Thermally-fluctuated single-flux-quantum pulse intervals reflected in input-output characteristics of a double-flux-quantum amplifier

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Abstract. A double-flux-quantum amplifier (DFQA) is a voltage multiplier of quantum accuracy, which we have employed at the final stage of a single-flux-quantum (SFQ) digital-to-analog converter (DAC). We recently found that experimental input–output (IO) characteristics of DFQAs were always slightly different from numerical results assuming ideally-periodic SFQ pulse trains. That is, experimental IO characteristics obtained using an over-biasing method were gradually deteriorated near their maximum operation voltages. Numerical simulation including the over-biasing method at a finite temperature suggested that the difference was likely to be attributed to thermally-fluctuated intervals of input SFQ pulses.

1. Introduction

Over-biasing a Josephson junction beyond its critical current is a simple method for feeding SFQ pulse trains into Josephson circuits, which we refer to as an "over-biasing method." We have employed the over-biasing method for evaluation of double-flux-quantum amplifiers (DFQAs) [1], especially for determination of their maximum input voltage $V_{\mathrm{IN-MAX}}$ and the corresponding Josephson frequency $f_{\mathrm{IN-MAX}} = V_{\mathrm{IN-MAX}}/\Phi_0$, where Φ_0 is a flux quantum [2, 3]. We have recently found that there is always slight difference between the experimental and numerical input-output (IO) characteristics near $V_{\mathrm{IN-MAX}}$ [4], for which the input SFQ pulses are fed by the over-biasing method in experiments and by periodic current pulses in simulation.

In this paper, we present experimental IO characteristics of a 100-fold DFQA and compare them with numerical results assuming different input methods of SFQ pulses. Numerical simulation including the over-biasing method at a finite temperature suggests that thermally-fluctuated pulse intervals gradually deteriorate IO characteristics of the DFQA near $V_{\mathrm{IN-MAX}}$ when the over-biasing method is employed for feeding SFQ pulses.

2. Experiments

We designed a 100-fold DFQA including 99-stacked three-junction loops (3JLs). The circuit configuration with the measurement set-up is shown in figure 1(a). In experiments, the input SFQ pulse train was generated by over-biasing a Josephson junction in front of a 10-stage Josephson transmission line (JTL). In other words, the input voltage V_{IN} was generated by

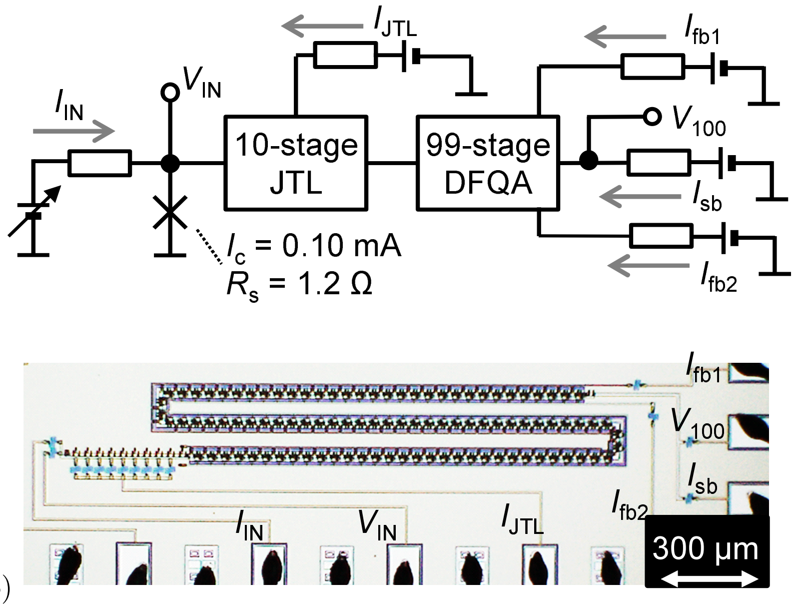
(a)
(b)


Figure 1. (a) Schematic configuration of a tested 100-fold DFQA comprising 99-stage 3JLs with the measurement set-up. (b) Photomicrograph of a test circuit fabricated using the AIST Nb STP2.

applying excess input current I_{IN} to the Josephson junction. The input and output voltages (V_{IN} and V_{100}) were measured using digital multi-meters.

Test chips were fabricated using the AIST 25-μA/μm^2 Nb standard process 2 (STP2). A photomicrograph of a fabricated circuit is shown in figure 1(b). In measurement, a test chip was cooled down in liquid helium.

