Simulations of chaos generation from Josephson junctions with various junction parameters

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Abstract. It is well known that voltage waveforms between electrodes of Josephson junctions under irradiation of a microwave behave chaos characteristics under appropriate conditions. In order to apply the chaos to a random number generator, we have been studying Josephson chaos by simulations. In the simulation, the Josephson junction is assumed to fabricate with YBCO materials. We used a RCSJ model in order to present an equivalent circuit of the Josephson junction, and derived a derivative equation. Lyapunov exponents, which determined if the state of the Josephson junction was chaotic or not, were calculated from time evolutions of voltages obtained from the equation. In the simulation, junction parameters were assigned feasible values for an actual YBCO Josephson junctions. As a result, we found that chaos can be generated by adjusting element parameters. Moreover, we found that there were lower limits in the resistance values for generation of chaos. In addition, we found that frequency margins, at which the chaos was obtained, were broadened by decrease of the resistance and increase of capacitance.

1. Introduction

A random number generator is used for cryptographic communication [1], Monte Carlo simulation [2], a probability model [3], and so on. In particular, high-quality random numbers are required for security applications. In general, there exists two ways to generate the random numbers; one is a pseudo random number, which is generated by a computer algorithm, and the other is a physical random number, which is generated from a physical random phenomenon. The pseudo-random number generator can be easily implemented on any platform, and a random number generation speed depends only on the processing hardware. But, since its random number series was always determined by an initial condition of the algorithm, it is reproducible. On the other hand, the physical random number generator was affected by probabilistic physical phenomena such as thermal noise [4] in resistors or diodes, radiations [5], photons [6-8], and so on. Therefore, the random number output from the physical random number generator has an unreproducible characteristic, and is suitable for security application. However, generally, the generation speed of the physical random number generator is relatively slow, and the size of the generator itself is relatively large. For example, the generation speed of random numbers using photons is reported of about 16 Mbit/s [9]. Some approaches to solve these problems have been reported. The physical random number generator using semiconductor laser chaos as chaotic phenomena have been studied, and random number generation at 12.5 Gbit/s has been successfully achieved [10]. However, there is a periodicity due to return light of the laser, and it can't be completely removed.

On the other hand, the chaos phenomenon is also observed in a voltage oscillation from a Josephson junction under irradiated with microwaves. Up to now, the studies of chaos in the Josephson junction have been reported so far [11-13]. Main purposes of these studies were to study chaos theory and to suppress occurrence of chaos in a Josephson voltage standard. But, it is expected that chaotic oscillation by Josephson junction can be applied as a high-quality random number source. In fact, it has been reported that high-quality white noise can be generated by chaotic oscillation from Josephson junction by a numerical simulation [14]. Therefore, it has been shown that chaos from the Josephson junction behaves highly randomly [15]. Moreover, as the voltage in the Josephson junctions are oscillates at the frequency of several hundreds of GHz, it has the potential to extract the high-speed voltage output [16].

Our research objective is to realize a physical random number generator using chaos oscillation by the Josephson junction. In order to realize that, it is necessary to know whether chaos can be generated by using actual junction parameters of the Josephson junctions. In this paper, we report on the chaos simulations for various junction parameters, which are realized in actual devices.

2. Simulation

2.1. Simulation method

Josephson junction is represented by an equivalent circuit, in which a term responsible for a current flowing through a resistance R and a capacitance C is connected in parallel to a term responsible for Josephson current according to $I_c \sin\varphi$. This circuit is called RCSJ model and is often used for analysis of Josephson junctions. An external microwave irradiation $I_{rf} \sin\omega t$ and current source I_0 are added to its equivalent circuit, and a model of a circuit for generating chaos is shown in Fig. 1. A circuit equation is written from the circuit of Fig. 1, and Eq. (1) is obtained by normalization of the circuit equation.

$$i_0 + i_{rf}\sin\Omega\tau = \sin\varphi + \frac{\mathrm{d}\varphi}{\mathrm{d}\tau} + \beta\frac{\mathrm{d}^2\varphi}{\mathrm{d}\tau^2} \tag{1}$$

