

# New interpretation of data of the Earth's solid core

**H H Guliyev**

Department of Tectonophysics and Geomechanics, Institute of Geology and Geophysics of Azerbaijan National Academy of Sciences, Ave. H. Javid 119, Baku, AZ 1143, Azerbaijan

E-mail: hatamguliyev@gmail.com

**Abstract.** The commonly accepted scientific opinions on the inner core as the deformable solid globe are based on the solution of the problem on the distribution of elastic parameters in the inner structures of the Earth. The given solution is obtained within the necessary integral conditions on its self-weight, moment of inertia concerning the axes of rotation and periods of free oscillations of the Earth. It is shown that this solution does not satisfy the mechanics of the deformable solid body with sufficient local conditions following from basic principles concerning the strength, stability and actuality of velocities of propagation of elastic waves. The violation of local conditions shows that the inner core cannot exist in the form of the deformable solid body within the commonly accepted elastic parameters.

## 1. Introduction

The results of geomechanical analysis of geophysical data are given within the non-classical linearized approach (NLA). At the same time numerical data of Preliminary Reference Earth Model (PREM) [1] are used considering the fact that the parameters of the inner core provided in this paper taken as a basis in all pre-proposed theoretical models of the medium [2].

The purpose of geomechanical analysis is to determine the conditions for pressure and deformation that ensures the accuracy of calculations of physical and mechanical parameters of the model of the Earth's solid core on the basis of geophysical dataset.

## 2. Achievements of theoretical limit of strength

Let's consider the case when the medium is evenly and uniformly deformed prior to the beginning of fracture. In this case, if the pressure value does not exceed the theoretical limit of strength, all calculations on the physico-mechanical parameters are correct. It is shown in the NLA [3] that the value of the theoretical limit of the strength of the medium is determined from  $P = \mu$  under the conditions of compression (we are interested in this variant of deformation) for a perfect elastic isotropic material. This is found from the condition of the loss of ellipticity of the basic equation of motion [3].

## 3. Instability of equilibrium state

The uniform distribution of deformation in the medium can also be violated as a result of the instability (in various forms) without fracture.

NLA allows defining the limits of change of deformation within the framework of which the equilibrium of uniformly deformed states is stable [4, 5, 6]. Deformation in the globe is nonuniformly distributed long before the limits of strength of material were reached in case of realization of instability. Stability of solid globe is considered to concretize the discussion [5]. It is considered that:

a)



elastic isotropic body with potential of harmonic type within the theory of large initial deformations and conditions of stability are obtained in the form

$$0 < \lambda_1 < 1; \left( \lambda + \frac{2}{3}\mu \right) \left( \lambda + \frac{4}{3}\mu \right)^{-1} < \lambda_1 < 1; \quad (1)$$

b) elastic body with quadratic potential within the theory of the second variant of small initial deformations and conditions of stability are obtained in the form

$$(2 - \lambda_1) \left( \lambda + \frac{2}{3}\mu \right) > 0; \mu + 3(\lambda_1 - 1) \left( \lambda + \frac{2}{3}\mu \right) > 0; \quad (2)$$

c) elasto-plastic body (deformation theory) within the theory of the second variant of small initial deformations and condition of stability are obtained in the form

$$P < \mu; \quad (3)$$

d) elasto-plastic body (Prandtl-Reuss theory of plasticity) within the theory of the second variant of small initial deformations is obtained in the form

$$P < \lambda_1^2 \mu. \quad (4)$$

It is shown for all the considered models of the medium that the equilibrium state is stable in case of setting follower loads on the surface of an isotropic globe under the fulfillment of conditions (1)-(4).

The stability of body in the form of the globe is considered in case of influence of "dead" loads on its surface. It is shown that there is a critical load  $P_{cr}$ . (according to the value this load is less than the value  $\mu$ ) in reaching of which uniform deformed state of equilibrium of the globe is unstable. The critical forces and deformation leading to the loss of stability of the equilibrium state of the globe are determined within the theory of the second variant of small initial deformations

$$P_{cr} = \frac{\mu}{4(1-2\nu)} \left( 5 - 4\nu - (16\nu^2 - 8\nu + 9)^{\frac{1}{2}} \right). \quad (5)$$

In case of the theory of large initial deformations and the use of harmonic elastic potential, critical value of shortening is defined in the form

$$(\lambda_1)_* = \frac{(3-2\nu)(1+\nu)}{(3-2\nu)(1+\nu) + (1-2\nu)}. \quad (6)$$

In equations (1)-(6),  $\lambda$  and  $\mu$  are Lamé's moduli,  $\nu$  is Poisson's ratio and  $P$  is pressure parameter.

#### 4. Internal instability

Critical values of stress and deformations leading to violation of conditions of ellipticity of basic equations cause the phenomenon in the body which is called the "internal" instability in the theory [5, 6].

In case of modelling the deformation process using harmonic elastic potential within the theory of large initial deformations, the limit value of elongation (shortening) coefficient  $\lambda_1^*$  is defined in the form

$$\lambda_1^* = \frac{1+\nu}{2-\nu}, \quad \varepsilon_0^* = \frac{3}{2} \frac{2\nu-1}{(2-\nu)^2}, \quad (7)$$

and in the following form in case of quadratic elastic potential within the theory of large initial deformations

$$\lambda_1^* = \left( \frac{1+\nu}{2-\nu} \right)^{\frac{1}{2}}, \quad \varepsilon_0^* = \frac{1}{2} \frac{2\nu-1}{2-\nu}. \quad (8)$$

In case of linear elastic isotropic material within the theory of the second variant of small initial deformations, we get

$$P_{cr.} = \mu, \quad \varepsilon_0^* = \frac{1}{2} \frac{2\nu - 1}{1 + \nu}. \quad (9)$$

$\varepsilon_0$  is a parameter of overall deformation in equations (7)-(9). It follows from the aforementioned equations (1)-(4) and (7)-(9) that the "internal" instability occurs within the NLA in overall uniform deformation (compression) of isotropic globe at a level of pressure comparable in value with shear moduli.