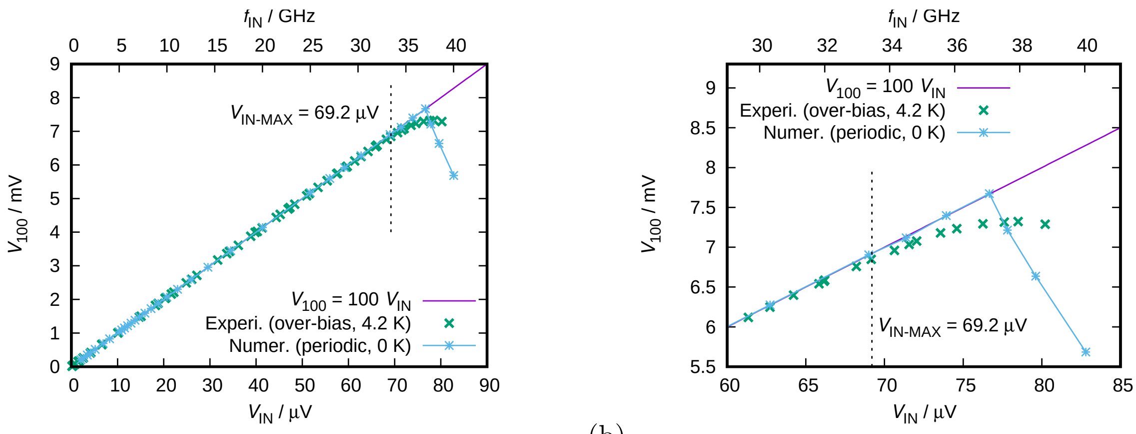
3. Results

The experimental IO (V_{IN}–V_{100}) relationships are presented in figure 2 as cross marks, where the ideal 100-fold voltage multiplication ($V_{100} = 100V_{\mathrm{IN}}$) is indicated by a solid straight line. It should be noted again that the over-biasing method was used for feeding SFQ input pulses. $V_{\mathrm{IN-MAX}}$ and $f_{\mathrm{IN-MAX}}$ are determined as 69.2 μV and 33.4 GHz, respectively, with multiplication errors less than 1%. These are typical values for our 100-fold DFQAs. It is confirmed that the experimental results gradually separated from the ideal $V_{100} = 100V_{\mathrm{IN}}$ line as V_{IN} increases beyond $V_{\mathrm{IN-MAX}}$.

Numerical simulation was executed assuming an ideally-periodic pulse current source for SFQ feeding. That is, periodic SFQ pulses were fed to the DFQA. The numerical IO characteristics are plotted as crosses combined with plus marks in figure 2. It is confirmed that numerical V_{100} follows the ideal relationship up to 77 μV, of which the corresponding frequency is 37 GHz, and that the numerical V_{100} values drop rapidly for $V_{\mathrm{IN}} > 77$ μV. Such rapid drop is not observed in the experimental results.

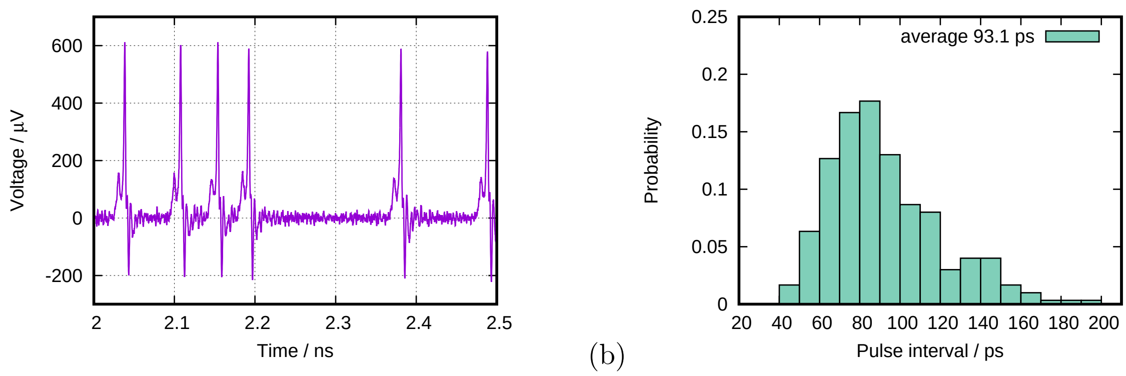
4. Discussion

To figure out the origin of such gradual degradation observed in the experimental IO characteristics, we employed the second simulation model where the over-biasing method was used for feeding the input SFQ pulses. Thermally-induced currents in resistors at 4.2 K were also included [5].



