

Thin accretion disks around a rotating compact object surrounded by quintessence matter

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Abstract. We study the influence of quintessential matter on the equations describing the properties of a thin accretion disk around a rotating compact object. From these, we show that quintessence matter may play an important role in the accretion process.

1. Introduction

Current astronomical observations suggest that the Universe is undergoing an accelerated expansion. One model used to explain this behavior is the existence of a dynamic scalar field known as quintessence. This field has been incorporated by Kiselev [1] in a static and spherically symmetric solution of the Einstein field equations, which has been generalized to a rotating solution in [2–4]. Some general aspects of spherical accretion onto static Kiselev black holes have been presented in [5]. In this work, we will derive the equations describing the properties of a thin disk [6–8] around a rotating compact object surrounded by quintessence matter to show that this field may play an important role in the accretion.

2. The Rotating Kiselev Solution

Recently, a rotating generalization of the Kiselev's metric [1] was obtained using the Newman-Janis algorithm [2–4] with a line element in Boyer-Lindquist coordinates written as

$$ds^2 = - \left(\frac{\Sigma \Delta}{\Xi} + \frac{\varpi^2 \Xi \sin^2 \theta}{\Sigma} \right) dt^2 - \frac{2\varpi \Xi \sin^2 \theta}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\Xi}{\Sigma} \sin^2 \theta d\phi^2, \quad (1)$$

where $\Delta = r^2 + a^2 - 2Mr - \alpha r^{1-3\omega}$, $\Sigma = r^2 + a^2 \cos^2 \theta$, $\Xi = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta$ and $\varpi = \frac{a}{\Xi} (2Mr + \alpha r^{1-3\omega})$. This solution describes a compact object characterized by its mass M , angular momentum a and a parameter α which measures the intensity of a quintessence field with equation of state $P_q = \omega \rho_q$, where P_q and ρ_q are its pressure and energy density [1, 9, 10] and the state parameter takes values in the range $-1 < \omega < -\frac{1}{3}$. In this work, we are interested in the particular value $\omega = -\frac{2}{3}$, which describes an object surrounded by quintessence matter. For black holes, the horizon structure of this solution is defined by the relation $\Delta = 0$, which gives, in general, three horizons: an inner r_- , an outer r_+ and a cosmological horizon r_q . The existence of a black hole spacetime (with horizons) is restricted by the conditions $\alpha \leq \frac{1}{6}$ and

$$a \leq a_{crit} = \sqrt{\frac{2(-1+9\alpha+(1-6\alpha)^{3/2})}{27\alpha^2}} \quad (\text{see [2] for the details}).$$



3. The accretion disk structure equations

Consider a thin accretion disk with a very small total mass in the equatorial plane around a rotating compact object described by equation (1). In order to study the dynamics of the gas flow, we will consider a general energy-momentum tensor of the form $T^{\mu\nu} = (\rho + \epsilon + P) U^\mu U^\nu + P g^{\mu\nu} + S^{\mu\nu} + U^\mu q^\nu + U^\nu q^\mu$, [7,8] where ρ is the rest-mass density, ϵ is the thermal energy density and P is the pressure, including gas and radiation contributions. U^μ represents the 4-velocity of the flow and contains the velocity 3-vector $\vec{u}$. q^ν is the energy-flux and $S^{\mu\nu}$ is the viscous stress tensor, written in terms of the bulk viscosity, ξ , and the dynamic viscosity, η , as in [11]: $S^{\mu\nu} = \eta (h^{\mu\sigma} \nabla_\sigma u^\nu + h^{\nu\sigma} \nabla_\sigma u^\mu) + \left(\xi - \frac{2}{3} \eta \right) h^{\mu\nu} \nabla_\sigma u^\sigma$, with $h^{\mu\nu} = U^\mu U^\nu + g^{\mu\nu}$. As usual, these contributions satisfy the constraints $S^{\mu\nu} U_\nu = 0$ and $q^\mu U_\mu = 0$.

The structure equations for the accretion disk are the energy momentum balance, $\nabla_\nu T^{\mu\nu} = 0$, together with the continuity equation (baryon conservation), $\nabla_\mu (\rho U^\mu) = 0$, an equation of state, a viscosity prescription and an equation describing the overall energy transport mechanism. In order to study these equations, we will consider some simplifying assumptions [12,13], in which the structure of the accretion disk is build using cylindrical coordinates (t, r, ϕ, z) and

- The disk is time independent and axisymmetric (derivatives with respect to t and ϕ vanish).
- The disk is geometrically thin, i.e. the proper height satisfies, $H(r) \ll r$ or $\frac{\partial_r P}{\partial_z P} \sim \frac{z}{r} \ll 1$.
- The gas is moving on almost Keplerian orbits with azimuthal velocity $u_\phi \approx \sqrt{\frac{GM}{r}}$, but also has a small radial drift velocity, u_r , and a smaller vertical velocity, u_z . From the continuity equation one can obtain the conditions $\frac{u_z}{u_r} \sim \frac{z}{r} \ll 1$ and $\frac{u_r}{u_\phi} \sim \frac{H^2}{r^2} \ll 1$ [6].

