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Higher order non-classicalities in the Kerr-like coupler systems

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Abstract. In this paper, we show the existence of various types of non-classical effects in the model of the nonlinear Kerr-coupler containing two quantum nonlinear oscilators mutually coupled by continuous nonlinear interaction, such as squeezing, anti-bunching, intermodal entanglement and their higher-order counterparts. By using unitary evolution operator formalism and the common inequalities expressed in term of various moments defined by products of creation and annihilation operators, we find numerically the "exact" solutions of these factors for non-dissipative. We show and discuss the parameters considered can be indicators of generation such nonclassical effects and hence, quantumness of the system.

1. Introduction

One of the key problems related to the quantum information theory is quantum state engineering which allows us generate the nonclassical states. These special features of the quantum systems bring the remarkable applications in quantum information theory in comparison with the classical one [1, 2, 3, 4, 5, 6]. The signature of non-classicalities can be detected in the numerous ways, which can characterize as squeezing, anti-bunching, steering, intermodal entanglement, and their higher-order counter parts. Squeezed states can be suggested for the implementation of continuous variable quantum cryptography and teleportation of coherent states [1, 2]. Anti-bunching is understood as the characteristic of a radiation field in whatever place the mean number of photon is greater than the variance of the photon number distribution. Thus, antibunched states can be employed to build a high-quality single photon sources [4]. Entangled states play a crucial role in implementation quantum cryptography, quantum teleportation, dense-coding, quantum key distribution [1, 3, 5]. The existence of those properties have been widely studied in various quantum optical systems [7, 8]. However, the researchers working in different aspects of quantum information theory and quantum optics are still interested in study the possibility of generation of different nonclassical characters. In this paper we shall show how to model quantum dynamics of nonlinear coupler system, nd time-evolution wave function and evolution of operator formalism and the common inequalities expressed in term of various moments defined by products of creation and annihilation operators. Then we will show that our system exhibits its non-classical effect and hence, quantumness of the system.

2. Model of Hamiltonian

In this paper we shall consider the model comprising two mutually nonlinear interacting quantum nonlinear oscillators labeled by a and b. They are described by the following Hamiltonian expressed in terms of boson creation and annihilation operators $\hat{a}^\dagger$ ($\hat{b}^\dagger$ and $\hat{a}(\hat{b})$ respectively:

$$\hat{H} = \hbar\chi_a \hat{a}^{\dagger 2}\hat{a}^2 + \hbar\chi_b \hat{b}^{\dagger 2}\hat{b}^2 + \hbar\varepsilon_{nl}\hat{a}^{\dagger 2}\hat{b}^2 + \hbar\varepsilon^*_{nl}\hat{b}^{\dagger 2}\hat{a}^2 + \hbar\tilde{\chi}\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b} \tag{1}$$

The parameters χ_a (χ_b) and $\tilde{\chi}$ correspond to the self- and cross-Kerr nonlinear interaction, whereas ϵ_{nl} is a strength of the nonlinear internal coupling between two oscillators. Moreover, we assuming here that two Kerr-like oscillators located inside the cavity are identical $\chi_a/2 = \chi_b/2 = \chi$.

Thank to *Quantum Mechanics*, the quantum problem can be solved in Schrödinger picture, thus the time-evolution of the system can be understood as the evolution of the vector in Hilbert space. For state vector $|\psi(t)\rangle$, it is held by Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle \tag{2}$$

The evolution of state vector now can be determined as

$$|\psi(t)\rangle = \hat{U}|\psi(0)\rangle \tag{3}$$

where $\hat{U} = \exp\left(-\frac{i}{\hbar}\hat{H}t\right)$ is the unitary operator.