Where $i_0 = I_0 / I_c$, $i_{rf} = I_{rf} / I_c$, $\Omega = \hbar\omega / 2eI_cR$, $\beta = 2eI_cCR^2 / \hbar$, $\tau = 2eI_cR / \hbar$. A simulation of the voltage behavior was carried out using by Eq. (1). A time evolution of the voltage was calculated by a 4th-order Runge-Kutta method, a Lyapunov exponent λ was derived based on it, and a chaotic state was judged. The parameters of the junctions used in the simulation were set to values that can be realized in actual Josephson junction parameters. Since an I_cR of a general YBCO Josephson junction is about 2 mV, the resistance value and the critical current value were set to I_c = 1 mA, R = 2 Ω in this simulation, respectively. The capacitance value was set as C = 82.24 fF so as to set a McCamber parameter to be 1. Since we focused on simple operation of chaos generation as a first trial, the bias current was set to 0 mA. Using these parameters, we investigated the chaotic state in range of the irradiation frequency from 100 GHz to 1 THz.

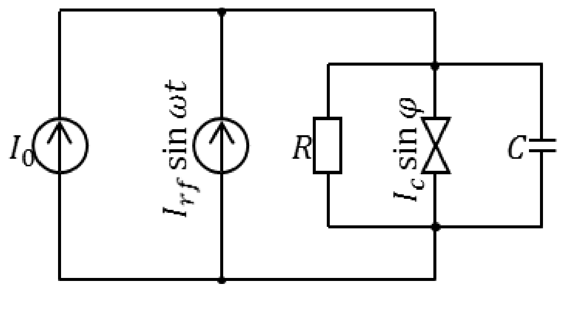

Figure 1. Resitively shunted junction model.

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2.2. Adjustment of resistance and capacitance value

First, the Lyapunov exponent was calculated using the initial parameters shown in the Table. 1, but the Lyapunov exponent was negative within the examined frequency. Therefore, we predicted that the frequency, at which the Lyapunov exponent becomes positive, would appear by changing the junction parameters. We tried to change the resistance, since it can be easily changed by resistive shunt on the Josephson junctions. Simulations were performed with resistance values of 2, 1.5, and 1 Ω. As the I_c value was unchanged, the I_cR_ns changed with the R_ns. The value of the Lyapunov exponent also decreased with decrease of the resistance value, and the frequency, at which the Lyapunov exponent was positive, did not appear. Next, we tried to change the capacitance value. As we supposed to use the shunt capacitance to the Josephson junction, the capacitance value would be increase because of parallel configuration. Therefore, simulation was performed by changing the capacitance to 164.4, 246.7, 328.9, and 411.2 fF, which are integer number times of initial capacitance value. Although we surveyed if the Josephson junctions in the chaos state at the capacitance value of 164.4 fF, the frequency, at which the

