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Inflationary magnetogensis in R^2-inflation on the light of Planck 2015

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Abstract. We study the primordial magnetic field (PMF) generated by the simple inflationary model f^2FF in the context of R^2-inflationary model, the favourite model of Planck 2015 results. The scale invariant PMF is achieved and the backreaction problem is avoided as long as, the rate of inflationary expansion, *H*, is less than the upper bound reported by Planck, 2015, in the observable scales of wave numbers, $k\eta$. Based on these results, we find that the upper limit of present magnetic field, $B_0 < 8.1\times10^{-9}\,\mathrm{G}$. It is in the same order of the one reported by Planck, 2015.

1. Introduction

Large scale magnetic fields are being observed in all kinds of galaxies and cluster of galaxies at wide range of redshifts. Moreover, some evidences for the presence of the magnetic fields in a very low density intergalactic medium (void) were reported [1]. The generation of such a large scale primordial magnetic fields (PMF) is still an open question in cosmology, see the reviews of this subject [2]. In this paper, the simple inflationary model, f^2FF, of PMF is investigated in the context of R^2-inflation [3], in the same way we did for NI and LFI [4-5]. Further, the reheating parameters are constrained in the same inflationary model by using the reported upper limit of PMF by Planck, 2015 [95]. Finally, the present PMF is constrained based on the scale invariant magnetic field generated during inflationary era.

2. Inflationary Model of PMF in R^2-inflation

In this model, a scalar (*inflaton*) field, ϕ, is coupled to the electromagnetic (vector) field A_μ [7-8] through an unspecified coupling function $f(\phi,t)$. The Lagrangian in this model can be written as,

$$\mathrm{L} = -\sqrt{-g}\left(\frac{1}{2}\left(\partial_\mu\phi\right)\left(\partial^\mu\phi\right)+V\left(\phi\right)+\frac{1}{4}g^{\alpha\beta}g^{\mu\nu}f^2\left(\phi,t\right)F_{\mu\alpha}F_{\nu\beta}\right) \tag{1}$$

where, $F_{\nu\beta}=\partial_\nu A_\beta-\partial_\beta A_\nu$ is the electromagnetic field tensor and g is the determinant of the spacetime metric $g_{\mu\nu}$. The first term in the Lagrangian is the standard kinetic part of the scalar field, and the second term, $V(\phi)$, is the potential, which decides the model of inflation.

The main reason behind this coupling is to break the conformal symmetry of electromagnetism and hence prevent the dilution of the seed of magnetic field as it is generated in the inflation era. The potential $V(\phi)$ of R^2-inflation can be written [9] as,

$$V(\phi) = M^4\left[1 - \exp\left(-\sqrt{2/3}\phi / M_{\rm Pl}\right)\right]^2, \tag{2}$$

where, M is the amplitude of the potential and it can be determined by the amplitude of CMB anisotropies ($M \simeq 4.0\times 10^{-5} M_{\rm Pl}$) [10]. Solving equation (1) for A_μ under slow roll approximations gives,

$$\mathrm{A}''(\eta,k) + \left(k^2 - \frac{f''}{f}\right)\mathrm{A}(\eta,k) = 0, \tag{3}$$

where $\mathrm{A}(\eta,k)$ is the Fourier transform of $A_\mu(\eta,x)$ and η is the conformal time and k is the comoving wave number. Similarly, in the slow roll limit and by assuming that the relation between couplings function and scale factor is in the power form, $f(\eta) \propto a^\alpha$ where, α is free index to be determined we can find,

$$f(\phi) \propto exp\left[-\frac{\alpha}{3M_{\rm Pl}{}^2}\int^\phi \frac{V(\phi)}{V'(\phi)}d\phi\right]. \tag{4}$$

Hence in a power law expansion, $f(N) = D\left[\left(\frac{4N}{3}\right)^{\frac{\alpha}{4}} \mathrm{e}^{-\frac{N\alpha}{3}}\right]$, where, N is the e-folds of inflation. Therefore, solving (3) by using (2) and (4) gives,

$$\mathrm{A}(\eta,k) = (k\eta)^{1/2}\left[C_1(k)\boldsymbol{J}_\chi(k\eta) + C_2(k)\boldsymbol{J}_{-\chi}(k\eta)\right], \tag{5}$$

where, $\boldsymbol{J}_\chi(k\eta)$, is the Bessel function of the first kind. The electric and magnetic spectra can be written respectively as,

$$\frac{d\rho_E}{dlnk} = \frac{f^2}{2\pi^2}\frac{k^3}{a^4}\left|\left[\frac{\mathrm{A}(\eta,\, k)}{f}\right]'\right|^2, \qquad \frac{d\rho_B}{dlnk} = \frac{1}{2\pi^2}\left(\frac{k}{a}\right)^4 k\left|\mathrm{A}(\eta,k)\right|^2. \tag{6}$$

