

Enhanced current fluctuation in Coulomb blockade regime of multilevel quantum dot

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Abstract. Nonequilibrium current fluctuation is theoretically studied through a multilevel quantum dot in the Coulomb blockade regime, where the higher-order tunneling processes, “cotunneling,” are dominant in the transport. The current fluctuation shows a Poisson (super-Poisson) noise in the elastic (inelastic) cotunneling regime when the bias voltage eV between the leads is smaller (larger) than the excitation energy Δ from the ground level in the quantum dot. The super-Poisson noise is remarkably enhanced by the degeneracy of the excited states around the beginning of inelastic cotunneling at $eV \approx \Delta$, qualitatively in good agreement with a recent experimental result. An analytical expression is given for the current fluctuation as a function of degeneracy.

1. Introduction

Nonequilibrium current fluctuation through a quantum dot yields information on the dynamic motion of single electron transport [1]. In the present paper, we theoretically examine the current fluctuation in the Coulomb blockade regime, where the transport takes place by the higher-order tunnel processes, cotunneling. Although the nonequilibrium fluctuation of the cotunneling current was reported by several groups [2, 3, 4], we pay special attention to the roles of excited levels in the quantum dot.

Let us consider a quantum dot with N energy levels ($E_1, E_2, \dots, E_N$) attached to two leads L and R , as shown in Fig. 1. The bias voltage V is applied, $eV = \mu_L - \mu_R$ with μ_L and μ_R being the electrochemical potentials in the leads ($\mu_R < \mu_L$). At low temperatures, only the elastic cotunneling through the ground level E_1 takes place when eV does not exceed $\Delta_{12} = E_2 - E_1$, whereas inelastic processes are also possible when $eV > \Delta_{12}$. In the latter regime, Okazaki *et al.* found a large current fluctuation when an excited level is within the bias window; $E_1 < \mu_R < E_2 < \mu_L$ [5]. This is attributable to the sequential transport through level E_2 , besides the cotunneling through level E_1 . A large difference in the time scale between sequential and cotunneling transports results in a super-Poisson noise (Fano factor ≈ 3) for the current fluctuation. Recently, an enhancement of the current fluctuation (Fano factor ≈ 1.5) was observed when the excited levels are out of the bias window ($E_1, \dots, E_N < \mu_R$) in the transport through a carbon-nanotube (CNT) quantum dot [6]. The degeneracy of the energy levels should play a role in this current fluctuation, which stems from the spin and valley degrees of freedom in the CNT.

To understand the mechanism of enhanced current fluctuation in CNT, we examine a simple model of a quantum dot with N energy levels for spinless electrons. We show that a super-Poisson noise in the inelastic regime can be significantly enhanced by the degeneracy of excited levels.



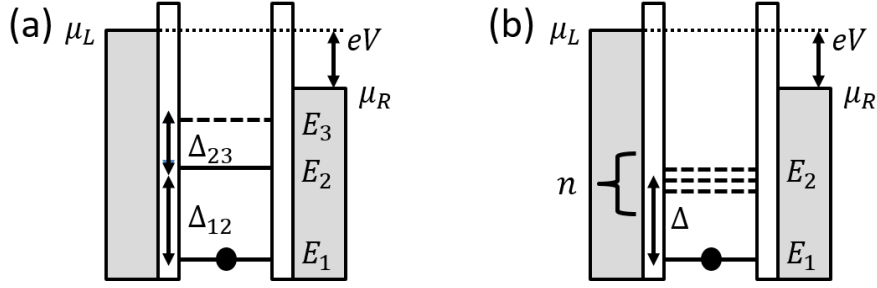


Figure 1. Two models for a quantum dot in the Coulomb blockade regime with one electron. Electrons are spinless. The quantum dot is attached to two leads with electro-chemical potentials, μ_L and μ_R ($\mu_L > \mu_R$). The bias voltage V is applied between the leads in such a way as $\mu_L = eV/2$ and $\mu_R = -eV/2$. (a) There are two or three energy levels in a quantum dot. The level spacings are denoted by $\Delta_{i,i+1} = E_{i+1} - E_i$. (b) A quantum dot possesses a non-degenerate ground level E_1 and an excited level E_2 with n -fold degeneracy. $\Delta = E_2 - E_1$.