Using n-photon basis, we define the annihilation and creation operators of each mode as square matrices in the Hilbert space $\mathcal{H} = \mathcal{H}_a \otimes \mathcal{H}_b$ as [10]

$$\breve{a} = \hat{a} \otimes \hat{I}_n \text{ and } \breve{a}^\dagger = \hat{a}^\dagger \otimes \hat{I}_n \tag{4}$$

$$\breve{b} = \hat{I}_m \otimes \hat{b} \text{ and } \breve{b}^\dagger = \hat{I}_m \otimes \hat{b}^\dagger \tag{5}$$

for the mode a and mode b, respectively. Here $\hat{I}_{m(n)}$ is an unity matrix with $m(n)$ dimension. Applying such operators we are able to construct the Hamiltonians $\hat{H}$ and the unitary operators $\hat{U}$ in the matrix representation. Setting initial condition is $|\alpha\rangle|\beta\rangle$ in two modes of the field. These initial states can be expressed in number state as [11]

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n_a=0}^{\infty}\frac{\alpha^{n_a}}{\sqrt{n_a!}}|n_a\rangle; \quad |\beta\rangle = \exp\left(-\frac{|\beta|^2}{2}\right)\sum_{n_b=0}^{\infty}\frac{\beta^{n_b}}{\sqrt{n_b!}}|n_b\rangle \tag{6}$$

Thus, we can express these states in the matrix presentation. Applying standard numerical procedure we can perform and manipulate evolution of the wave-functions of the system and then calculate indicators of quantum behavior of the system expressed in terms of expectation values of functions of annihilation and creation operators of various modes.

3. Non-classicality criteria

3.1. Squeezing and higher-order squeezing

Squeezed states of electromagnetic field are defined on the basis of the generalized Heisenberg's uncertainty principle [11]. Contrary to the coherent states, they exhibit strong quantum properties. Therefore squeezing effect can be applied as indicators of the quantumness of the system. For the cases of the *one mode squeezing*, the fluctuations of two quadrature operators of a single mode a are given by the variances $\langle(\triangle\hat{X}_a)^2\rangle = \langle\hat{X}_a^2\rangle - \langle\hat{X}_a\rangle^2$ and $\langle(\triangle\hat{Y}_a)^2\rangle = \langle\hat{X}_a^2\rangle - \langle\hat{Y}_a\rangle^2$, with $\hat{X}_a$ and $\hat{Y}_a$ defined two quadrature operators [11]:

$$\hat{X}_a = \frac{\hat{a} + \hat{a}^\dagger}{2}; \quad \hat{Y}_a = -i\frac{\hat{a} - \hat{a}^\dagger}{2} \tag{7}$$

The system is squeezed if the fluctuation satisfies one of two of the follow conditions

$$4\langle(\triangle\hat{X}_a)^2\rangle - 1 < 0, \quad 4\langle(\triangle\hat{Y}_a)^2\rangle - 1 < 0. \tag{8}$$

If we express appeared above variances in terms of the creation and annihilation operators, the conditions can be written as [9]:

$$S_a = \frac{1}{2}[\langle\triangle\hat{a}^\dagger\triangle\hat{a}\rangle + \mathrm{Re}\langle(\triangle\hat{a})^2\rangle] < 0, \tag{9}$$

$$S'_a = \frac{1}{2}[\langle\triangle\hat{a}^\dagger\triangle\hat{a}\rangle - \mathrm{Re}\langle(\triangle\hat{a})^2\rangle] < 0. \tag{10}$$

More general definitions of the squeezing were defined in [9, 13], we can see as *principal squeezing.* It can be measured by the following factor

$$\lambda_a = \langle\triangle\hat{a}^\dagger\triangle\hat{a}\rangle - |\langle(\triangle\hat{a})^2\rangle| < 0 \tag{11}$$

If the fluctuation $\langle(\triangle\hat{a})^2\rangle$ is real, then two criteria are equivalent.

Two mode squeezing Analogously to the single-mode case we can define the quadrature parameters when two modes a and b are considered [14]:

$$X_{ab} = \frac{\hat{a} + \hat{a}^\dagger + \hat{b} + \hat{b}^\dagger}{2\sqrt{2}}, \qquad Y_{ab} = \frac{\hat{a} - \hat{a}^\dagger + \hat{b} - \hat{b}^\dagger}{i2\sqrt{2}} \tag{12}$$

