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Impurity effect on entanglement asymptotic state in one-dimensional Ising system coupled to a dissipative environment

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Abstract. We consider a finite one-dimensional Ising spin chain under the influence of a dissipative Lindblad environment obeying the Born-Markovian constrain in presence of an external magnetic field with open boundary conditions. We study the effect of a single impurity, located at the terminal or center of the chain, on the time evolution and asymptotic steady state of the bipartite entanglement in the chain starting from a maximally entangled initial state. We found that the impurity has a significant effect on the bipartite entanglement of its nearest spins and can be used to tune their steady state value but has almost no noticeable impact on the far ones. At finite temperature, the thermal excitations suppress the dynamics of the system and reduce the value of the steady state and may completely wipe it out as the temperature is increased, which eliminates the effect of the impurity in that case.

1. Introduction

Quantum entanglement represents the main barrier between the quantum and classical worlds with no classical analog [1]. It is considered as a crucial resource in quantum teleportation, cryptography, and quantum computation and many other fields of interest [2]. Deep analysis and good quantification of entanglement are needed to explain the different behaviors of many body quantum systems. Particularly, there has been great interest in studying the different sources of errors in quantum computing and their effect on quantum gate operations [3]. Different approaches have been proposed for protecting quantum systems during the computational implementation of algorithms, such as quantum error correction [4] and decoherence-free subspace [5]. Therefore, studying the effect of naturally existing sources of errors, such as impurities and dissipative environments in systems of interest is a must. In a previous work we have shown how the entanglement dynamics and ergodicity in Heisenberg spin systems can be controlled using impurities and anisotropy [6]. Very recently, we have investigated the impact of a thermal dissipative environment on the dynamic behavior of one-dimensional Heisenberg spin systems [7]. We demonstrated that the system entanglement reaches asymptotically a steady state that varies significantly depending on the degree of anisotropy and the environment temperature. In this paper, we show how we can control the asymptotic behavior and the entanglement steady state value in a one-dimensional open boundary Ising spin system coupled to a dissipative environment at finite temperature using magnetic impurities.

2. The Model

We consider a set of 7 localized spin-$\frac{1}{2}$ particles in a one-dimensional open boundary Ising chain coupled through exchange interaction $J_{i,j}$ in the x-direction and subject to an external magnetic field of strength h applied in the z-direction. All the particles are identical except one, which is considered as an impurity. The Hamiltonian for such a system is given by

$$H = -\sum_{<i,j>} J_{i,j}\sigma_i^x\sigma_j^x - h\sum_i \sigma_i^z, \tag{1}$$

where σ_i^α ($\alpha = x$, y or z) are the Pauli operators (for convenience we set $\hbar = k = 1$) and $< i, j >$ is a pair of nearest-neighbors sites on the lattice, with $J_{i,j} = J$ for all sites except the sites nearest to an impurity. The coupling between the impurity and its neighbors is $J_{i,j} = J' = (\alpha+1)J$, where α is a parameter that tunes the strength of the impurity. The dynamics of an isolated quantum system is described by the time evolution of its density matrix $\rho(t)$ according to the quantum Liouville equation $\dot{\rho}\,(t) = -i\,[H, \rho]$. For an open quantum system interacting weakly with an environment that has a short relaxation time for its excitation modes, the Born-Markovian approximation can be applied and the time evolution of the system is best described by the Lindblad Master equation [8], which is given by

$$\dot{\rho}\,(t) = -i\,[H, \rho] + \mathcal{D}_\rho\ , \tag{2}$$

where $\mathcal{D}_\rho$ is the extra term that describes the dissipative dynamics and is represented in the Lindblad form as

$$\mathcal{D}_\rho = -\frac{1}{2}\sum_{k=1} \left\{[L_k\rho, L_k^\dagger] + [L_k, \rho L_k^\dagger]\right\}\ , \tag{3}$$

where the Lindblad operator L_k represents all the effects of the considered environment on the system site k. It is more convenient to work in the Liouville space, where Eq. (2) can be reformulated into the matrix equation form $\vec{\rho}(t) = (\hat{\mathcal{L}}^H + \hat{\mathcal{L}}^D)\vec{\rho} = \hat{\mathcal{L}}\vec{\rho}$, where $\hat{\mathcal{L}}^H$ and $\hat{\mathcal{L}}^D$ are superoperators acting on the vector ρ in the Liouville space. The exact numerical solution of the last equation yields the density vector $\vec{\rho(t)}$, which can be rearranged into the customary density matrix form $\rho(t)$. The effect of the dissipative thermal environment on the considered spin system is given by the local Lindblad operator [8]