Table 1. The initial parameters used to calculate the Lyapunov exponent

I_0 [mA]	I_{rf} [mA]	R [Ω]	I_c [mA]	C [fF]
0	1	2	1	82.24

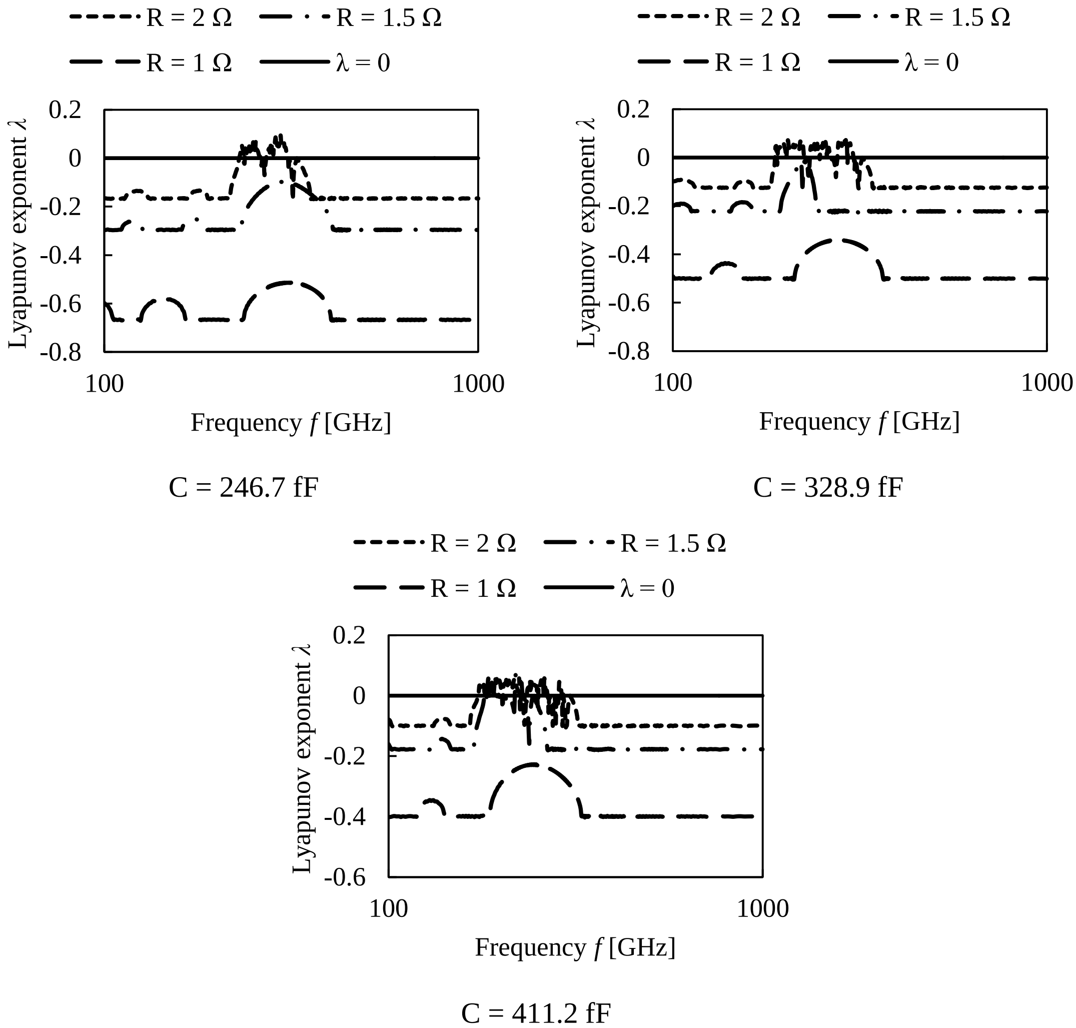

Figure 2. Frequency dependences of Lyapunov exponent and frequency when resistance values are 2, 1.5, and 1 Ω.

Lyapunov exponent was positive, did not appear. On the other hand, when we surveyed it as the same way at the capacitance values of the other values, we found the chaos state. The result is shown in Fig.