As it was shown in [15], two-mode squeezing appears when the following conditions are fulfilled

$$\begin{aligned} S_{ab} = 2[1 &+ \langle\triangle\hat{a}^\dagger\triangle a\rangle + \langle\triangle b^\dagger\triangle b\rangle \\ &+2\mathrm{Re}\langle\triangle\hat{a}^\dagger\triangle b\rangle + \mathrm{Re}(\langle(\triangle a)^2\rangle + \langle(\triangle b)^2\rangle + 2\langle\triangle a\triangle b\rangle)] < 2. \end{aligned} \tag{13}$$

$$\begin{aligned} S'_{ab} = 2[1 &+ \langle\triangle\hat{a}^\dagger\triangle a\rangle + \langle\triangle b^\dagger\triangle b\rangle \\ &+2\mathrm{Re}\langle\triangle\hat{a}^\dagger\triangle b\rangle - \mathrm{Re}(\langle(\triangle a)^2\rangle + \langle(\triangle b)^2\rangle + 2\langle\triangle a\triangle b\rangle)] < 2. \end{aligned} \tag{14}$$

Moreover, we can apply the analogous condition for the two-mode *principal squeezing*

$$\begin{aligned} \lambda_{ab} = 2[1 &+ \langle\triangle\hat{a}^\dagger\triangle a\rangle + \langle\triangle b^\dagger\triangle b\rangle \\ &+2\mathrm{Re}\langle\triangle\hat{a}^\dagger\triangle b\rangle - |\langle(\triangle a)^2\rangle + \langle(\triangle b)^2\rangle + 2\langle\triangle a\triangle b\rangle|] < 2 \end{aligned} \tag{15}$$

where

$$\langle\triangle\hat{a}^\dagger\triangle b\rangle = \langle\hat{a}^\dagger b\rangle - \langle\hat{a}^\dagger\rangle\langle b\rangle; \qquad \langle\triangle a\triangle b\rangle = \langle ab\rangle - \langle a\rangle\langle b\rangle \tag{16}$$

One- and two-mode squeezing can be treated as the lowest order non-classicality indicators, whereas there appear other criteria which are can be applied to test *higher-order squeezing* effects. The concept of k-th order squeezing and its criterion was introduced for the first time by Hong and Mandel [16, 17]. However, in this part we shall concentrate on the definition proposed by Hillery [18], where higher-order squeezing is defined with use of the variances of the quadrature operators defined with use of squared creation and annihilation operators. This idea can be extended to the cases when we take into account higher powers, and two amplitude powered quadrature variables can be defined as

$$X_{h,a} = \frac{a^k + \hat{a}^{\dagger k}}{2}, \quad Y_{h,a} = -i\frac{a^k - \hat{a}^{\dagger k}}{2}. \tag{17}$$

Two operators $X_{h,a}$ and $Y_{h,a}$ do not commute and therefore, from the uncertainty relation the criterion for kth-order squeezing can be written as:

$$H_{1,a} = \langle(\triangle X_{h,a})^2\rangle - \frac{1}{2}|\langle C\rangle| < 0, \tag{18}$$

$$H_{2,a} = \langle(\triangle Y_{h,a})^2\rangle - \frac{1}{2}|\langle C\rangle| < 0, \tag{19}$$

where $[X_{h,a}, Y_{h,a}] = iC$

3.2. Normal and higher-order anti-bunching

Anti-bunching is understood as the characteristic of a radiation field in whatever place the mean number of photon is greater than the variance of the photon number distribution. In quantum statistics, for the single-mode case photon anti-bunching effect can be described in terms of the normalized second-order correlation function [19], later defined as

$$\mathcal{D}_a^2 = \langle\hat{a}^{\dagger 2}(t)a^2(t)\rangle - \langle\hat{a}^\dagger(t)a(t)\rangle^2 < 0. \tag{20}$$

For the case when two modes of the field a and b are considered, one can generalize two-mode anti-bunching concept and define as

$$\mathcal{D}_{ab} = \langle(\hat{a}^\dagger a)^2(b^\dagger b)^2\rangle - \langle\hat{a}^\dagger a\rangle^2\langle b^\dagger b\rangle^2 < 0. \tag{21}$$

The inequalities shown here are anti-bunching criteria determining two-mode anti-bunching, for the situation when two modes are involved, one can also apply single-mode criteria for the particular modes. Beside usual (single- and two-mode) anti-bunching effects one can consider more general case of *higher-order anti-bunching* (HOA). Single mode HOA criteria were proposed by Lee [22, 23]. After generalization, kth order anti-bunching criterion can be expressed in terms of the creation and annihilation operators as [26]:

$$\mathcal{D}_a^k = \langle\hat{a}^{\dagger k}\hat{a}^k\rangle - \langle\hat{a}^\dagger\hat{a}\rangle^k < 0. \tag{22}$$

From (22), it is clear that if a state is anti-bunched in kth order, it has to be anti-bunched in $(k-1)$th order, as well.