$$L_k = \Gamma\ \left\{\frac{(\bar{n}+1)}{2}\ S_k^- + \bar{n}\ S_k^\dagger\right\}\ , \tag{4}$$

where S^+ and S^- are the spin raising and lowering operators, $S^\pm{=}S^x \pm iS^y$. Γ is a phenomenological parameter that determines the strength of the coupling between the environment and the system and is assumed to be the same for all spins. The thermal parameter $\bar{n}$ is proportional to the temperature of the environment. We consider $0 \leq \bar{n} \leq 0.1(\approx 41$ mK). Due to the Born-Markovian approximation, we consider values of Γ and J such that Γ and $J << \omega$. We adopt the concurrence $C_{i,j}$ as a measure of the bipartite entanglement between any two spins i and j in the system [9]. We study the time evolution of the system starting from the maximally entangled state, $|\psi_m\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)|\downarrow\downarrow\cdots\downarrow\rangle$.

3. Results and discussion

We study the bipartite entanglements along the 7 spins Ising chain for two different impurity location scenarios. In the first one, we consider an impurity at the terminal (spin 1), which is discussed and illustrated in figure 1. In the second case the impurity is assumed to be in the center (spin 4), shown in figure 2. In both cases, the system is initiated in a global maximally entangled state where the two spins 1 and 2 are maximally entangled with each other but are disentangled from the rest of the spins in the chain, where they are in a separable state themselves. To briefly discuss the overall dynamics we focus on and illustrate only the bipartite entanglement of the terminal pair (1 and 2), the central pair (3 and 4) and the other terminal pair (6 and 7) at zero and finite temperature. For convenience we consider the time evolution of the system in terms of the dimensionless time $T = \omega\, t$. As can be noticed in figure 1(a), C_{12}

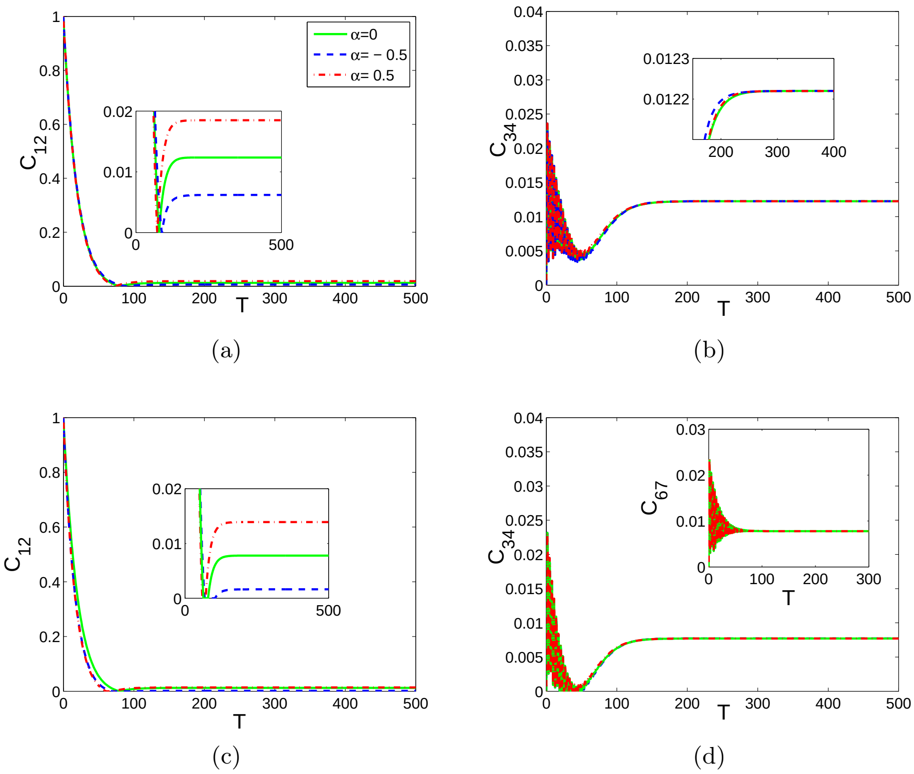

Figure 1. Dynamics of the entanglements C_{12} and C_{34} with a terminal impurity in the Ising chain at $\bar{n} = 0$ in (a) and (b) and $\bar{n} = 0.05$ in (c) and (d). In the inner panel of (d) C_{67} is depicted versus T. The legend is as shown in panel (a).

