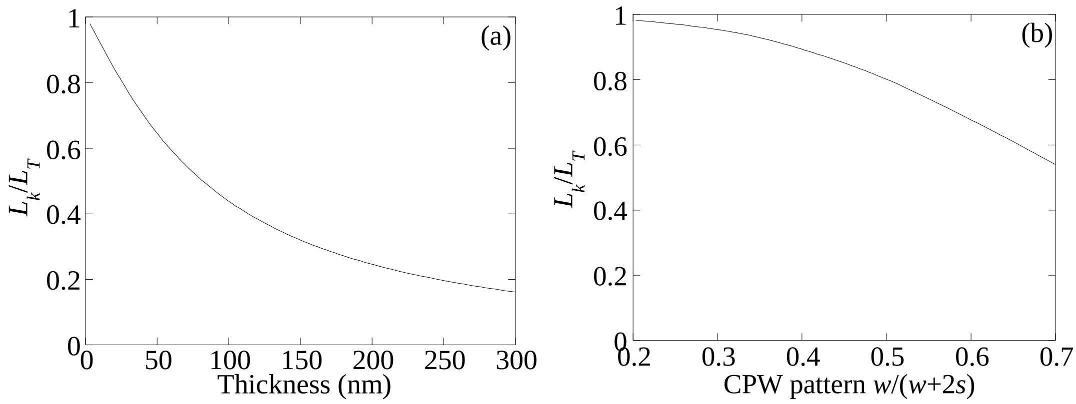

Figure 2. Kinetic inductance in superconducting CPW. The kinetic inductance was normalized with the total inductance. The vertical axes are (a) the film thickness d dependences (w = 4 μm, s = 2 μm), and (b) the line pattern $w/(w+s)$ dependences (d = 40 nm).

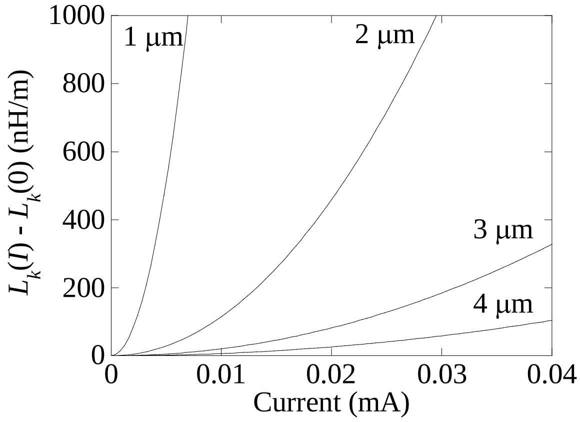

Figure 3. Kinetic inductance variations with respect to applied current.

Next, we calculated the kinetic inductance nonlinearity when the current was applied. In the calculation, the dimensions of the transmission line were set to the film thickness d = 40 nm, the width of signal line w = 1~4 μm, and the gap width between the signal line and the ground line s = 2 μm. In any case, since $L_k/L_T > 0.7$ is satisfied, the kinetic inductance is dominant in the total inductance. When the external current is applied to the superconducting transmission line, the kinetic inductance would change as shown in Eq. (1). Figure 3 shows the calculation results of the kinetic inductance variation $L_k(I) - L_k(0)$ in the existence of the applied current. Here, since $I^* \approx I_C$, calculations were implemented with assuming that I^* in Eq. (1) was I_C of the YBCO thin film. The I_C was calculated from J_C obtained in Section 2. The YBCO penetration depth λ at zero-temperature was set to 135 nm from the data of the reference [11], and an operating temperature was set to 77 K. Here, the longer the penetration depth, the stronger the kinetic inductance nonlinearity. From the Fig. 3, it is indicated that the reduction of the width of the signal line made, it possible to obtain a large change in the kinetic inductance with respect to the applied current. We fitted the curves in Fig. 3 to a polynomial, and derived their quadratic coefficient, which was defined as χ. In the case of the line width of 4 μm, the quadratic coefficient χ was 6.504×10^{-2} H/A^{2}m. In the previous study, the quadratic coefficients of the single crystal NbTiN thin film and the polycrystalline NbTiN thin film of the same pattern were 3.436 $\times 10^{-4}$ H/A^{2}m and 1.202×10^{-4} H/A^{2}m, respectively. Therefore, the nonlinearity of the YBCO thin film was stronger than the single crystal NbTiN thin film. That is, if the YBCO thin film is used, the high parametric gain can be obtained at the high temperature and a small applied current.

3.2. Calculation of amplification

The amplification process of the superconducting parametric amplifier is the degenerated four-wave mixing. Therefore, the amplification gain can be calculated by solving a coupled wave equation of the optical fiber parametric amplifier [12]. We assumed $2f_p = f_s + f_i$ where f_p, f_s and f_i, are a pump frequency, a signal frequency and an idler frequency, respectively. In the superconducting transmission line,

$$\Delta\beta = \beta(f_s) + \beta(f_i) - 2\beta\left(f_p\right) \tag{7}$$

is established in the linear dispersion relation, where $\beta(f)$ is a propagation constant. The parametric amplification gain derived from the coupled wave equation is

$$G_s = 1 + (\Delta\theta)^2. \tag{8}$$

At this point, $\Delta\theta$ is the amount of a nonlinear phase shift caused by the pump wave current.

$$\begin{aligned}\Delta\theta &= [\beta(0) - \beta(I)]l \\ &= \omega\left[\sqrt{L_T(0)C} - \sqrt{L_T(I)C}\right]l\end{aligned} \tag{9}$$

