Effect of magnetic reconnection in stellar plasma

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Abstract. An important phenomenon in Astrophysics is the process of magnetic reconnection (MGR), which is envisaged to understand the solar flares, coronal mass ejection, interaction of the solar wind with the Earth's magnetic field (so called geomagnetic storm) and other phenomena. In addition, it plays a role in the formation of stars. MGR involves topological change of a set of magnetic field lines leading to a new equilibrium configuration of lower magnetic energy. The MGR is basically described in the framework of the Maxwell's equations linked to Navier-Stockes equations. Nevertheless, many details are still not understood. In this paper, we investigate the MGR process in the framework of the Magnetohydrodynamic (MHD) model of a single conducting fluid using a modern powerful computational tool (OpenFOAM). We will show that the MGR process takes place only if resistivity exists. However, despite the high conductivity of the plasma, resistivity becomes effective in a very thin layer generating sharp gradients of the magnetic field, and thus accelerating the reconnection process. The net effect of MGR is that magnetic energy is converted into thermal and kinetic energies leading to heating and acceleration of charged particles. The Sun's coronal ejection is an example of the MGR process.

1. Introduction

MGR is a process of rearranging the magnetic field-lines topology leading to a restructuring of macroscopic quantities of plasma such as flow and thermal energy. It's observed in both astrophysical and laboratory plasmas. This phenomenon is important from the microphysics point of view, and for the wide range of its applications. Applying the resistive MHD approach to MGR is a challenging task, but it is worth doing, since MGR is important for the evolution of solar flares, coronal mass ejection, and is considered to occur in the formation of stars [1–5]. In addition, MGR plays a decisive role in understanding the interaction of the solar wind with the Earth's magnetosphere leading to geomagnetic storms [6], and to the formation of the auroras. NASA has launched a mission, namely "MMS", aimed at collecting data of the MGR process that takes place in the Earth's magnetosphere [7]. The theoretical approach of MHD model aims at seeking better description of the diffusion process during the MGR. In section 2, we show why resistive MHD is essential for the MGR process. In section 3, we present the simulation model. Results and conclusion are discussed in section 4.

2. Why resistive MHD?

When plasma moves, it carries the magnetic field lines along with it or vice-versa. This is what is known as the frozen-flux constraint. As shown in figure (1-a), the magnetic flux through a surface S, enclosed by a closed contour C, is given by
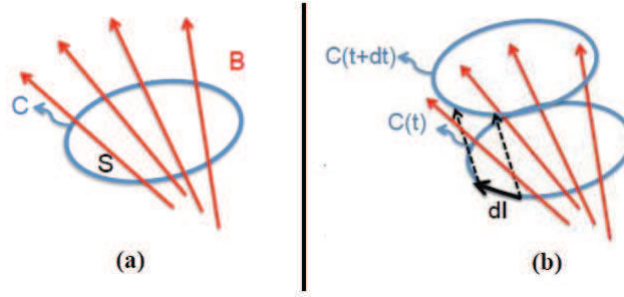
Figure 1. (a) The magnetic flux Ψ through a surface S enclosed by a contour C. (b) Varying the magnetic flux Ψ by moving the surface S with an arbitrary velocity $\vec{u} = \frac{\vec{dl}}{dt}$

$$\Psi = \int_S \vec{B}.\vec{dS} \tag{1}$$

If we want to change this magnetic flux with time, two possibilities are present:

$$\left(\frac{\partial \Psi}{\partial t}\right)_1 = \int_S \frac{\partial \vec{B}}{\partial t}.\vec{dS} = -\int_S \vec{\nabla} \times \vec{E}.\vec{dS} \tag{2}$$

or

$$\left(\frac{\partial \Psi}{\partial t}\right)_2 = \int_C \vec{B}.\vec{u} \times \vec{dl} = \int_S \vec{\nabla} \times (\vec{B} \times \vec{u}).\vec{dS} \tag{3}$$

In equation (2), the magnetic field $\vec{B}$ itself can change in time. While, in equation (3) and figure (1-b), the surface S can move with an arbitrary velocity $\vec{u}$. Combining the two contributions, we get the full time-varying magnetic flux,

$$\frac{d\Psi}{dt} = -\int_S \vec{\nabla} \times (\vec{E} + \vec{u} \times \vec{B}).\vec{dS} \tag{4}$$