2. Model and Calculation Method

In our model, spinless electrons are described by the Hamiltonian, $H = H_0 + H_R + H_T$:

$$H_0 + H_R = \sum_{i=1}^N E_i n_i + U \sum_{i>j} n_i n_j + \sum_{\alpha=L,R} \sum_{i=1}^N \sum_k \epsilon_k c_{\alpha ki}^\dagger c_{\alpha ki}, \quad (1)$$

$$H_T = \sum_{\alpha=L,R} \sum_{i=1}^N V_{\alpha i} d_i^\dagger c_{\alpha ki} + \text{h.c.}, \quad (2)$$

with $n_i = d_i^\dagger d_i$, where d_i and $d_i^\dagger$ are annihilation and creation operators of an electron in the quantum dot with energy level E_i ($i = 1, \dots, N$), respectively. $c_{\alpha ki}$ and $c_{\alpha ki}^\dagger$ are those in lead α with wavenumber k and channel i . (We assume that level i in the quantum dot is coupled to channel i in lead α by $V_{\alpha i}$ in the tunnel Hamiltonian H_T , which is the situation in CNT with spin and valley degrees of freedom.) The Coulomb interaction is denoted by U in the quantum dot.

We examine the Coulomb blockade regime with one electron in the quantum dot, $E_1 < \mu_L, \mu_R < E_1 + U$. The bias voltage $eV = \mu_L - \mu_R$ is so small that all the excited levels are out of the bias window, $E_i < \mu_R$ ($i = 1, \dots, N$). First, we study a model for the quantum dot with two or three levels [Fig. 1(a)]. The level spacings are denoted by $\Delta_{i,i+1} = E_{i+1} - E_i$. Second, the degeneracy of the excited levels is studied by another model in Fig. 1(b).

The current $\langle I \rangle$ and current fluctuation $S(\omega = 0)$, which is the zero-frequency component of Fourier transformation of the current-current correlation function, $\langle I(t)I(0) \rangle - \langle I \rangle^2$, are evaluated using the full counting statistics to solve the master equation for the stationary state [4]. The transition rate $\Gamma_{i \rightarrow j}^{\alpha \rightarrow \beta}$ from level E_i to level E_j in the quantum dot, accompanied by the single-electron transport from lead α to β is given by the second-order perturbation with respect to H_T . For $i = j$, it is

$$\Gamma_{i \rightarrow i}^{\alpha \rightarrow \beta} = \frac{2\pi}{\hbar} \nu^2 \int d\epsilon_k \left[\frac{|V_{\alpha i}|^2 |V_{\beta i}|^2}{(E_i - \epsilon_k)^2} + \sum_{l \neq i} \frac{|V_{\alpha l}|^2 |V_{\beta l}|^2}{(\epsilon_k - E_l - U)^2} \right] f_\alpha(\epsilon_k) [1 - f_\beta(\epsilon_k)], \quad (3)$$

where ν is the density of states in the leads and $f_\alpha(\epsilon)$ is the Fermi distribution function in lead α . For $i \neq j$

$$\Gamma_{i \rightarrow j}^{\alpha \rightarrow \beta} = \frac{2\pi}{\hbar} \nu^2 |V_{\alpha j}|^2 |V_{\beta i}|^2 \int d\epsilon_k \left(\frac{1}{E_i - \epsilon_k} + \frac{1}{\epsilon_k - E_j - U} \right)^2 f_\alpha(\epsilon_k) [1 - f_\beta(\epsilon_k - E_j + E_i)]. \quad (4)$$

