

Magnetic field dependence of most stable vortex states in the chiral helimagnet / superconductor bilayer system

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Abstract. We have studied vortex states in chiral helimagnet / superconductor bilayer systems. An effect of a chiral helimagnet on a superconductor is taken as an external magnetic field. Solving the Ginzburg-Landau equations, we obtain various vortex configurations. In order to find a stable vortex state, we compare the Ginzburg-Landau free energies of these states. Under the large magnetic field from the chiral helimagnet, the state in which vortices appear in a row becomes more stable.

1. Introduction

A vortex state in a superconductor is affected by a magnet, in particular, a ferromagnet [1]. In a ferromagnet / superconductor bilayer structure, a magnetic structure of a ferromagnet causes a magnetic field. In a superconducting layer, vortices appear due to the magnetic field from the ferromagnet and vortex configuration is affected by the magnetic domain of the ferromagnet. So, the ferromagnet / superconductor bilayer structure enhances superconductivity, in particular, its critical current due to a pinning of vortices.

We focus on a chiral helimagnet / superconductor bilayer system. A chiral helimagnet has spins that form a helical rotation along a helical axis [2],[3], which is shown in figure 1. This helical configuration of spins comes from a competition between the ferromagnetic exchange interaction and the Dzyaloshinsky-Moriya interaction. The ferromagnetic exchange interaction causes nearest neighbor spins to be parallel. On the other hand, the Dzyaloshinsky-Moriya interaction causes nearest neighbor spins to be perpendicular to each other [4],[5]. Due to the competition between these two interactions, the helical configuration of spins is slightly deviated from the ferromagnetic configuration of spins. Then, all spins rotate along the helical axis. We expect that this magnetic structure of the chiral helimagnet affects the superconductor strongly. These influences may be different from those of the ferromagnet.

In this paper, we investigate effects of the chiral helimagnet on the superconductor in the chiral helimagnet / superconductor bilayer system. In particular, we focus on vortex states of a superconducting layer. To achieve our purpose, we solve the Ginzburg-Landau equations with the finite element method [6]-[8].



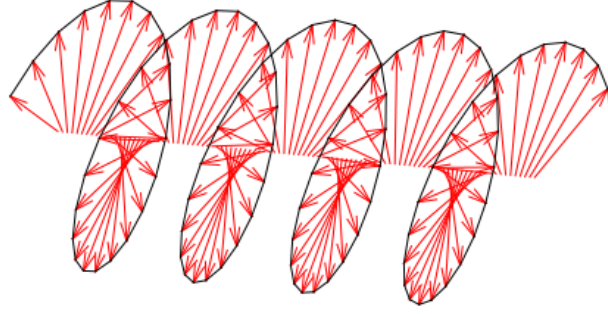


Figure 1. Spin configurations in the chiral helimagnet.

2. Methods

To obtain vortex states, we solve the Ginzburg-Landau equations,

$$\alpha|\psi|^2 + \beta|\psi|^2\psi + \frac{1}{2m^*} \left(\frac{\hbar}{i} \nabla - \frac{e^*}{c} \mathbf{A} \right)^2 \psi = 0, \quad (1)$$

$$\text{curl}(\text{curl} \mathbf{A} - \mathbf{H}_{\text{ext}}) = \frac{4\pi}{c} \mathbf{J} = \frac{4\pi}{c} \left\{ \frac{e^* \hbar}{2m^* i} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^{*2}}{m^* c} \psi^* \psi \mathbf{A} \right\}, \quad (2)$$

where $\alpha = \alpha_0(T - T_c)$, T is a temperature, T_c is a critical temperature. α_0 , β are constants. ψ is an superconducting order parameter, m^* is an effective mass, e^* is an effective charge, $\mathbf{A}$ is a magnetic vector potential, and $\mathbf{J}$ is a supercurrent density. As a system, we consider a chiral helimagnet / superconductor bilayer. The effect of the chiral helimagnet on the superconductor is taken as an external magnetic field $\mathbf{H}_{\text{ext}}$. Effects of the superconductor on the chiral helimagnet are neglected. In this study, we take into account only the perpendicular component of $\mathbf{H}_{\text{ext}}$, $(\mathbf{H}_{\text{ext}})_z$. Then, our model is equivalent to a superconductor under an oscillating magnetic field $(\mathbf{H}_{\text{ext}})_z$ in figure 2.