Recalling Ohm's law from Maxwell's equations, $\vec{E} + \vec{u} \times \vec{B} = \eta \vec{J}$, where η and $\vec{J}$ are respectively the plasma resistivity and current density. If we neglect collisions, plasma resistivity vanishes, and magnetic flux is thus time-invariant. Therefore, the magnetic field lines must move with the plasma and the frozen-flux constraint holds. In contrast, with resistivity the plasma particles leave their own magnetic field line to get stuck on another field line, and hence the MGR mechanism occurs. So, plasma resistivity is a necessary condition to allow for MGR. However; plasma is a very good conductor, even though resistivity becomes efficient in a very fine region giving rise to sharp gradients of the magnetic field. Such a region, according to Ampère's law, holds then a large current, hence, the term current layer or current sheet.

3. Simulation model
Our simulation is carried out in a two-dimensional domain where $x \in [-16\delta, +16\delta]$ and $y \in [-32\delta, +32\delta]$. $\delta = 1.5 \times 10^6$ m is the half width of the current layer. The astrophysical plasma is treated here as a conducting fluid, so coupling between Navier-Stokes and Maxwell's equations is essential in the simulation of MGR. The obtained equations are the so-called: "Resistive MHD Model". It has four principal equations, continuity, momentum, induction and energy equations.

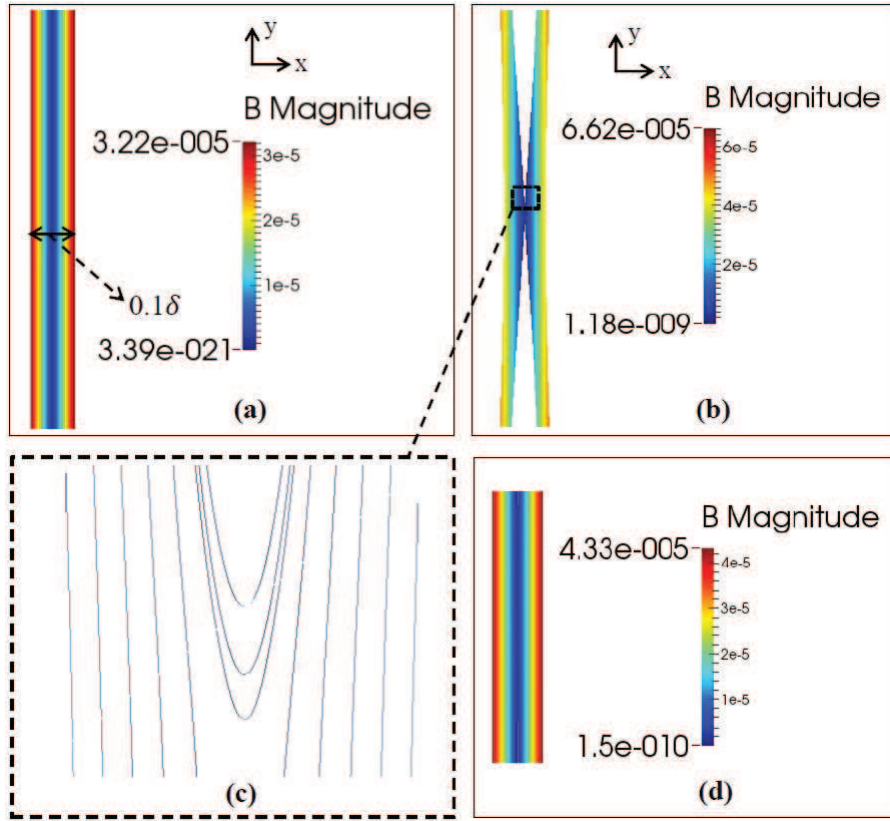$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.(\rho \vec{u}) = 0 \tag{5}$$

