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Reconstructing the distortion function for non-local gravity

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Abstract. We develop Maggiore and Mancarella non-local model by using new function, $f(m^2 \frac{R}{\Box^2})$ and we obtained analytical hyperbolic tangent function. This new model has expansion history exactly as same as ΛCDM (Lambda Cold Dark Matter) with same matter content, but without cosmological constant or dark energy. However, background evolution in our model and ΛCDM are the same, but these models may be distinguishable in structure formation or observation that will contain additional information more than background level.

1. Introduction

Universe expansion was discovered by Edwin Hubble [1] in 1929. Multiple evidences such as cosmic microwave background [2], supernova observations [3] and galaxy distribution [4] lead to the discovery of accelerating expansion of the Universe. The standard interpretation of this acceleration, in the form of a cosmological constant [5] raises some fundamental problems and questions. Why the cosmological constant should be much smaller than any other scale in physics and why it should have the right magnitude have all recently came into question [6]. Modified gravity theories have been also considered for Universe accelerating expansion [7]. For example, $f(R)$ theories [8] which generalize the Einstein-Hilbert Lagrangian from R to $f(R)$ can be used to create a model which exactly reproduces the ΛCDM expansion history just by choosing $f(R) = R - 2\Lambda$ [9]. More generally, $f(R)$ theories can deviate from observations even with no perturbation consideration.

In 2007 Deser & Woodard [10], proposed a new class of modified gravity models, which add some inverse differential operators of the curvature invariants terms to the Einstein-Hilbert action. The simplest choice is the inverse of the d'Alembert operator operator acting on the Ricci scalar, $f\left(\frac{1}{\Box}R\right)$. The function $f\left(\frac{1}{\Box}R\right)$ is dimensionless obviously, therefore this model does not have explicit mass scale in contrast to the non-local models that we will discuss later. In the literature, more activity is dedicated for identifying the form of the function $f\left(\frac{1}{\Box}R\right)$ in order to obtain background evolution of ΛCDM. The final result for, $f(\frac{1}{\Box}R) = a_1[\tanh(a_2Y + a_3Y^2 + a_4Y^3) - 1]$ (where $Y = \frac{1}{\Box}R + a_5$, and $a_1, \ldots a_5$ are coefficients obtained by fitting $f(X)$ to the observed expansion history) [11] which does not agree with observations. A good review about this model can be found [12].

Another choice is adding directly a term involving $\left(R\frac{1}{\Box^2}R\right)$ to the Einstein-Hilbert Lagrangian and it will reproduce dark energy dynamically again. This modification has been argued as a consistent infrared modification of General Relativity (GR) theory and also it is ghost–free [13]. This scenario has been greatly studied in the field of cosmological perturbations and other observable imprints [14].

Therefore, we focus in particular on the generalized $R\frac{1}{\Box^2}R$ model, and the paper is organized as follows: We introduce a generalized non-local gravity model in section 2, and in section 3 equations of motion (EOM) are derived. We find distortion function by reconstructing the ΛCDM expansion history in section 4. Finally, in section 5 conclusions and discussion are given.

2. The model

A non-local modified GR model proposed by Maggiore and Mancarella was based on the following action [13]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - \frac{1}{6}R\frac{m^2}{\Box^2}R\right], \tag{1}$$

in order to reproduce the observed amount of dark energy. Where $\frac{1}{\Box}$ is formal inverse of d'Alembertian operator ,$\Box$, in the scalar representation which can be expressed as the convolution with a bi-scalar Green's function $G(x,y)$:

$$\left(\frac{1}{\Box}R\right)(x) \equiv \int d^4y\sqrt{-g(y)}G(x,y)R(y) \quad , \quad \Box_x G(x,y) = \frac{1}{\sqrt{-g(x)}}\delta^4(x-y) \tag{2}$$

This model seems to have advantages to other non-local models that have already been confronted with observations [10-12,15]. The non-local term in equation (1) is controlled by a mass scale, m, in contrast to the non-local models of Deser & Woodard [10] and Barvinsky [16], which does not have such mass scale. The parameter m is approximately of the order of H_0. As a result, this modification will not affect the solar system tests and therefore in small scale, we cannot observe any deviation from the GR. This model will generate dark energy dynamically and by choosing $m \approx 0.28H_0$, we can reproduce the present observed value of cosmological constant [13]. Additional features of $\frac{1}{\Box}$R and $\frac{1}{\Box^2}$R models, is that by choosing t_s () to lie inside the radiation-dominated era one obtains a natural onset for the appearance of dark energy. Indeed, since $R = 0$ during radiation dominated era, the deviation from GR starts in matter domination era and the precise value of t_s (start time of the accelerating expansion of the universe) is not important.