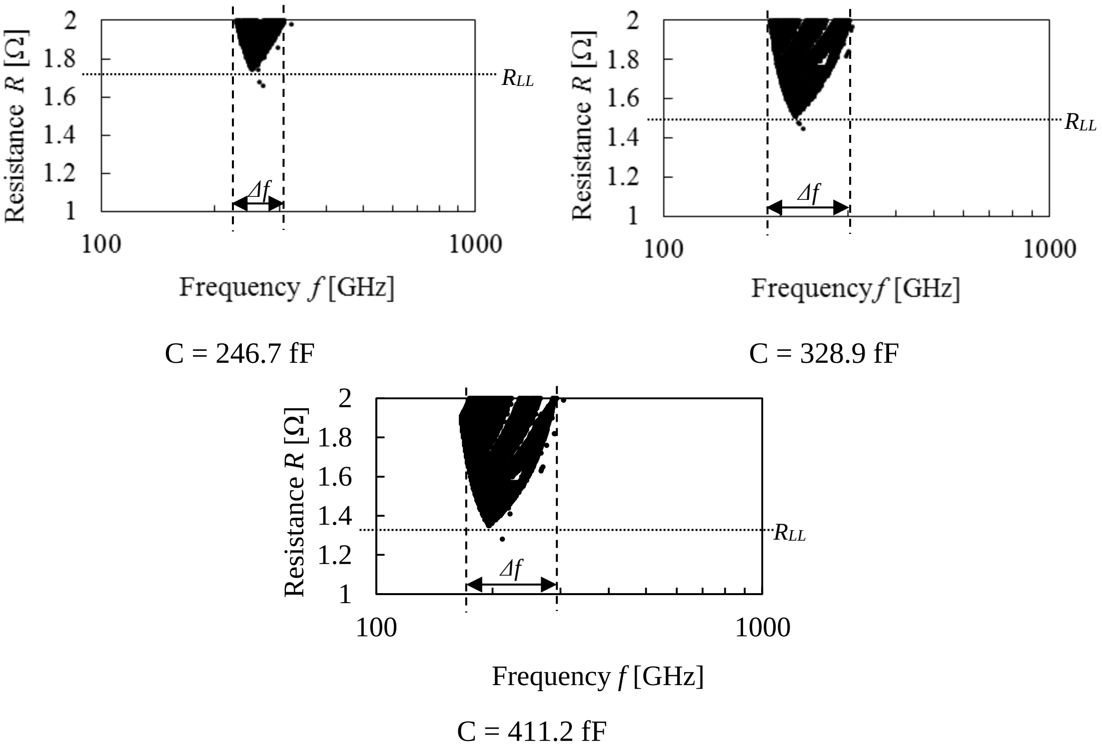

Figure 3. Results of mapping of the regions of chaos generation.

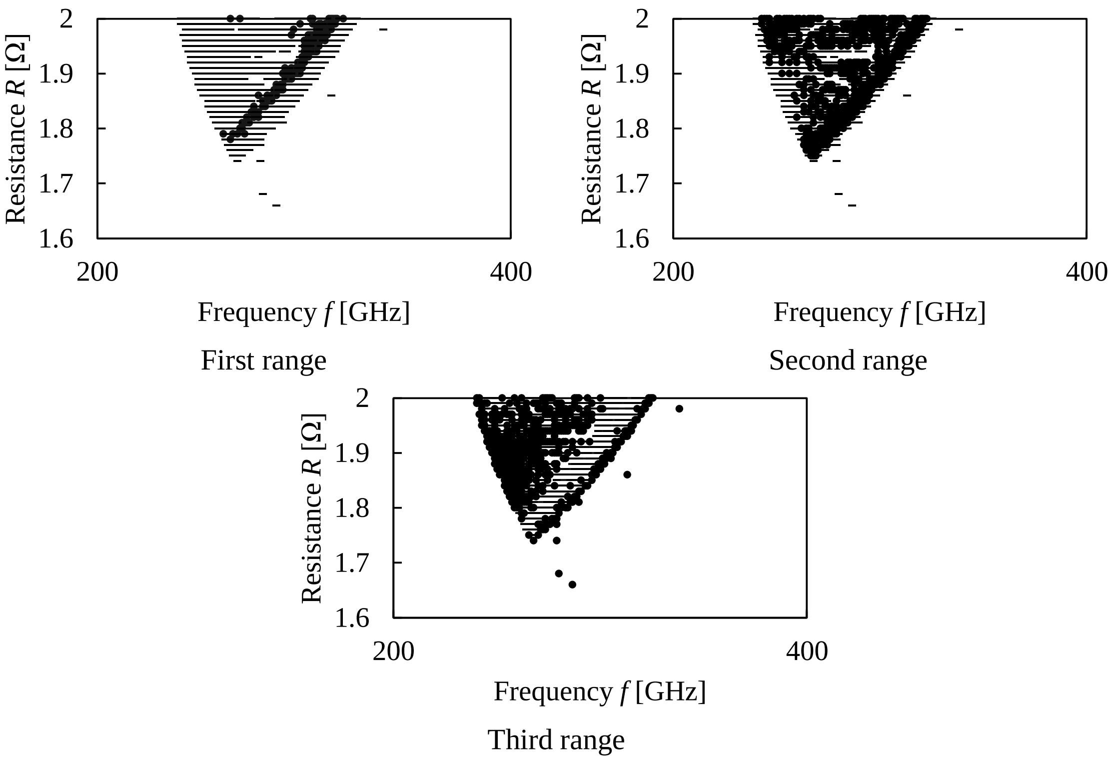

Figure 4. Results of mapping of the regions of chaos generation. The points in each map were plotted for the first range ($0.063795<\lambda\leq0.095696$), the second range ($0.031898<\lambda\leq0.063795$), and the third range ($0.000003<\lambda\leq0.031898$).