(a) (b)

Figure 2. Experimental and numerical V_{IN}–V_{100} characteristics. The lower and upper horizontal axis represent the input voltage V_{IN} and the corresponding SFQ repetition frequency f_{IN}, respectively. The vertical axis represents the output voltage V_{100}. The over-biasing method was used in experiments for feeding SFQ input pulses. In simulation, periodic SFQ pulses were fed into the 100-fold DFQA. (a) Wide range. (b) Narrow range near $V_{\mathrm{IN-MAX}}$.



(a) (b)

Figure 3. (a) Numerical input voltage waveform generated using the over-biasing method at 4.2 K. I_{IN} was adjusted to realize SFQ pulse intervals of 93.1 ps on average. SFQ pusles are not exactly periodic. (b) Histogram of SFQ pulse intervals for $V_{\mathrm{IN}} = 22.2$ μV, for which the corresponding f_{IN} and $1/f_{\mathrm{IN}}$ are 10.7 GHz and 93.1 ps. The pulse intervals deviate between 40 and 200 ps.

An example of input voltage waveforms generated using the over-biasing method is shown in figure 3(a). I_{IN} was adjusted to realize SFQ pulse intervals of 93.1 ps on average, for which V_{IN} was calculated to be 22.2 μV. It is confirmed that pulse intervals are fluctuated. Figure 3(b) shows a histogram of 300 SFQ pulse intervals. (We have confirmed that the number of pulse intervals should be more than 100 for the statistical processing in this work.) The pulse intervals deviate between 40 and 200 ps. That is, SFQ pulses generated using the over-biasing method at 4.2 K are not exactly periodic.

Such deviated pulse intervals should be reflected in the IO characteristics of DFQAs near $V_{\mathrm{IN-MAX}}$. Open squares in figure 4(a) represent the IO characteristics obtained using the second simulation model, that is, the circuit model including the over-biasing method at 4.2 K. Although there are still quantitative difference for the voltage range of $V_{\mathrm{IN}} > 77$ μV, gradual

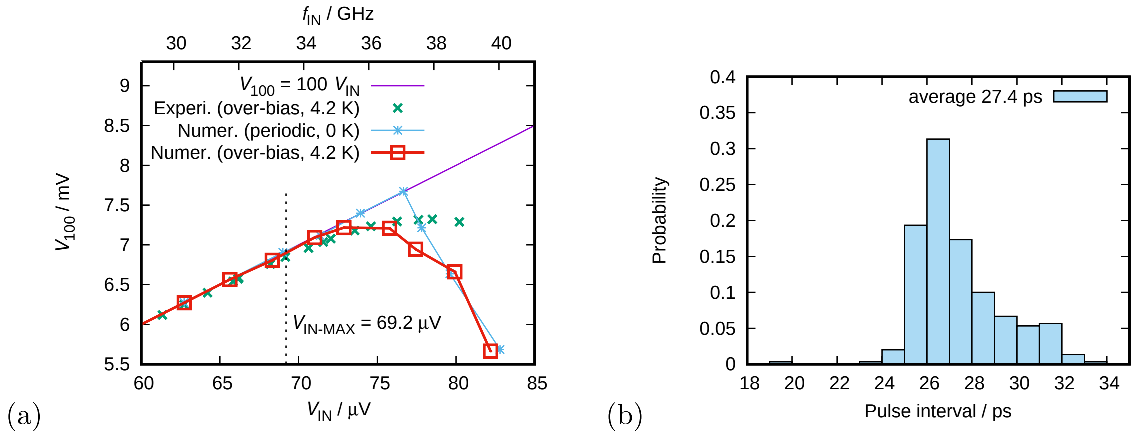


Figure 4. (a) Numerical V_{IN}–V_{100} characteristics of the second simulation model shown with other data on figure 2(b). The over-biasing method at 4.2 K was included in simulation. (b) Histogram of SFQ pulse intervals for V_{IN} = 75.6 μV (f_{IN} = 36.5 GHz, $1/f_{IN}$ = 27.4 ps). Pulse intervals shorter than 27 ps are out of the correct operation range of the DFQA.

degradation similar to the experimental V_{IN}–V_{100} characteristics are reproduced.