We develop the Maggiore and Mancarella model by replacing $\left(-\frac{1}{6}R\frac{m^2}{\Box^2}R\right)$ term with $f\left(m^2\frac{R}{\Box^2}\right)$ in equation (1). As a consequence, we use the following action

$$S = \int\left[\frac{R}{16\pi G}\left[1 + f\left(m^2\frac{R}{\Box^2}\right)\right] + \mathcal{L}_{matter}\right]\sqrt{-g}d^4x. \tag{3}$$

The free parameter $f(X)$ in equation (3) is known as the nonlocal distortion function and it is function of its dimensionless argument which will be determined by matching with expansion history of ΛCDM model.

3. Equations of motion

In all models field equations are obtained by varying the actions with respect to the metric, $g_{\mu\nu}$. For flat Friedmann–Lemaitre–Robertson–Walker (FLRW) metric, the EOM will take the following forms

$$3H^2 + \Delta G_{tt} = 8\pi G\rho \tag{4}$$

$$-2\dot{H} - 3H^2 + \frac{1}{3a^2}\delta^{ij}\Delta G_{ij} = 8\pi GP \tag{5}$$

where ρ and P are the total energy density and pressure of matter respectively and the non-local terms, ΔG_{tt} & $\frac{1}{3a^2}\delta^{ij}\Delta G_{ij}$, are

$$\begin{aligned}
&\Delta G_{tt}\\
&= (3H^2 + 3H\partial_t)\left[f\left(m^2\frac{R}{\Box^2}\right) + \frac{1}{\Box^2}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right]\\
&+ \frac{1}{2}m^2\left[\partial_t\left(\frac{R}{\Box^2}\right)\partial_t\left(\frac{1}{\Box}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right) + \partial_t\left(\frac{R}{\Box}\right)\partial_t\left(\frac{1}{\Box^2}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right)\right]\\
&+ \frac{1}{2}m^2\left(\frac{R}{\Box}\right)\left[\frac{1}{\Box}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right]
\end{aligned} \tag{6}$$

$$\frac{1}{3a^2}\delta^{ij}\Delta G_{ij}$$
$$= \left(-2\dot{H} - 3H^2 - \partial_t^2 - 2H\partial_t\right)\left[f\left(m^2\frac{R}{\Box^2}\right) + m^2\frac{1}{\Box^2}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right]$$
$$+\frac{1}{2}m^2\left[\partial_t\left(\frac{R}{\Box^2}\right)\partial_t\left(\frac{1}{\Box}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right) + \partial_t\left(\frac{R}{\Box}\right)\partial_t\left(\frac{1}{\Box^2}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right)\right]$$
$$-\frac{1}{2}m^2\left(\frac{R}{\Box}\right)\left[\frac{1}{\Box}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right], \tag{7}$$

where prime denotes the derivative respect to argument of function.

4. Reconstruction function

We will use Woodard and Deffayet method [11] in order to find the reconstruction function. By subtracting equation (5) from equation (4), we obtain following second order differential equation

$$6H^2 + 2\dot{H} + \left\{\left(6H^2 + 2\dot{H} + 5H\partial_t + \partial_t^2\right)\left[f\left(m^2\frac{R}{\Box^2}\right) + m^2\frac{1}{\Box^2}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right]\right.$$
$$\left. + m^2\left(\frac{R}{\Box}\right)\left[\frac{1}{\Box}\left[Rf'\left(m^2\frac{R}{\Box^2}\right)\right]\right]\right\} = 8\pi G(\rho - P) \tag{8}$$

Because we want to reproduce ΛCDM cosmology with the same matter content, but without any cosmological constant, therefore the Hubble parameter, H, appearing in equation (8), is a solution of the famous standard Friedmann's equations. We can compute $8\pi G(\rho - P)$ term from Friedmann's equations and it can be plugged in to equation (8)

$$\left(6H^2 + 2\dot{H} + 5H\partial_t + \partial_t^2\right)\left[f(X) + m^2\frac{1}{\Box^2}[Rf'(X)]\right] + m^2\left(\frac{R}{\Box}\right)\left[\frac{1}{\Box}[Rf'(X)]\right] = -6H_0^2\Omega_\Lambda \tag{9}$$

Where, $\Omega_\Lambda = \frac{\Lambda}{3H_0^2}$ and $X = m^2\frac{R}{\Box^2}$.