Figure 2. (a) Oppositely directed magnetic fields (downward magnetic fields in the left side and upward fields in the right side), expressed in Tesla, at time t = 0. (b) The MGR process takes place around the origin at t = 60 sec for $\tilde{\eta}$ = 200 Ω.m. (c) A zoom of the reconnection region around the origin at time t = 60 sec for $\tilde{\eta}$ = 200 Ω.m. (d) The current layer does not show any MGR process at time t = 60 sec for $\eta = 0$.

$$\frac{\partial(\rho\vec{u})}{\partial t} + \vec{\nabla}.(\rho\vec{u}\vec{u}) = -\vec{\nabla}(p + \frac{B^2}{2\mu_0}) + \vec{\nabla}.(\frac{\vec{B}\vec{B}}{\mu_0}) \tag{6}$$

$$\frac{\partial\vec{B}}{\partial t} + \vec{\nabla}.(\vec{u}\vec{B}) - \frac{\eta}{\mu_0}\vec{\nabla}^2\vec{B} = \vec{\nabla}.(\vec{B}\vec{u}) + \vec{J} \times \vec{\nabla}\eta \tag{7}$$

$$\frac{\partial\rho E}{\partial t} + \vec{\nabla}.(\rho\vec{u}E) = -\vec{\nabla}.[(p + \frac{B^2}{2\mu_0})\vec{u}] + \frac{1}{\mu_0}\vec{\nabla}.[(\vec{B}.\vec{u})\vec{B}] - \frac{1}{\mu_0}\vec{\nabla}.(\eta\vec{J} \times \vec{B}) \tag{8}$$

$E = e + \frac{1}{2}u^2 + \frac{B^2}{2\rho\mu_0}$ is the total energy per unit mass. e is the internal energy per unit mass, ρ is the plasma density (ρ_0=1.7 × 10^{-12} kg.m^{-3} for hydrogen plasma), $\vec{u}$ is the plasma velocity, $\vec{B}$ is the magnetic field ($B_0 = 5$ Gauss is the typical magnetic field) and μ_0 is the permeability (=4π × 10^{-7} SI units). The plasma here is treated as an ideal gas where plasma pressure can be written in terms of internal energy according to $p = (\gamma - 1)\rho e$ ($\gamma = 5/3$ for hydrogen plasma). The functional form of resistivity used is $\eta = \eta_0 + \tilde{\eta}e^{-(x^2+y^2)}$ [8], where $\eta_0 = 1$ Ω.m is the background resistivity and $\tilde{\eta} = 200$ Ω.m is the enhanced resistivity around the origin to ensure the MGR process. The simulation of MGR is done using non-uniform grid spacing, from 0.3δ at boundaries down to $5\mu\delta$ around origin. The time step is 10 ms. As initial conditions, we

use uniform density ρ_0, uniform temperature T_0 (= 2 Mk), we start with exactly zero-velocity and the initial thermal pressure is obtained by solving the equilibrium equations. The initial profile of magnetic field is chosen to be along y-direction, $B_y = B_0\tanh(x)$. As for boundary conditions, a periodic boundary condition is used in the y-direction. In the x-direction, magnetic pressure (of value 0.1 Pascal) is imposed at boundaries, while a zero-gradient condition is used for velocity, temperature and magnetic field.

4. Results and conclusion

Eqs.(5, 6, 7, 8) are four nonlinearly coupled equations. Numerical simulation of the governing equations are carried out using the open source OpenFOAM toolbox [9]. The plasma dynamics is treated here as a compressible fluid motion and the equations are solved in an Eulerian frame using the finite volume method [10].

Two runs are done to see the effect of resistivity on the MGR process. The initial profile of the magnetic field is presented by figure (2-a). Figures (2-b) and (2-c) show evidence of reconnecting field lines at time t = 60 sec if the resistivity is enhanced around the origin. However; MGR disappears in figure(2-d) where resistivity is zero. These results support what we have discussed in section 2 where the resistive MHD model is an important candidate for the MGR process. Of course, this is not the end, we still need to test the energy released during MGR [11], because it is the clue beyond some of the solar activities (solar wind, coronal mass ejection, geomagnetic storm, etc... .). But, energy analysis needs a lot of work and results, and this will be left for future work.

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