### 5. Elastic wave propagation in the deformed medium

Such problems are intensively studied in [4, 6,7]. The fulfillment of condition of ellipticity of basic equations also provides an actuality (not negative value) of velocities of propagation of small perturbations.

In case of overall uniform pre-compression of isotropic medium, "true" velocities of propagation of elastic waves in it are defined by expressions [6]

$$\rho C_l^2 = \lambda + 2\mu - PK_p^R; \quad \rho C_s^2 = \mu - PK_s^R. \quad (10)$$

Where  $C_l, C_s$  are "true" velocities of quasi-pressure and quasi-shear elastic waves;  $K_p^R, K_s^R$  are coefficients of nonlinear action of isotropic medium [7].

In cases of the theory of the second variant of small and large initial deformations, pressure elastic wave with true velocity cannot be propagated in the quadratic elastic potential under the fulfillment of condition

$$\frac{P}{\mu} \geq \frac{2(1-\nu^2)}{(1-2\nu)(3-\nu)}; \quad \frac{P}{\mu} \geq \frac{2(1-\nu^2)}{(1-2\nu)(5-3\nu)} \quad (11)$$

in the stressed isotropic medium. This condition takes the following form for shear elastic waves

$$\frac{P}{\mu} \geq \frac{1+\nu}{2-\nu}; \quad \frac{P}{\mu} \geq \frac{1+\nu}{3(1-\nu)}. \quad (12)$$

### 6. Numerical results and discussions

The calculated values of the critical forces and elongations leading to the loss of stability of the equilibrium state of the deformed globe are given in table 1. The values of  $P^*/\mu$  are calculated according to equation (9). "Internal" instability is realized in reaching these values of surface loads in the globe in case of small initial deformations.

**Table 1.** Critical parameters of instability.

$\nu$	0	0.1	0.2	0.3	0.4	0.41	0.45
$P^*/\mu$	1	1	1	1	1	1	1
$P_*/\mu$	0.5	0.53	0.57	0.60	0.64	0.64	0.65
$\lambda_1^*$	0.5	0.58	0.67	0.76	0.88	0.89	0.94
$(\lambda_1)_*$	0.75	0.79	0.84	0.89	0.94	0.94	0.97

The values of  $\lambda_1^*$  calculated according to the equation (7) shows that similar process also occurs in the case of large initial deformations. The values of  $P_*/\mu$  and  $(\lambda_1)_*$  which causes the loss of stability in the globe on the geometric form change are calculated according to equations (5) and (6) respectively.

Comparisons of the values  $P^*/\mu$  with  $P_*/\mu$  and  $\lambda_1^*$  with  $(\lambda_1)_*$  shows that the process of loss of stability of the globe on the geometric form change precedes the process of the "internal" instability.

Calculations are also performed according to the equations (11) and (12). Elastic pressure waves with true velocity cannot propagate in the globe in reaching the numerical values of  $P_l/\mu$  given in table 2. Similar results for  $P_s/\mu$  are also obtained in the case of shear elastic waves. The data obtained for small initial deformations are given in the numerators and the data obtained for large initial deformations are given in the denominators.

**Table 2.** Critical values  $P_l/\mu$  and  $P_s/\mu$  for different Poisson's ratio.

$\nu$	0	0.1	0.2	0.3	0.4	0.41	0.45
$P_l/\mu$	0.6667	0.8534	1.1429	1.6852	3.2308	3.5689	6.2549
	0.4	0.5266	0.7273	1.1098	2.2105	2.4518	4.3699
$P_s/\mu$	0.5	0.5789	0.6667	0.7647	0.875	0.8868	0.9355
	0.3333	0.4074	0.5	0.6190	0.7778	0.7966	0.8788

Contrary to the results of table 2, data given in PREM [1] shows that elastic waves propagate in the solid core of the Earth even under higher baric conditions.

## 7. Conclusions

The establishment of features of deformation process in the baric conditions of the internal solid core related to reaching the ultimate strength of the medium, forms of instability of the isotropic globe and the impossibility of propagation of elastic waves in the medium is new and not taken into account in the existing theoretical models of the Earth [1, 2].

## References

- [1] Dziewonski A M and Anderson D L 1981 Preliminary reference Earth model *Physics of the Earth and Planetary Interiors* **25** (4) pp 297–356
- [2] Sumita I and Bergman M I 2007 Inner-Core Dynamics *In Book: Treatise on Geophysics, 1st Edition (Editor-in-Chief: Gerald Schubert)*, **8** pp 299–318
- [3] Kuliev G G 1988 *Strength of Materials* **20** (5) 623–629
- [4] Akbarov S D 2015 *Dynamics of pre-strained bi-material elastic systems: Linearized three-dimensional approach* (Switzerland: Springer) p 1004
- [5] Guz A N 1986 *Fundamentals of three-dimensional theory of stability of deformable bodies* (Kyiv: Vishcha shkola Head Publishing House) p 511
- [6] Guz A N 1986 *Elastic waves in bodies with initial stresses, propagation patterns* (2<sup>nd</sup> Volume) (Kyiv: Naukova Dumka) p 536
- [7] Guliyev H H 2009 Nonlinear actions of elastic medium and their effect on the propagation velocity of elastic waves *Proceedings of the NAS of Azerbaijan Earth Sciences* **2** pp 31–39.