If we define a new function $Y(t) = \frac{1}{\Box^2}[Rf'(X)]$, we obtain an ODE for $Y(t)$

$$C_6(t)Y^{(6)} + C_5(t)Y^{(5)} + C_4(t)Y^{(4)} + C_3(t)Y^{(3)} + C_2(t)Y^{(2)} + C_1(t)Y^{(1)} + C_0(t) = 0 \tag{10}$$

Where $= \frac{1}{\Box}R$, $Y^{(n)} = \frac{d^n}{dt^n}Y$ and C_n coefficients are function of (H, b, X) which we don't show them here.

For solving equation (10) numerically, we have to calculate first the C_n coefficients. Therefore, we wrote a program in Fortran language for computing C_n coefficients, and then equation (10) can be solved with the Runge-Kutta method. In this step we need to know the initial condition of $Y^{(n)}\big|_{t_0}, n = 1, 2, \dots, 5$, where t_0 is the initial starting point. We can choose all initial conditions be zero, because $Rf\left(m^2\frac{R}{\Box^2}\right)$ as well as $f\left(m^2\frac{R}{\Box^2}\right)$ must tend to zero in the initial time. After finding $Y^{(1)}(t)$, we can obtain $f(t)$ with the condition $f(t_0) = 0$. Finally, by computing $t(X)$ we can find distortion function $f(X)$. The numerical solution for distortion function $f(X)$ is shown in figure 1. For finding a suitable function that will be fitted with the numerical results of figure 1, we tried to use many functions and finally, we found the following best analytic function

$$f(X) = -0.48999 \times \tanh(1.32489 \times X^{0.8} - 1.24632 \times X + 0.35843 \times X^{1.2}) \tag{11}$$

Moreover, the fitted curve and regular residuals are demonstrated in figure 2.

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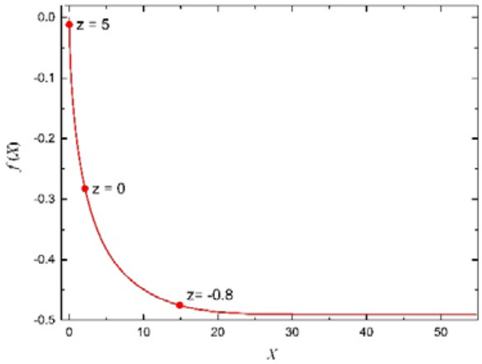

Figure 1. Numerical result for distortion function $f(X)$.

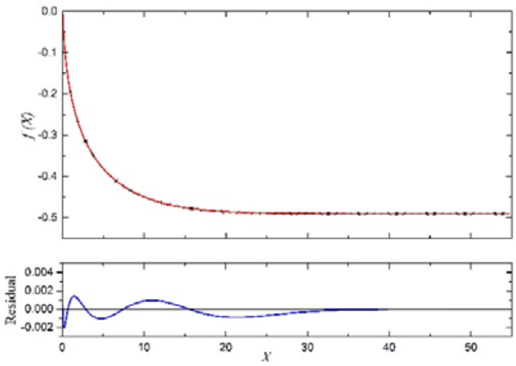

Figure 2. Fitted curve (red line) and regular residuals (blue line) for distortion function $f(X)$.

5. Results and discussion

We chose Maggiore and Mancarella model and develop it in the form of equation (3) to reproduce ΛCDM expansion history. For this purpose, we used Woodard and Deffayet method to find distortion function $f(X)$ numerically, and found the analytic function in the form of equation (11).

This model has expansion history exactly as same as ΛCDM with same matter content, but without cosmological constant or dark energy. However, background evolution in our model and ΛCDM are the same, but these models may be distinguishable in structure formation or observation that will contain additional information more than background level. Derivative of function $f(X)$ at $X = 0$, is infinite. As a result, this term, can make significant corrections to GR theory when it expands around flat space. In equation (4) we have only $Rf'(X)$, however, $f'(X)$ will be very large at $X \approx 0$, but R is very small and consequently $Rf'(X)$ will be finite. It would be extremely interesting to apply those tests for close massive binary, solar system and gravitational lensing.

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