at $\bar{n} = 0$ starts with a maximum value but vanishes within a short time before reviving again and reaching asymptotically a steady state value. The steady state value depends significantly on the impurity strength, represented by α and is enhanced as it increases. The central pair entanglement C_{34}, illustrated in figure 1(b) behaves differently where it starts from zero value then strongly oscillates before reaching a steady state value that is very slightly affected by the terminal impurity. C_{67} was found to show a very similar behavior to C_{34}. The effect of the temperature on C_{12} and C_{34} is explored in figure 1(c) and (d) respectively. As expected the steady state values of C_{12} are reduced due the thermal excitations but the impurity impact is still effective. The thermal effect on C_{34} is very similar to what was observed for C_{12}, where the steady state values is reduced but the impurity effect is still slim. A similar behavior to C_{34} is shown by the far terminal pair C_{67}. The effect of a central impurity is explored in figure 2. As one can see, it has no noticeable impact on C_{12} neither at zero temperature (panel (a)) nor at finite temperature (panel(c)). On the other hand, the central spin pair are the most affected one by the central impurity as shown in panel (b) for zero temperature, where higher impurity strength enhances the steady state value. At finite temperature, the thermal excitations again reduces the steady state values but impurity is still effective as can be concluded from panel (d).

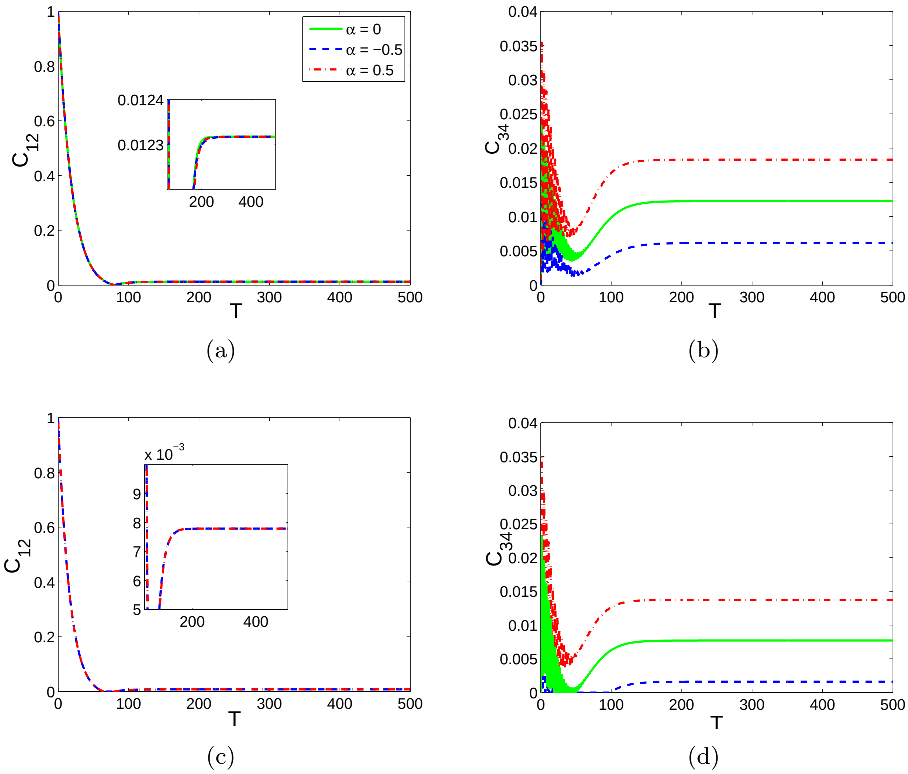

Figure 2. Dynamics of the entanglements C_{12} and C_{34} with a central impurity in the Ising chain at $\bar{n} = 0$ in (a) and (b) and $\bar{n} = 0.05$ in (c) and (d). The legend is as shown in panel (a).

4. Conclusion

We studied the effect of an impurity in a one-dimensional Ising spin chain in presence of a Lindblad environment on the asymptotic behavior of the system. The asymptotic steady state value of the bipartite entanglement of the nearest neighbor spins of the impurity was found to be strongly dependent on the impurity strength and is enhanced as it increases regardless of the impurity location. As the temperature is raised the steady state value is suppressed and the impurity effect is reduced significantly.

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