3.3. Intermodal entanglement

It is highly desirable to find inseparability criteria which would be directly applicable for multimode problems. The examples of them are those proposed by Hillery and Zubairy (HZ) [27, 28], and by Duan *at al.* [29]. The HZ inseparability criteria refer to some expectation values, variances and higher-order moments of observable operators. The first HZ criterion can be expressed in terms of the creation $\hat{a}^\dagger$ ($\hat{b}^\dagger$) and annihilation operators $\hat{a}$ ($\hat{b}$) in the following way:

$$\langle\hat{a}^\dagger a b^\dagger b\rangle - |\langle ab^\dagger\rangle|^2 < 0, \tag{23}$$

When this inequality is fulfilled, the state is entangled. This criterion can be generalized to the cases when higher moments are considered. For such a case it can be written as [27]

$$E_{ab}^{kl} = \langle(\hat{a}^\dagger)^k a^k (b^\dagger)^l b^l\rangle - |\langle a^k (b^\dagger)^l\rangle|^2 < 0. \tag{24}$$

where k and l are non-zero positive integers.

The second HZ criterion, which is fulfilled for the separability states, can be expressed as:

$$|\langle\hat{a}\hat{b}\rangle| \leq [\langle\hat{a}^\dagger\hat{a}\rangle\langle\hat{b}^\dagger\hat{b}\rangle]^{1/2}, \tag{25}$$

and a violation of it implies that the considered state is entangled. The same way as previously, this criterion can be generalized for higher-order moments. Thus, it reads [27]:

$$E_{ab}^{\prime kl} = \langle\hat{a}^{\dagger k}a^k\rangle\langle b^{\dagger l}b^l\rangle - |\langle a^k b^l\rangle|^2 < 0 \tag{26}$$

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4. Witnesses of non-classicality

4.1. Single-mode squeezing

From the solutions found in the previous section we can determine the dynamics of our quantum system. In consequence we are able to find various witnesses of non-classical phenomena. We start here from discussion of the one-mode squeezing. In Fig.1 and Fig. 2 we show the time-evolution of the squeezing parameters $S_{a(b)}$ (solid curve), $S_{a(b)}$ (dashed curve) and that of principal squeezing $\lambda_{a(b)}$. Assuming that the amplitudes α and β for the initial coherent states are real and equal each other. Because of the equivalence, the lines for two modes are identical. From the behavior of squeezing factors and principle squeezing, we see that the our system can give single mode squeezing in a and b. However, for the chosen values of the parameters, squeezing can not be created in $S'_a(b)$ factors.

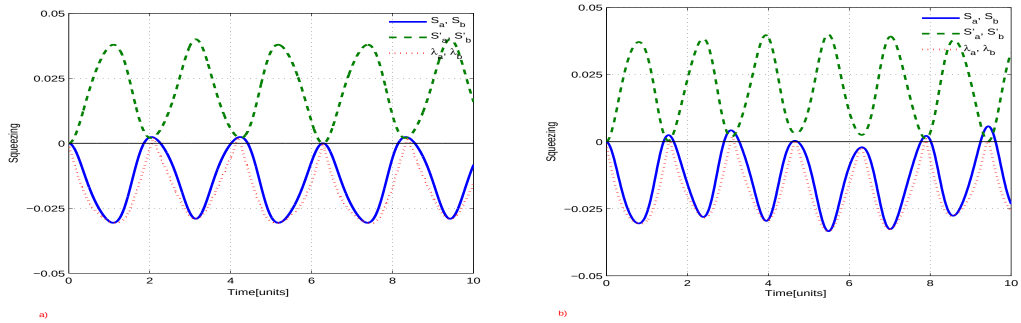

Figure 1. Evolution of squeezing factor when initial coherent state are $\alpha = 0.2$, $\beta = 0.2$. Figures a) and b) correspond to the parameters $\chi = 1$, $\tilde{\chi} = 1$, $\epsilon_{nl} = 0.5$ and $\chi = 1$, $\tilde{\chi} = 1$, $\epsilon_{nl} = 1$, respectively.