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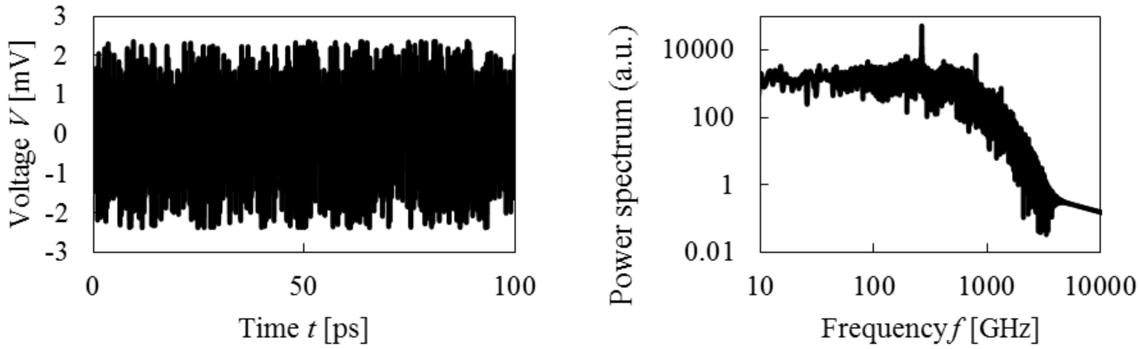
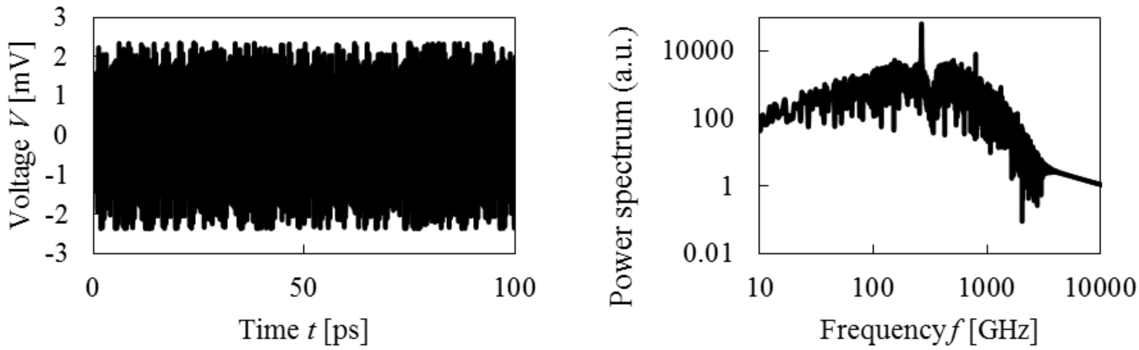
2. The positive Lyapunov exponents were obtained intermittently for any resistance values. To clarify the details of the region, at which the Josephson junctions were in the chaos state, we mapped the points of the positive Lyapunov exponents in Fig. 3. Here, in Fig.3 a vertical and horizontal axes are the frequency and the resistance, respectively. From the data of first range, second range, third range in Fig.3, the lower limit of the resistance values (R_{LL}s) were 1.74, 1.51, and 1.35 Ω, respectively. As the