Figure 4(b) shows a histogram of 300 intervals in the input SFQ pulse train for V_{IN} = 75.6 μV. The corresponding f_{IN} and $1/f_{IN}$ are 36.5 GHz and 27.4 ps, respectively. It is found that the pulse intervals deviate between 19 and 34 ps, of which the corresponding frequencies are 52 and 29 GHz. As described in the previous section, the numerical simulation demonstrates that the DFQA operates for periodic SFQ pulses up to 37 GHz, and thus, pulse intervals shorter than 27 ps (= 1/37 GHz) are out of the correct operation range. The percentage of intervals shorter than 27 ps is as much as 22%, which cannot be ignored from the viewpoint of DFQA operation. That is, thermally-fluctuated SFQ pulse intervals should be a reason for the gradual degradation observed in experiments.

5. Conclusion

We presented experimental IO (V_{IN}–V_{100}) characteristics of a 100-fold DFQA fabricated using the Nb integration technology. The voltage multiplication was gradually deteriorated near the maximum input voltage. Numerical simulation assuming periodic input SFQ pulses did not exhibit such gradual degradation, whereas another numerical simulation assuming the over-biasing method with thermal noise current demonstrated gradual degradation similar to the experimental results. The numerical results suggested that SFQ pulse trains generated by the over-biasing method contained non-negligible deviation of SFQ pulse intervals.

Acknowledgments

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Appendix A. Distribution of SFQ pulse intervals generated using the over-biasing method

Although timing jitters induced by thermal noise have been investigated for two decades in the field of SFQ digital electronics [6], over-biased conditions are outside the scope of digital circuit operations. In this appendix, we describe the distributions of SFQ pulse intervals generated using the over-biasing method at 4.2 K.

Fitting results for the SFQ pulse intervals shown in figures 4(b) and 3(b) are respectively presented in figures A1(a) and A1(b). The normal distribution $N(\mu_{\rm N}, \sigma_{\rm N}^2)$ and the logarithmic normal distribution $LN(\mu_{\rm LN}, \sigma_{\rm LN}^2)$ are employed for fitting. Their probability density functions are expressed as $P_{\rm N}(x) = \left(\sqrt{2\pi}\sigma_{\rm N}\right)^{-1} \exp\left\{-(x-\mu_{\rm N})^2 / (2\sigma_{\rm N}^2)\right\}$ and $P_{\rm LN}(x) = \left(\sqrt{2\pi}\sigma_{\rm LN} x\right)^{-1} \exp\left\{-(\ln x-\mu_{\rm LN})^2 / (2\sigma_{\rm LN}^2)\right\}$, respectively.

In figure A1(a), there is no significant difference in the both fitting results for the average interval of 27.4 ps. On the other hand, the pulse intervals for the average interval of 93.1 ps are fitted better by using the logarithmic normal distribution, as shown in figure A1(b). Another fitting for the average interval of 215 ps (figure A1(c)) clearly demonstrates the adequacy of the logarithmic normal distribution.

In addition, the variances $\sigma_{\rm N}^2$ and $\sigma_{\rm LN}^2$ increase rapidly as the average intervals increases. This feature should be related to nonlinear characteristics of a Josephson junction. The I–V curve of a resistively-shunted-junction (RSJ) model is presented in figure A1(d), where the