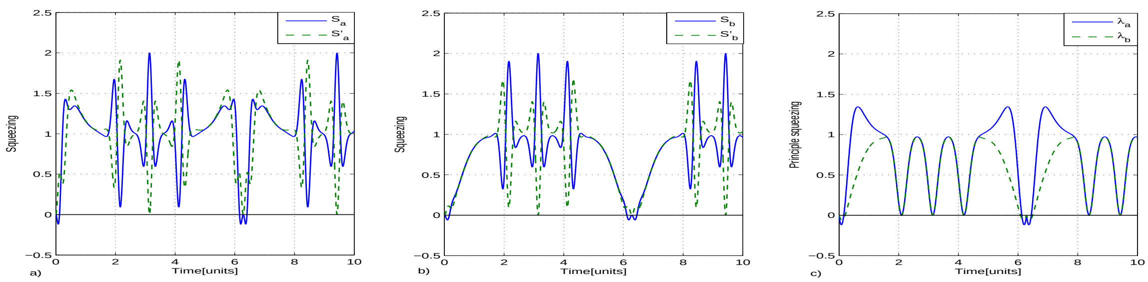

Figure 2. The time-evolution of the squeezing parameters: a) S_a, S'_a b) S_b, S'_b, and c) λ_a, λ_b. Initial coherent states are $\alpha = 2$, $\beta = 0$. Other parameters: $\chi = 1, \tilde{\chi} = 1$ and $\epsilon_{nl} = 0.5$.

4.2. Two-mode squeezing

In Fig.3 two-mode quadrature variances S_{ab} (solid curve), S'_{ab} (dash curve) and two-mode principle squeezing parameter λ_{ab} are depicted. For the initial coherent states defined by $\alpha = 0.2$, $\beta = 0.2$ (see Fig. 3b)), we see that the squeezing of the quadrature S'_{ab} does not appear, contrary to that corresponding to S_{ab} which appear with a quite high intensity. Additionally,

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two-mode squeezing is present practically all the time in our system. When we change the initial states and assume that $\alpha = 2$ and $\beta = 0$ (see Fig. 3a), non-classical effect disappears almost completely. The most interesting point is that S'_{ab} does not give any signature of squeezing.

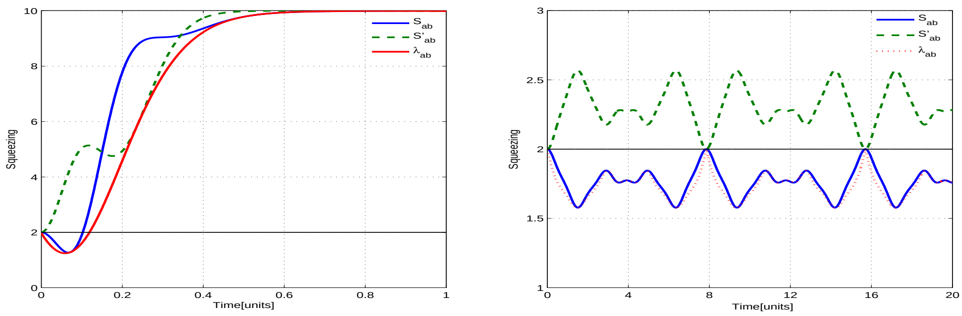

Figure 3. The time-evolution of two-mode quadrature variances: a) Initial coherent states are $\alpha = 2$, $\beta = 0$, other parameters: $\chi = 1, \tilde{\chi} = 1$ and $\epsilon_{nl} = 0.5$. b) $\alpha = 0.2$, $\beta = 0.2$, other parameters $\chi = 1, \tilde{\chi} = 1.6$ and $\epsilon_{nl} = 0.2$.