3. Scale invariant PMF

As magnetic fields are being observed in all scales of the universe, it can be considered as a scale invariant quantity. It implies that the magnetic spectrum can be assumed as constant. Thus, the main task is to investigate the conditions that keep this spectrum constant. One of the main challenges to this model is the backreaction problem, at which the scale of the electromagnetic field can exceed the scale of inflation itself. This problem may spoil the inflation. Hence, any viable model should be free from this problem.

In order to achieve the scale invariance condition in interesting values of N, the values of $\chi(\alpha,N) = 5/2$ and $\alpha \simeq \{-7.4,\, 7.4\}$. Therefore, the scale invariant PMF can be obtained in the long wavelength regime, $k\eta \ll 1$ (outside Hubble radius) around these values of α. It can be depicted along with energy density of the inflation, $\rho_{\rm Inf}$ in figure.1.

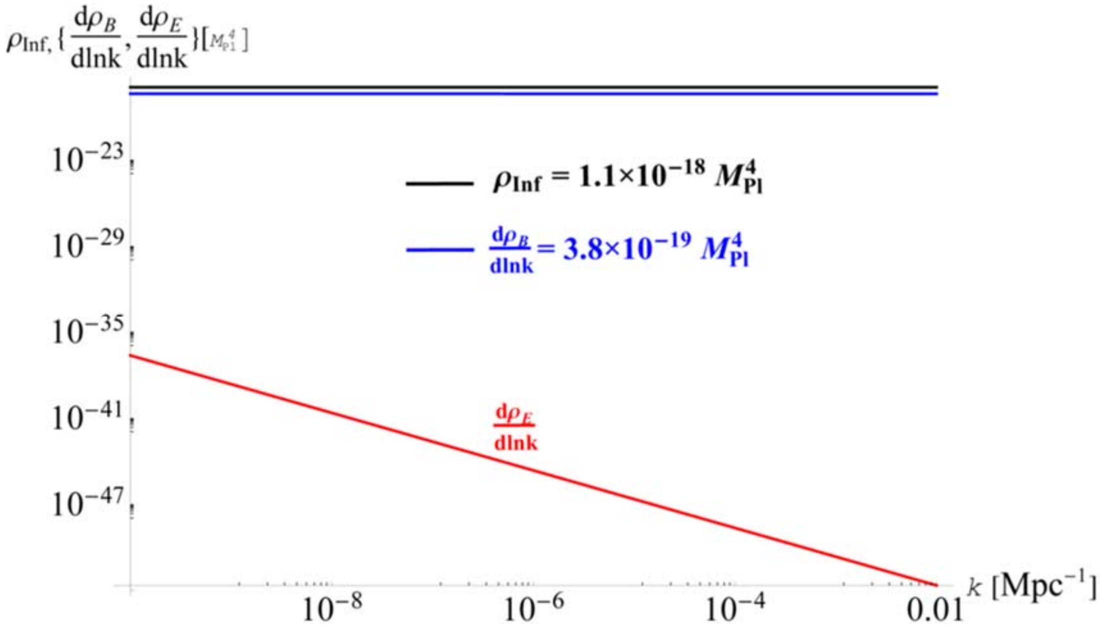

Figure 1. The EM spectra for $\alpha \approx 7.43$, $N = 64$, $\eta = -20$ and the expansion rate (PL) at the pivot scale, $H \approx 3.6 \times 10^{-5} M_{\rm Pl}$, and $k\eta \ll 1$.