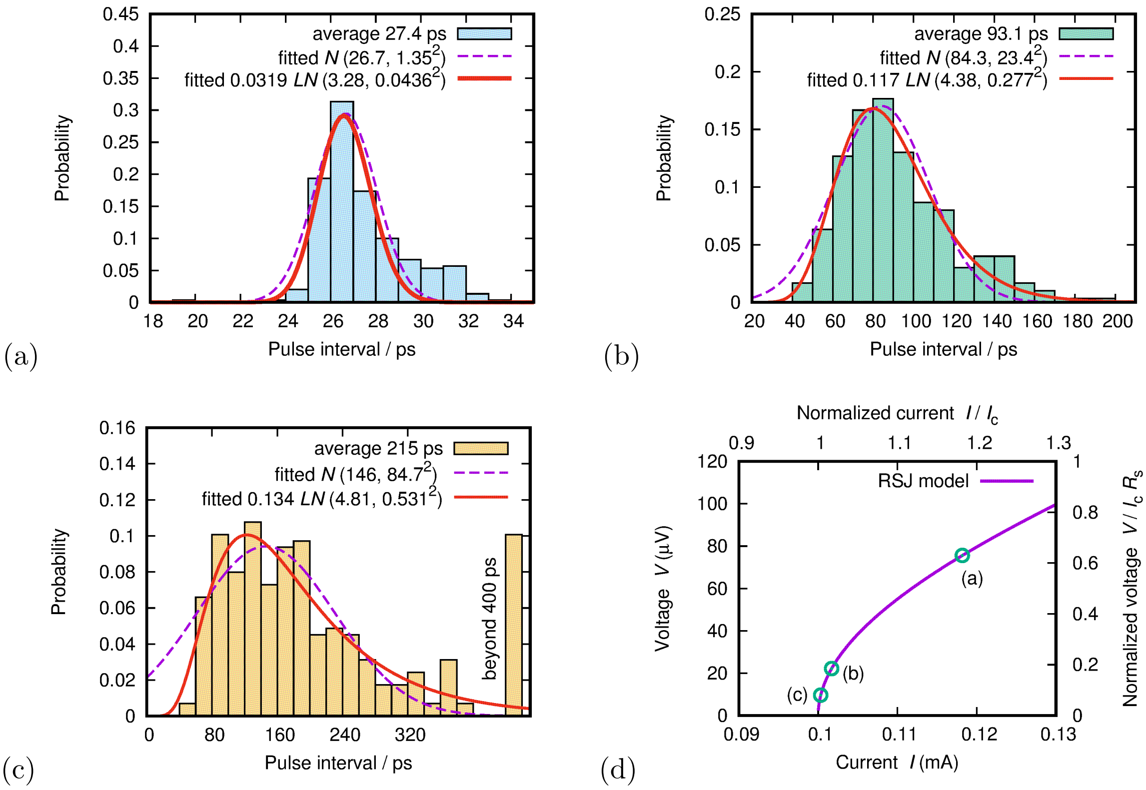

Figure A1. Probability distributions fitted to the SFQ pulse intervals of (a) 27.4, (b) 93.1, and (c) 215 ps on average. Both the normal distribution and the logarithmic normal distribution are employed for fitting. (d) I–V curve of an RSJ model. The average voltages corresponding to the SFQ pulse intervals of (a), (b), and (c) are indicated by open circles.

critical current and the shunting resistance are assumed to be 0.10 mA and 1.2 Ω. The average voltages corresponding to the average intervals for figures A1(a), A1(b), and A1(c) are indicated by open circles. The I–V curve is expressed as $V = R\sqrt{I^2 - I_c^2}$, whereas its derivative dV/dI is given by $RI/\sqrt{I^2 - I_c^2}$. (Since I_c is assumed to be much larger than the thermal current $2\pi k_B T/\Phi_0$ of 0.18 μA, the rounding effect around $I = I_c$ can be neglected [7].) dV/dI increases as I decreases down to I_c, which means that the voltage fluctuation is enhanced as the voltage decreases to zero. The voltage fluctuation is proportional to the frequency fluctuation, resulting in the enhancement of the variances σ_N^2 and σ_{LN}^2 at low voltages.

It should be noted that the large fluctuations at low voltages (long pulse intervals) do not modify the IO charcteristics of DFQAs unless they exceed the operation margins of DFQAs.

References

[1] Herr Q P 2005 Stacked double-flux-quantum output amplifier *IEEE Trans. Appl. Supercond.* **15** 259–62
[2] Sato Y, Moriya M, Shimada H, Mizugaki Y and Maezawa M 2013 Design and operation of 1000-fold voltage multiplier based on double-flux-quantum amplifier *Physics Procedia* **45** 221–4
[3] Mizugaki Y, Sato Y, Shimada H and Maezawa M 2014 Input-output characteristics of a 999-stage double-flux-quantum amplifier designed for 1000-fold voltage multiplication *Japan. J. Appl. Phys.* **53** 053101-1–4
[4] Urai Y, Shimada H and Mizugaki Y 2015 Feeding methods of SFQ pulse trains in evaluation of DFQ amplifiers *IEICE Technical Report* SCE2015-11 (in Japanese).
[5] Satchell J 1997 Stochastic simulation of SFQ logic *IEEE Trans. Appl. Supercond.* **7** 3315–8
[6] e.g., Ortlepp T and Uhlmann F H 2005 Noise induced timing jitter: a general restriction for high speed RSFQ devices *IEEE Trans. Appl. Supercond.* **15** 344–7
[7] Amegaokar V and Halperin B I 1969 Voltage due to thermal noise in the dc Josephson effect *Phys. Rev. Lett.* **22** 1364–6