4.3. Higher order squeezing

In the preceding section, we have found the single-modes and coupled-mode squeezing effects existing in our system. In this part, we seek for the possibility of providing higher-order squeezing effect based on Hillery criteria [16, 17]. If we assume $k = 2$, the second order squeezing criteria are constructed.

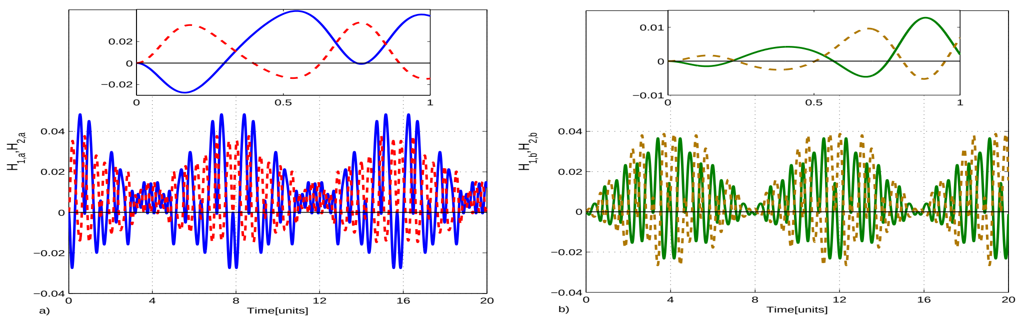

Figure 4. The time evolution of second order squeezing for mode a (Fig. a) and mode b (Fig. b) where $\alpha = 0.5$ $\beta = 0.3$, $\chi = 1, \tilde{\chi} = 1.6$ and $\epsilon_{nl} = 0.2$.

In Fig.4 and Fig. 5 we plot higher-order squeezing factor in short an long time evolution and see the negative regions of these two plots clearly illustrate the existence of this effect. Similar to the single mode and two mode squeezing, we show that the system can provide higher order squeezing and the appearance of effect in a particular quadrature depends on the choice of input coherent state.

4.4. Normal and higher-order anti-bunching effects

In this part, we proceed with the calculations of normal and higher-order anti-bunching. When the coherent single modes are equal ($\alpha = \beta$), there does not exist any signature of normal and

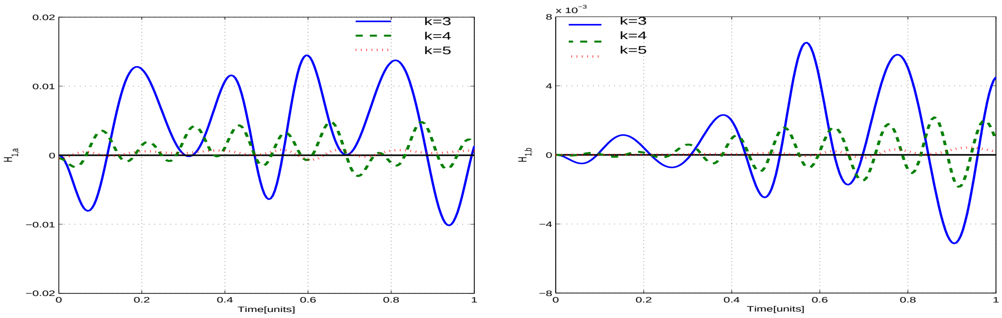

Figure 5. Short time evolution of higher-order squeezing where $\alpha = 0.5$ $\beta = 0.3$, $\chi = 1$, $\tilde{\chi} = 1.6$ and $\epsilon_{nl} = 0.2$

higher-order anti-bunching. If two initial coherent states are not equal, these effects might be pronounced.
Existence of normal anti-bunching is obtained in single-mode a for $\alpha = 2$, $\beta = 0$ and $\alpha = 0.5$, $\beta = 0.3$ is demonstrated in figures 6a) and 6b). It is easy to recognize that for our system this non-classical property is evident when the initial states are setting up with smaller values of mean number of photons. In figure 7, the results of our calculations in the factors $\mathcal{D}_a(k)$ when $\alpha = 0.5$, $\beta = 0.3$ (Fig.7a) and $\alpha = 0.5$, $\beta = 0.2$ (Fig.7 b) with $\tilde{\chi} = 1.6$, $\chi = 1$, $\epsilon_{nl} = 0.2$ for different values of k are shown. It is seen that we can only detect signals of higher-order anti-bunching in the mode a (note that the initial coherent state of mode a has greater mean number of photons than for the mode b *i.e.* $|\alpha| > |\beta|$). Otherwise, there is no any signature of higher-order anti-bunching.