As seen from figure.1 the relation, $\rho_{Inf} > \frac{d\rho_B}{d\ln k} \gg \frac{d\rho_E}{d\ln k}$ is preserved at the rate of expansion, $H \simeq H_* = 3.6 \times 10^{-5} M_{\rm Pl}$. In fact that value of the Hubble parameter is the upper value during inflation or at the time of pivot scale, at which the space time exits the Hubble radius [11]. Therefore, the problem of the backreaction can be avoided. In order to make sure, the above relation stays valid throughout the inflationary era, we can plot the EM spectra versus η, N, H and α. All of these plots show that the problem of backreaction can be avoided in the interesting cosmological parameters [3].

4. Constraining reheating parameters by means of PMF

The second main result is the constraining of reheating parameters by using R^2-inflation and the present upper limit of PMF reported by Planck, 2015 [6]. In this part, the effect of the scale invariant PMF on the parameters of reheating is investigated. That is basically similar to Ref [12] but by considering R^2-inflationary model instead. The three main values that specify the reheating era are the reheating temperature, $T_{\rm reh}$, the reheating parameter, $R_{\rm rad}$, and the equation of state parameter, $\omega_{\rm reh}$. All of them are constrained by PMF, see how $T_{\rm reh}$ are constrained for different $\omega_{\rm reh}$, in figure.2. It fits well with the range of spectral index reported by Planck 2015 [11]. Also, we calculate the lower limits, $R_{\rm rad} > 6.9$.

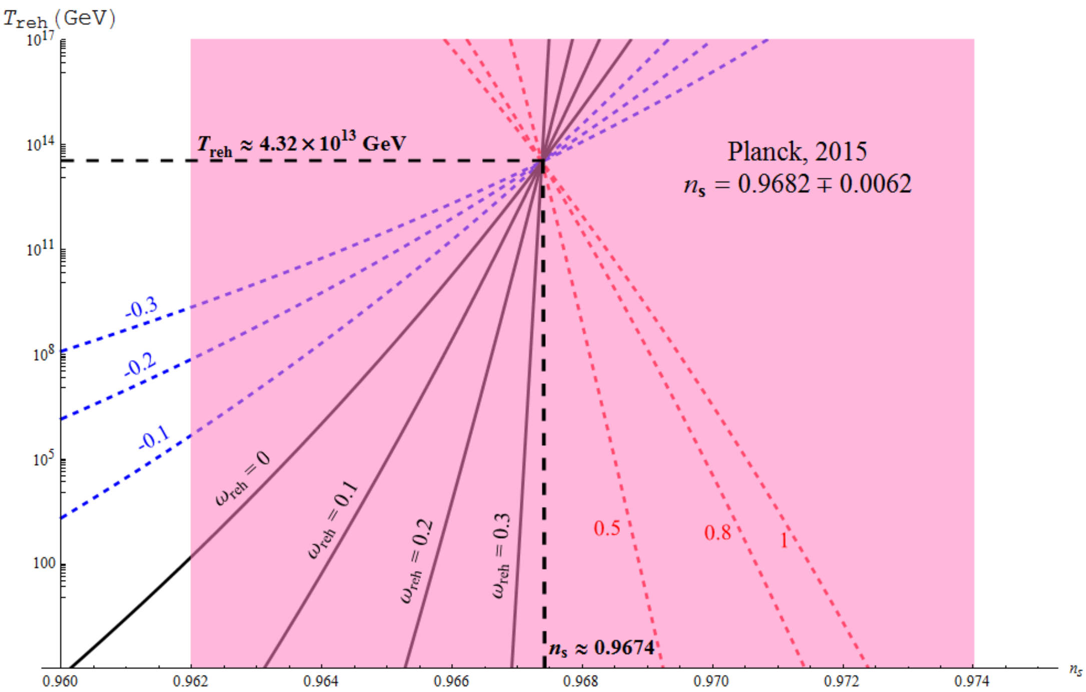

Figure 2. The reheating temperature, $T_{\rm reh}$, versus spectral index, n_s, at the end of R^2-inflation, for some values in $-0.3 < \omega_{\rm reh} < 1$. They all intersect into $T_{\rm reh} \sim 4.32\times10^{13}\,\mathrm{GeV}$ at $n_s \approx 0.9674$.

5. Constraining the present value of PMF

The main result is the constraining of the value of the present PMF, B_0. Based on the results of the first two parts, we calculate the upper limit of the present PMF as, $B_0 < 8.058\times10^{-9}\,\mathrm{G}$. It is in the order of nG, which is the upper bound of PMF reported by Planck 2015 [6].

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