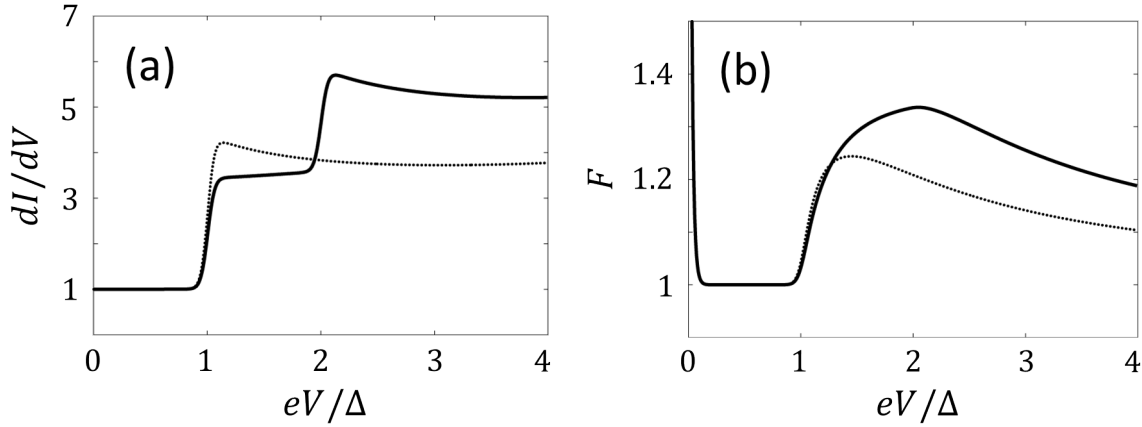


Figure 2. (a) dI/dV and (b) Fano factor F as a function of the bias voltage eV for the model in Fig. 1(a). The case of two levels ($\Delta_{12} = \Delta$) is denoted by dotted lines, whereas that of three levels ($\Delta_{12} = \Delta_{23} = \Delta$) is by solid lines. dI/dV is normalized by its value at $eV/\Delta = 0$. $E_1 = -10\Delta$, $U = 20\Delta$, and temperature $k_B T = 0.02\Delta$. All the tunnel couplings $V_{\alpha i}$ are identical to each other.

The Fano factor, $F = S(\omega = 0)/2e\langle I \rangle$, is useful to characterize the nonequilibrium current fluctuation at $eV > k_B T$, so-called shot noise. $F = 1$ for the Poisson processes, whereas $F > 1$ ($F < 1$) for the super- (sub-) Poisson processes.

3. Calculated Results

Figure 2 presents (a) dI/dV and (b) Fano factor F as a function of bias voltage eV for the model in Fig. 1(a). In the case of two levels ($\Delta_{12} = \Delta$; dotted line), dI/dV is almost constant at $eV < \Delta$, reflecting the elastic cotunneling through the lower level E_1 only, whereas dI/dV suddenly increases around $eV = \Delta$ owing to the possible inelastic processes at $eV > \Delta$. A small peak in the dI/dV curve is seen at the beginning of inelastic cotunneling, which is caused by the increase in the probability P_2 that an electron occupies level 2 in the quantum dot with increasing eV ($dP_2/dV > 0$) [7]. The Fano factor is almost unity at $k_B T < eV < \Delta$ (large F at $eV < k_B T$ is due to the thermal noise). $F = 1$ in the elastic cotunneling regime means that the cotunneling takes place as a Poisson process from lead L to R . F is enhanced to 1.25 at $eV > \Delta$. This super-Poisson noise is caused by more than one time-scale for both elastic and inelastic processes.

In the case of three equi-distant levels ($\Delta_{12} = \Delta_{23} = \Delta$; solid line), dI/dV shows two rapid increases around $eV = \Delta$ and 2Δ . Note that the Fano factor is larger than that in the two-level system at $\Delta < eV < 2\Delta$, which is ascribable to the additional transport accompanied by the transition from level E_2 to E_3 . Since the probability P_2 to occupy level E_2 is finite, an electron can be excited to level E_3 from E_2 , which enlarges the current fluctuation further.