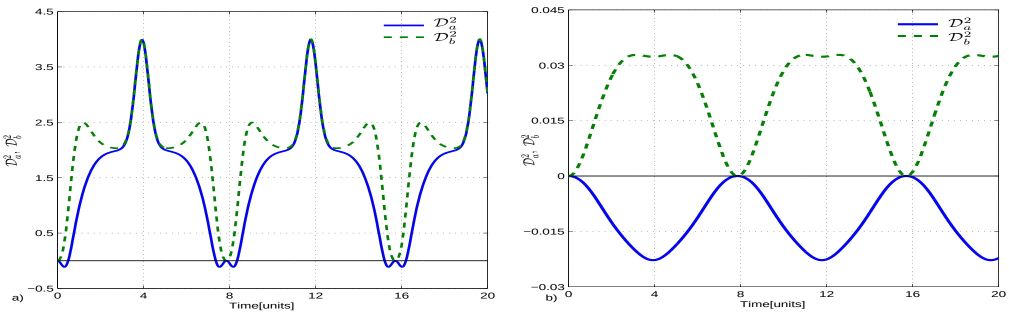

Figure 6. Normal order anti-bunching for the parameters a) $\tilde{\chi} = 1.6$, $\chi = 1$, $\epsilon_{nl} = 0.2$, $\alpha = 2$, $\beta = 0$, b) $\tilde{\chi} = 1.6$, $\chi = 1$, $\epsilon_{nl} = 0.2$, $\alpha = 0.5$, $\beta = 0.3$

4.5. Intermodal entanglement

At this point we apply two higher-order intermodal entanglement criteria for two-mode state which were introduced by Hillery and Zubairy in [27, 28].
Thus, in Fig.8 we plot $E_{ab}^{1,1}$, $E_{ab}^{2,1}$, and $E_{ab}^{2,2}$ for several different values of the nonlinear interaction strength. It is seen that two modes are not always entangled in time domain. In addition, when we increase the nonlinear interaction between two modes, we obtain stronger intermodal entanglement (see dashed lines with deeper minima). Moreover, although we observe a week

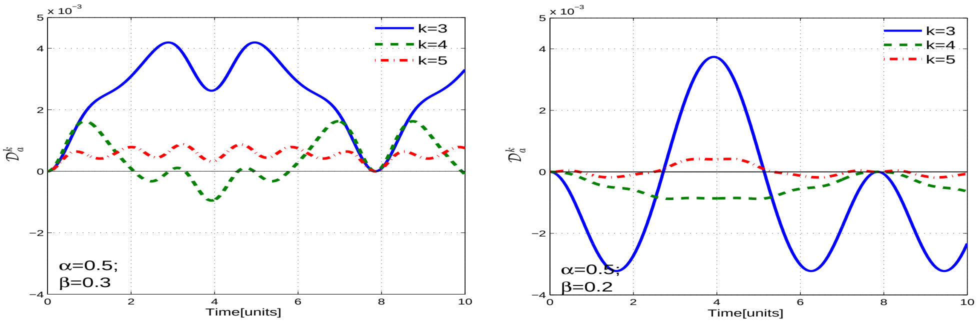

Figure 7. Evolution of higher-order anti-bunching for the parameters $\tilde{\chi} = 1.6$, $\chi = 1$, $\epsilon_{nl} = 0.2$