In order to write the structure equations for the thin disk, we introduce the radial functions

$$A = 1 - \frac{2M}{r} - \frac{\alpha}{r^{3\omega+1}} + \frac{a^2}{r^2}, \quad (2)$$

$$B = 1 - \frac{3M}{r} - \frac{3\alpha}{2r^{3\omega+1}} + \frac{2a}{r} \sqrt{\frac{M}{r} + \frac{\alpha}{2r^{3\omega+1}}}, \quad (3)$$

$$C = 1 - \frac{4a}{r} \sqrt{\frac{M}{r} + \frac{\alpha}{2r^{3\omega+1}}} + \frac{3a^2}{r^2}, \quad (4)$$

$$D = \frac{1}{2\sqrt{r}} \int_{r_0}^r \frac{x^2 - 6Mx - 3\alpha x^{1-3\omega} + 8a\sqrt{Mx + \frac{\alpha}{2}x^{1-3\omega}} - 3a^2}{x^2 - 3Mx - \frac{3\alpha}{2}x^{1-3\omega} + 2a\sqrt{Mx + \frac{\alpha}{2}x^{1-3\omega}}} \frac{dx}{\sqrt{x}}, \quad (5)$$

$$F = 1 + \frac{\alpha}{2r^{3\omega+1}} \left(\frac{M}{r} \right)^{-1}, \quad (6)$$

where r_0 is the inner boundary of the disk (marginally stable circular orbit for a black hole [14]).

Considering a non-relativistic gas, $\rho \gg \epsilon$ and $\rho \gg P$ [15], we project $\nabla_\nu T^{\mu\nu} = 0$ onto the 4-velocity and onto the 3-space orthogonal to U^μ to obtain the equations

$$U^\nu \partial_\nu \epsilon + (P + \epsilon) \nabla_\nu U^\nu + S^{\mu\nu} \nabla_\mu U_\nu + \nabla_\nu q^\nu + q^\mu U^\nu \nabla_\mu U_\nu = 0 \quad (7)$$

and

$$[U^i U^\mu + g^{i\mu}] \partial_\mu P + \rho U^\mu \nabla_\mu U^i + \nabla_\mu S^{i\mu} + U^i U_\nu \nabla_\mu S^{i\mu} + q^i \nabla_\mu U^\mu + q^\mu \nabla_\mu U^i + U^\mu \nabla_\mu q^i - U^i U^\mu q^\nu \nabla_\mu U_\nu = 0. \quad (8)$$

The velocity U^μ defines a local co-rotating reference frame from which it is possible to define an orthonormal tetrad $b_{(\alpha)}^\mu$ with $b_{(0)}^\mu = U^\mu$ and $g_{\mu\nu} b_{(\alpha)}^\mu b_{(\beta)}^\nu = \eta_{\alpha\beta}$ (details in [7,8,16]). Using the $b_{(\alpha)}^\mu$, we define the tetrad components of $S^{\mu\nu}$ and q^μ , denoted as $-t_{\alpha\beta}$ and F_α , respectively.

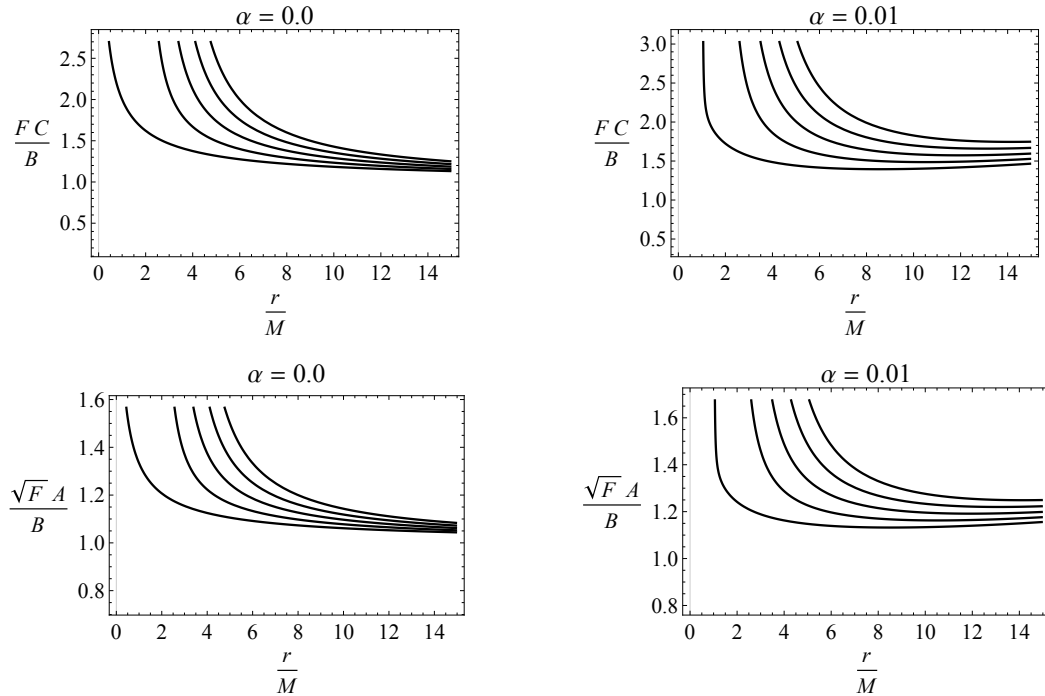


Figure 1. Relativistic correction factors $\frac{FC}{B}$, which affects the hydrostatic equilibrium equation (10) and $\frac{\sqrt{F}A}{B}$ appearing in the energy balance in tetrad components. We plot Kerr, $\alpha = 0$, and the $\alpha = 0.01$ rotating metric. Curves represent $\frac{a}{M} = 0, 0.25, 0.5, 0.75$ and 1.0 from right to left.