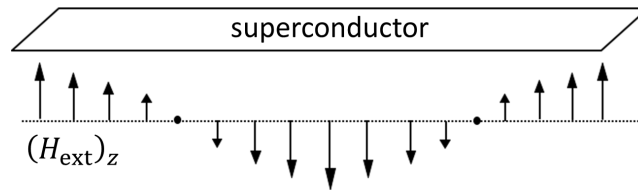


Figure 2. The superconductor under the magnetic field from the chiral helimagnet $(\mathbf{H}_{\text{ext}})_z$

We solve the Ginzburg-Landau equations (1) and (2) with the finite element method [9]. First, we give the initial values of the order parameter ψ randomly. Then, we obtain several stable solutions by iterations, which leads to various vortex states. In order to find the most stable state, we calculate the Ginzburg-Landau free energy,

$$\mathcal{F}(\psi, \mathbf{A}) = \int_{\Omega} \left[\frac{1}{\xi(T)^2} \frac{1}{2} (|\psi|^2 - 1)^2 + |(i\nabla - \tilde{\mathbf{A}})\psi|^2 \right] d\Omega + \kappa^2 \xi^2(T) \int_{\Omega} \left| \text{curl} \mathbf{A} - \frac{2\pi}{\Phi_0} \mathbf{H}_{\text{ext}} \right|^2 d\Omega, \quad (3)$$

where $\xi(T)$ is a coherence length and $\tilde{\mathbf{A}} = (2\pi/\Phi_0)\mathbf{A}$.

$\mathbf{H}_{\text{ext}}$ is expressed with the analytical solution of the chiral helimagnet, which is obtained from the hamiltonian[3],

$$\mathcal{H} = -J \sum_n \mathbf{S}_n \cdot \mathbf{S}_{n+1} + \mathbf{D} \cdot \sum_n \mathbf{S} \times \mathbf{S}_{n+1} + 2\mu_B \mathbf{H}_{\text{appl}} \cdot \sum_n \mathbf{S}_n^z, \quad (4)$$

where J is a strength of the exchange interaction, $\mathbf{S}_n$ is a n -th spin, $\mathbf{D} = (D, 0, 0)$ is a DM vector, μ_B is the Borh magneton, and $\mathbf{H}_{\text{appl}} = (0, 0, H_{\text{appl}})$ is a uniform applied magnetic field. The three terms represent the ferromagnetic exchange interaction, the Dzyaloshinsky-Moriya interaction, and the Zeeman energy, respectively. From this Hamiltonian, we obtain an angle between nearest spins $\theta(x)$,

$$\theta(x) = 2 \sin^{-1} \left[\text{sn} \left(\frac{(H^*)^{1/2}}{k} x \mid k \right) \right] + \pi, \quad (5)$$

where $\text{sn}(u|k)$ is the Jacobi's elliptic function and k is a modulus of the elliptic function. k is determined from the following relation,

$$\frac{\pi\alpha}{4(H^*)^{1/2}} = \frac{E(k)}{k}, \quad (6)$$

where $\alpha = \tan^{-1}(D/J)$ and $E(k)$ is the complete elliptic integral of the second kind. H^* is normalized applid magnetic field,

$$H^* = \frac{2\mu_B H_{\text{appl}}}{a^2 S^2 (J^2 + D^2)^{1/2}}, \quad (7)$$

where a is a lattice constant. We assume that the order of the lattice constant a is equal to the cohelence length at $T = 0$, ξ_0 . Using equation (5), we obtain $(\mathbf{H}_{\text{ext}})_z$,

$$(\mathbf{H}_{\text{ext}})_z(x) = H_0 \cos \theta(x) + H_{\text{appl}}. \quad (8)$$

In this equation, the first term is a magnetic field from the chiral helimagnet and the second term is the uniform applied magnetic field. The period of the helical rotation L' is given by,

$$L' = \frac{2kK(k)}{(H^*)^{1/2}}, \quad (9)$$

where $K(k)$ is the complete elliptic integral of the first kind.

3. Results and Discussions

We solve the Ginzburg-Landau equations and obtain various vortex states. We take the Ginzburg-Landau parameter $\kappa = \lambda_0/\xi_0 = 10$ where λ_0 is a penetration depth at $T = 0$. The ratio between two interaction strengths is taken from the $\text{Cr}_{1/3}\text{NbS}_2$, $D/J = 0.16$ [10]. The temperature T is $0.3T_c$. In this study, we use only the magnetic field from the chiral helimagnet. System size is $5.0L'\xi_0 \times 20\xi_0$, where L' is given by equation (9). For $D/J = 0.16$ and $H_{\text{appl}}/(\Phi_0/\xi_0^2) \sim 0.00$, L'/ξ_0 becomes 39.2699.

We show vortex states without the applied magnetic field in figure 3. These figures represent distributions of order parameter, phase, and magnetic field. In equation (8), $H_0/(\Phi_0/\xi_0^2)$ and $H_{\text{appl}}/(\Phi_0/\xi_0^2)$ are 0.04, 0.00, respectively. $\Phi_0 = \frac{hc}{2e}$ is an quantum flux. Figures 3 (a)-(d) are obtained from different initial distributions for the order parameter. In these states in figure 3, the numbers of vortices are (a) 0, (b) 2, (c) 4, and (d) 9. Each vortices appear around the peaks