Table 2. The parameters to simulate the voltage waveforms

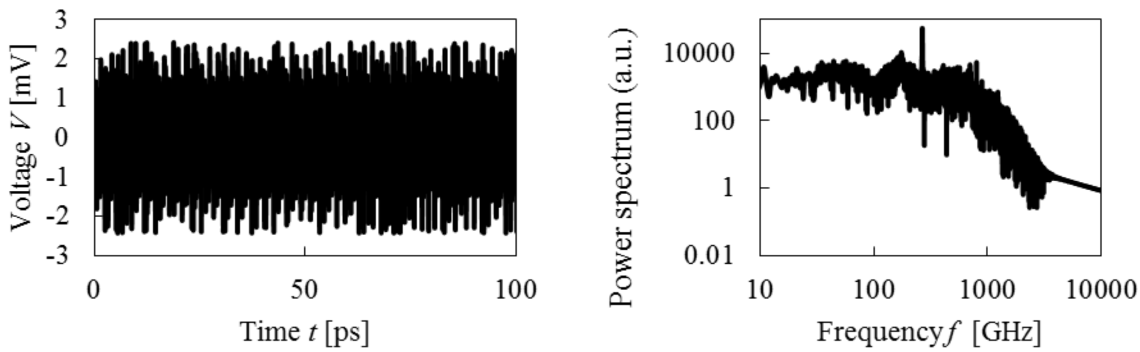
λ	I_0 [mA]	I_{rf} [mA]	f [GHz]	R [Ω]	I_c [mA]	C [fF]
0.095696	0	1	265	1.85	1	246.7
0.063205	0	1	265	1.83	1	246.7
0.031892	0	1	267	1.96	1	246.7



First range λ = 0.095696



Second range λ = 0.063205



Third range λ = 0.031892

Figure 5. Voltage waveforms and power spectra.

clear chaos regions were found in Fig. 3, we defined Δf as a frequency bandwidth at the resistance of 2 Ω. From the data of first range, second range, third range in Fig.3, Δf were 230-309, 188-303, and 174-293 GHz, respectively. As larger the capacitance value was, as broader the frequency bandwidth was, which indicated that a broad frequency margin would be utilized for a real random number generation. In addition, it was also found that increase of the capacity value made the chaos occurrence region shift to a lower frequency region, which indicated that we can use electromagnetic waves of relatively available GHz order.

2.3. Relationship between a magnitude of Lyapunov exponent and white noise

In general, if a Lyapunov exponent is positive, that is, the system is in the chaos state, the signal should be irregular. However, in some case, power spectra of waveforms may not be white-noise like. As the magnitude of the Lyapunov exponent might affect the power spectrum, we investigated an influence of the Lyapunov exponent for the voltage waveform and the power spectrum. Since it is difficult to set the capacitance to extremely large values for the actual Josephson junction, we investigated the influence only when the capacitance value is 246.7 fF. In order to clarify the influence of the Lyapunov exponent, a range from the maximum value 0.095696 to the minimum value 0.000003 of the Lyapunov exponent, which are obtained in our simulation, was equally divided into three. A first range is $0.063795 < \lambda \leq 0.095696$, a second range is $0.031898 < \lambda \leq 0.063795$, and a third range is $0.000003 < \lambda \leq 0.031898$. Figure 4 represents mappings of the regions of the chaos generation at the capacitance of 246.7 fF. Points in each map were plotted for the first, second, and third ranges. It was found that the region having a large Lyapunov exponent exists on the high frequency side as compared with the region having a small one. In addition, the voltage waveforms and the power spectra at the largest Lyapunov exponent λ = 0.095696(first range), 0.063205(second range), 0.031892(third range) in each range were calculated with the parameters shown in the Table 2., and are shown in Fig. 5. Although there were no clear significant differences in appearance in the waveforms due to differences of the Lyapunov exponents, clear differences below the irradiation frequency of 265, 265, 267 GHz was found in the power spectra. In the power spectrum of the second range, the power was not flat below the irradiation frequency. In the power spectrum of the third range, a dip around 100 GHz was observed. On the other hand, it was considered that the power spectrum of the first range was white noise. Therefore, we found that chaos can be generated with the available parameters in actual elements by adjusting element parameters

3. Conclusion and discussion

We described the simulations of chaos generation by using superconducting Josephson junctions in order to create a circuit to generate chaos. It was found that the range of irradiation frequency, in which the chaos generate, can be broaden by increase of the capacitance of the Josephson junction. However, chaos that can be used as white noise is a region with a large Lyapunov exponent. Therefore, when creating an actual element, it is necessary to operate in the region where the Lyapunov exponent having a large value concentrates. As a future schedule, we will consider how to increase the capacitance value in the actual element. Moreover, we plan to create the chaos generation circuit for the actual element.

Acknowledgement

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