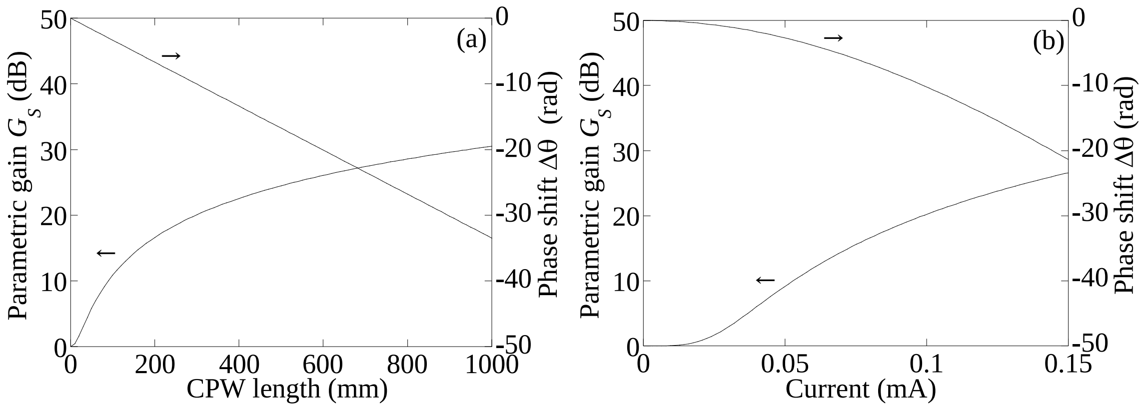

Figure 4. (a) Parametric gain and nonlinear phase shift as a function of the line length. Applied current was set to be 0.08 mA. (b) Parametric gain and nonlinear phase shift as a function of pump current. Line length was set to be 200 mm.

where l is the line length of the CPW. We calculated the amplification gain using the above equations. The CPW patterns were set to w = 4 μm, s = 2 μm and d = 40 nm. We assumed f_p = 4 GHz and $f_s \approx f_p$. Figure 4(a) is the parametric gain G_s and the nonlinear phase shift $\Delta\theta$ as a function of the line length l when the pump wave current is 0.08 mA. The longer the line length is, the larger the phase shift is, which indicated that the parametric gain also increases. In addition, we described that the YBCO thin film CPW has strong nonlinearity of the kinetic inductance in section 3.1, high parametric gain can be expected even with a short line length. Figure 4(b) shows the parametric gain as a function of pump current. The CPW length was set to be 200 mm. It is possible to realize high parametric gain even with a small pump current. As a result, a large parametric gain was expected for the YBCO, even if their configurations were with shorter line lengths and smaller pump current.

4. Conclusion

In order to realize the superconducting parametric amplifier, we calculated the kinetic inductance nonlinearity and the parametric gain in the YBCO thin film CPW. The kinetic inductance nonlinearity of YBCO was stronger than the single crystal NbTiN having strong nonlinearity. Therefore, the YBCO has a potential to be realized the high parametric gain, and it can be expected to operate in the range of the high frequency band, at the high temperature, and the small applied current. Moreover, from the fact that YBCO thin film CPW's kinetic inductance nonlinearity is strong, the use of the YBCO thin film CPW is considered to be useful for the realization of a other kinetic inductance devices.

Although some calculations were performed assuming that the phase matching condition was satisfied, the transmission line configurations have to be properly designed to realize the phase matching in the real operation. We are also studying for the realization of the phase matching for YBCO transmission lines, the results will be described elsewhere in the future. Moreover, we are planning to examine the nonlinearity and amplification gain in CPW more accurately for the parametric amplifier, and proceed the fabrication of superconducting CPW element.

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References

[1] Clarke J and Wilhelm F K 2008 *Nature* **453** 817.
[2] Lupascu A, Saito S, Picot T, de Groot P C, Harmans C J P M and Mooij J E 2007 *Nature. Phys* **3** 1031.
[3] Day P K, LeDuc H G, Mazin B A, Vayonakis A and Zmuidzinas J 2003 *Nature* **425** 817.

[4] Baselmans J 2012 *J. Low. Temp. Phys* **167** 292.
[5] Clerk A A, Devoret M H, Girvin S M, Marquardt F and Schoelkopf R J 2010 *Rev. Mod. Phys* **82** 1155.
[6] Muck M and Welzel C 2003 *Appl. Phys. Lett* **82** 3266.
[7] Eom B H, Day P K, LeDuc H G and Zmuidzinas J 2012 *Nature. Phys* **8** 623.
[8] Bockstiegel C, Gao J, Vissere M R, Sandberg M, Chaudhuri S, Sanders A, Vale L R, Irwin K D and Pappas D P 2014 *J. Low. Temp. Phys* **176** 476.
[9] Takeda M, Kojima T, Saito A, Makise K and Shimakage H 2016 *JJAP. Conf. Proc* **4** 011502.
[10] Watanabe K, Yoshida K, Aoki T and Kohjiro S 1994 *Jpn. J. Appl. Phys* **33** 5708.
[11] Pond J M, Carroll K R, Horwitz J H, Chrisey D B, Osofsky M S and Cestone V C 1991 *Appl. Phys. Lett* **59** 3033.
[12] Hansryd J, Andrekson P A, Westlund M, Li J and Hedekvist P O 2002 *IEEE J. Selected Topics in Quantum Electron* **8** 506.