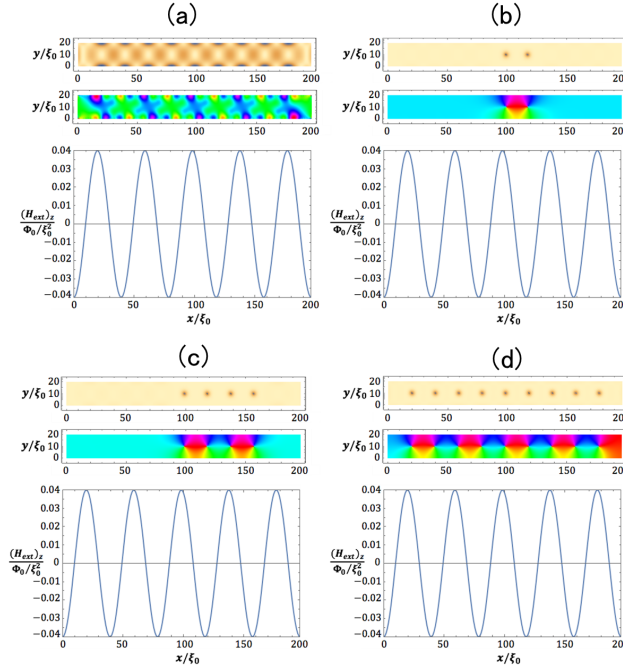


Figure 3. Distributions of order parameter, phase, and magnetic field. Magnetic fields in equation (8) are $(H_0/(\Phi_0/\xi_0^2))_z = 0.04$, $H_{\text{appl}} = 0.00$. (a)-(d) are different from initial values of the order parameter.

of the magnetic field $|(\mathbf{H}_{\text{ext}})_z/(\Phi_0/\xi_0^2)| = 0.04$ and these vortices are parallel to directions of the magnetic field.

Next, we calculate the Ginzburg-Landau free energy (equation (3)) in all states of figure 3. Dependences of free energies on magnetic fields are shown in figure 4. In figure 4, (a) red, (b) green, (c) blue, and (d) purple lines correspond to the states in figures 3 (a)-(d), respectively. For the small magnetic field $(\mathbf{H}_0/(\Phi_0/\xi_0^2))_z$, the state without vortices (= the Meissner state) has minimum free energy. However, for $(\mathbf{H}_0/(\Phi_0/\xi_0^2))_z \geq 0.64$, the state with nine vortices has minimum free energy.

From this result, we find that the most stable state of figures 3 (a)-(d) is the state without vortices (a) for the small magnetic field. Then, the most stable state changes into the state with vortices in a row (d) for the large magnetic field. Moreover, the states with two (b) and four (c) vortices are not the most stable states for all magnetic field. This transition of the most stable vortex state is different from those under the uniform magnetic field. Increasing the uniform magnetic field, the number of vortices increases as one, two, three, and so on. This difference can be explained considering the distributions of the magnetic field in two cases. In our model, the magnetic field $(\mathbf{H}_{\text{ext}})_z$ sinusoidally oscillates. Therefore, in the superconductor, there are two types of domain where the magnetic field is upward or downward. In other words, we regard this superconductor layer as an ensemble of small superconductors, whose size is $\frac{1}{2}L'\xi_0 \times 20\xi_0$. Each small superconductor is equivalent. When vortices appear in the superconductor, the state with a vortex in each domain has the same free energy, which is lower than that for Meissner state. Therefore, the state with vortices in a row (figure 3 (d)) is more stable than states with two and four vortices (figure 3 (b), (c)).

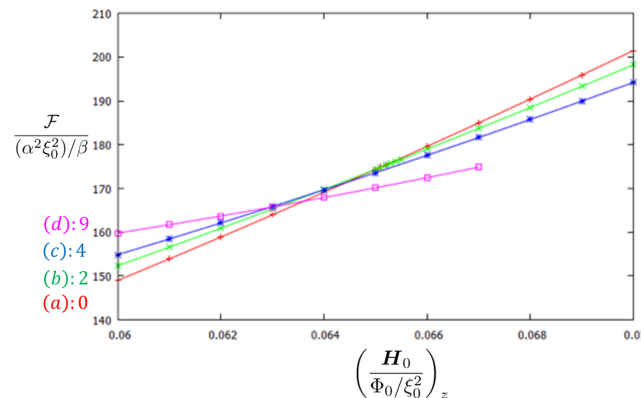


Figure 4. Magnetic field dependences of free energies. (a) Red, (b) green, (c) blue, and (d) purple lines correspond to the states in figure 3 (a)-(d), respectively.

4. Summary and Future

We have studied vortex states in the chiral helimagnet / superconductor bilayer system numerically. To investigate vortex states, we have solved the Ginzburg-Landau equations with the finite element method. We have obtained various vortex configurations. In order to find stable state, we have calculated the Ginzburg-Landau free energy. For the small magnetic field, the state without vortices is the stable state (Meissner state). On the other hand, for the large magnetic field, the state with vortices in all domains becomes stable.

Acknowledgments

This work was supported by JSPS KAKENHI Grant Number JP26400367.

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