The z - component of equation (8) gives the vertical momentum balance,

$$\partial_z P = -\rho z \frac{M}{r^3} \frac{FC}{B}. \quad (9)$$

Introducing the radial component of the 4-velocity, written through the continuity equation as $r \int_{-H(r)}^{H(r)} (\rho U^1) dz = r \langle \rho U^1 \rangle = -\frac{\dot{M}}{2\pi}$ with $\dot{M}$ is the total accreted mass flux, the azimuthal component of equation (8) is written in terms of the tetrad components of the stress tensor. After integrating over the height of the disk and using equation (3), it gives

$$\langle t_{r\phi} \rangle = \frac{\dot{M}}{2\pi} \sqrt{\frac{M}{r^3}} \frac{\sqrt{F}D}{A}. \quad (10)$$

Finally, from the energy balance equation (7) in tetrad components we obtain $\frac{\partial F_z}{\partial z} = \frac{3}{4} \sqrt{\frac{M}{r^3}} \frac{\sqrt{F}A}{B} t_{r\phi}$, which integrated over the disk height and using (10) becomes $F_z = \frac{3M\dot{M}}{8\pi r^3} \frac{\sqrt{F}D}{B}$.

When comparing the obtained equations with the non-relativistic results, it is easy to identify that the relativistic corrections are condensed into functions A, B, C, D and F . In Figures 1 and 2 we show the behavior of four of the relativistic correction factors appearing in the structure equations for the particular case of quintessence, $\omega = -\frac{2}{3}$. It is important to note that the existence of quintessence, even for small α , changes the behavior of the corrections importantly.

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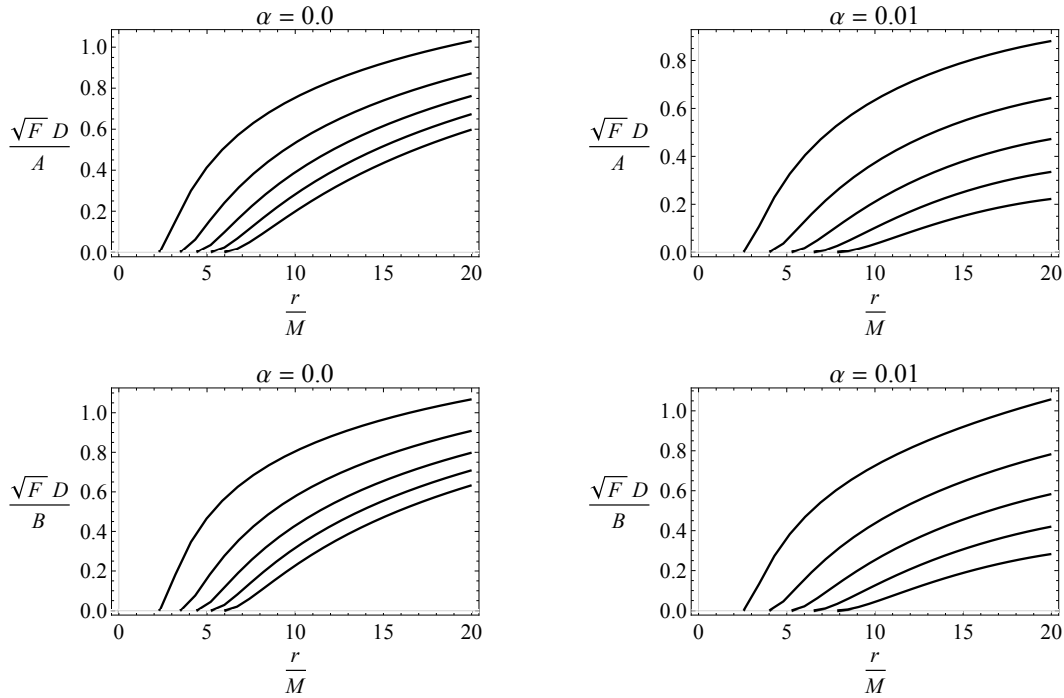


Figure 2. Relativistic correction factors $\frac{\sqrt{F}D}{A}$ affecting the tetrad components of the stress tensor equation (10) and $\frac{\sqrt{F}D}{B}$ characterizing the energy loss. We consider Kerr solution, $\alpha = 0$, and the rotating metric with $\alpha = 0.01$. Curves represent $\frac{a}{M} = 0, 0.25, 0.5, 0.75$ and 1.0 , from right to left.

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