Figure 3 is devoted to the results for the model in Fig. 1(b), where the higher level is n -fold degenerate in the quantum dot ($n = 1, 2$, and 4). The peak in the dI/dV curve at the beginning of inelastic cotunneling becomes larger for larger n . This is because the probability to occupy the higher level, P_2 , increases with V more rapidly (dP_2/dV is larger) for larger n .

We find a larger peak in the Fano factor at $\Delta < eV < 2\Delta$ for larger n . This is due to the larger difference in the time scales for the transport through level E_1 and E_2 for larger n . In contrast, the Fano factor becomes smaller for larger n at $eV > 3\Delta$, where the role of the lower level in the transport is less for larger n .

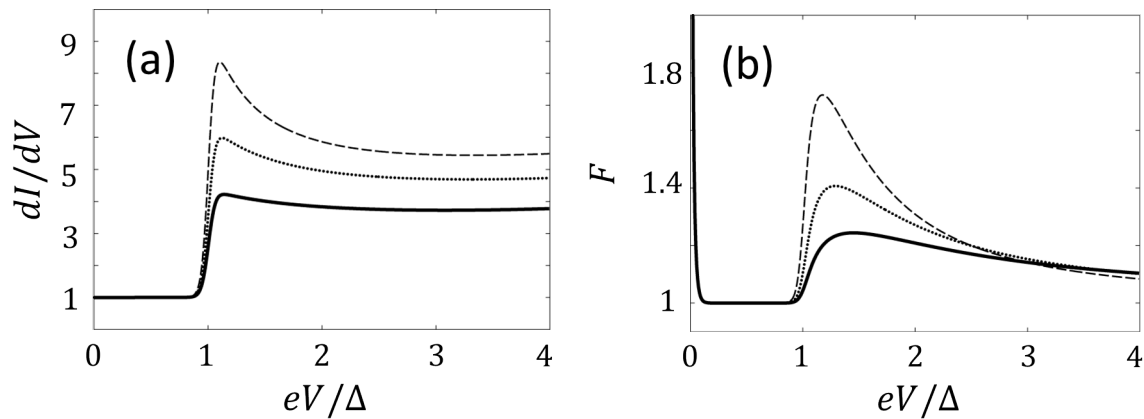


Figure 3. (a) dI/dV and (b) Fano factor F as a function of the bias voltage eV for the model in Fig. 1(b). The degeneracy of the higher level is $n = 1$ (solid line), 2 (dotted line), and 4 (broken line). dI/dV is normalized by its value at $eV/\Delta = 0$. $E_1 = -10\Delta$, $U = 20\Delta$, and temperature $k_B T = 0.02\Delta$. All the tunnel couplings V_{ai} are identical to each other.

We obtain an analytical expression for the Fano factor at $eV > \Delta$:

$$F = 1 + \frac{8n(n+1)(x-1)[3(n+1)x+n+9]}{[(n+1)x+3-n]^2[(5n+1)(n+1)x^2 - (5n+1)(n-3)x - 16n]}, \quad (5)$$

where $x = eV/\Delta$. This expression is justified when $-E_1, E_1 + U \gg \Delta, eV$. The curves in Fig. 3(b) are well reproduced by this expression for $eV > \Delta$.

4. Conclusion

We have studied the nonequilibrium current and its fluctuation through a multilevel quantum dot in the Coulomb blockade regime, where the cotunneling processes are dominant in the transport. The current fluctuation shows a Poisson noise in the elastic regime and a super-Poisson noise in the inelastic regime. The super-Poisson noise is remarkably enhanced by the degeneracy of the excited states around the beginning of inelastic cotunneling, which is qualitatively in good agreement with a recent experimental result using a CNT quantum dot. The analytical expression for this enhanced noise has been given as a function of degeneracy.

Acknowledgments

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