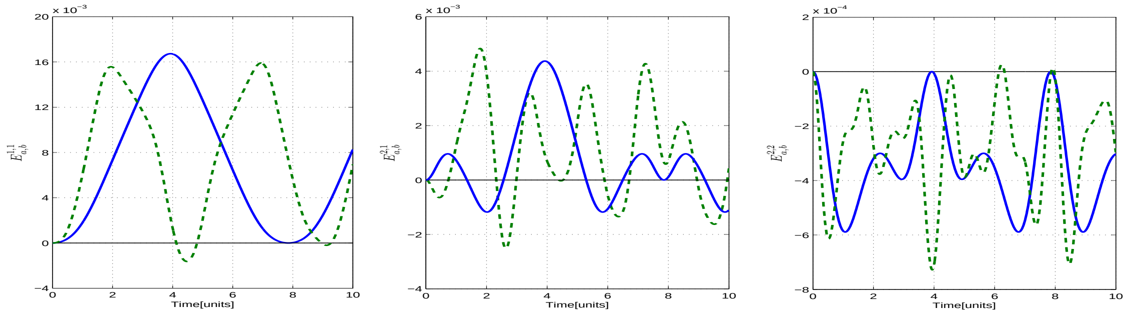

Figure 8. *Higher order intermodal entanglement for initial coherent states with* $\alpha = 0.5$, $\beta = 0.3$, *other parameters* $\tilde{\chi} = 1.6$, $\chi = 1$, $\epsilon_{nl} = 0.2$ *(solid line) and* $\epsilon_{nl} = 0.5$ *(dashed line).*

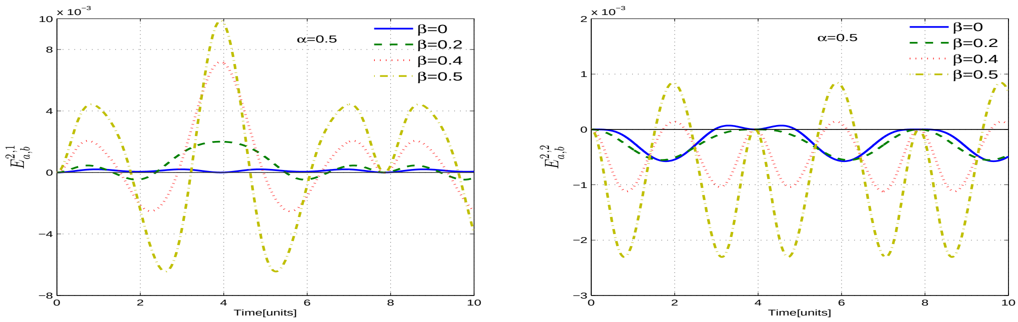

Figure 9. *Higher order intermodal entanglement for input parameters* $\tilde{\chi} = 1.6$, $\chi = 1$, $\epsilon_{nl} = 0.2$, *initial coherent state with* $\alpha = 0.5$, *and* $\beta = 0$ *(solid line),* $\beta = 0.2$ *(dashed line),* $\beta = 0.4$ *(dotted line),* $\beta = 0.5$ *(dashed-dotted line).*

signature of normal intermodal entanglement with lower value of the nonlinear interaction strength, higher-order entanglement is more pronounced (see evolution of $E^{2,1}_{ab}$, and $E^{2,2}_{ab}$). Especially, if we look at the plot corresponding to $E^{2,2}_{ab}$, we see that it is negative during the almost whole time-evolution.

In figure 9, we draw the plots of $E^{2,1}_{ab}$ ($k = 2,\ l = 1$), and $E^{2,2}_{ab}$ ($k = 2,\ l = 2$) where $\tilde{\chi} = 1.6$,

$\chi = 1$, $\epsilon_{nl} = 0.2$ for various cases of the initial coherent states. From the negative regions of the plots, we detect the signature of higher-order entanglement existing in our the system. It is seen that all plots exhibit periodic behavior. One can note that for the chosen here values of the parameters the signature of the lowest order intermodal entanglement (E_{ab}^{11}) is not detected. However, our Kerr-like coupler nonlinear system with nonlinear interaction term still can be seen as a source of intermodal entanglement and other higher-order ones in general.

5. Conclusions

Various types of non-classicality indicators of two-mode system, such as: squeezing, anti-bunching, intermodal entanglement and their higher-order counterparts were discussed. We derived the exact numerical solutions for witnesses of nonclassicality. We have analyzed the revealed effects caused by contributions of different input parameters. It is showed that the parameters considered here can be an indicators of the generation such non-classical effects and hence, quantumness of the system. The nonclassical properties of the system were more evident when we assumed small values of mean number of photons for the initial coherent states.

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