

# Future Challenges for Nuclear Clustering

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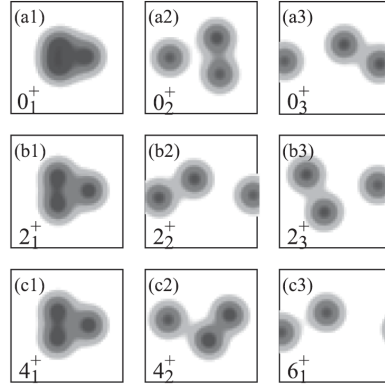
**Abstract.** This contribution will focus on some of the advances that have been made since the last Cluster Conference in Debrecen, Hungary, and the challenges that remain for the subject before the next conference in Brazil. It addresses the question of cluster structure above and below the cluster decay threshold, dynamical symmetries, molecular structures and clustering at the neutron drip-line.

## 1. Introduction

The subject of nuclear clustering is almost as old as nuclear physics itself, and for those reasons and the fact that it has its own conference series, it is often forgotten why it is important to understand *nuclear* clustering. After all, clustering appears in many systems from biological, chemical and physical and can result in structures which are highly appealing to the human sense of organization and symmetry. However, to think only of the cluster arrangements would make nuclear clustering no more than a curio. Clustering reveals much about the nature of the force through which the constituent components interact and the symmetries that result. It is important to remember this connection in order to maintain the relevance of the subject to nuclear physics. The nuclear strong interaction is clearly complex and this is revealed in the detail of the unbound and bound light nuclei. The  $\alpha$ -particle is one of the most highly bound light nuclei with a very high-lying,  $\sim 20$  MeV, first excited state, and here the array of correlations, that include not only  $n-n$  and  $p-p$  but  $n-p$ , maximize the binding energy. The tendency of other nuclei to optimize their own binding by generating spatial and momentum correlations induces the formation of clusters. This is responsible for clustering in  $\alpha$ -conjugate nuclei, Borromean and molecular systems, alike.

How the clusters arrange themselves in part derives from the minimization of the total energy, but is also strongly influenced by the fact that the cluster structures grow from the single-particle nature of the nucleus, driven by the mean-field. The mean-field generates magic numbers and symmetries - which may or may not be those of  $\alpha$ -particles. This tension is seen in approaches such as the no core shell model [1] where a realistic nucleon-nucleon interaction is employed within a harmonic oscillator basis. In some instances, the cluster structure emerges naturally from the available, low  $\hbar\omega$ , valence space, for example  $^8\text{Be}$ . The contrasting case is the Hoyle-state in  $^{12}\text{C}$ , which explores much more fully the entire  $\hbar\omega$  spectrum of the harmonic oscillator. Herein lies the signature that the clustering degree of freedom has taken over and the correlations outweigh the mean-field. Understanding the interplay between the limiting cases requires experimental intervention and in particular precision measurements. The following sections explore some outstanding challenges for the field which might be addressed along the path to the next Cluster Conference.





**Figure 1.** AMD densities of the ground state band,  $0_1^+$ : a1, b1 and c1, Hoyle-band,  $0_2^+$  a2, b2, c2 and  $0_3^+$  band a3, b3, c3, from [2].

## 2. Clustering above and below the decay threshold and dynamical symmetries

It is recognised and accepted that clustering plays a crucial role above the cluster-decay threshold. Here this is well illustrated by the AMD calculations for  $^{12}\text{C}$  (Fig. 1). The densities associated with the structures above the  $3\alpha$  decay threshold demonstrate that the nucleus precipitates into 3 clusters. However, below the decay threshold, and even in the ground-state, clustering also appears to have a structural influence. Demonstrating such a structure exists, experimentally, is not simple. The ground-state is compact, though still reveals the  $3\alpha$  structure, which is also strongly influenced by the  $^8\text{Be}+\alpha$  substructure. The collective excitations of this state (b1) and (c1)  $2^+$  and  $4^+$  states. Ultimately, measurements of the electromagnetic transition strengths,  $B(E2)$ , between the states above the  $\alpha$ -decay threshold are the litmus test of which states may be strongly structurally linked.

Such measurements have been performed for  $^8\text{Be}$  between the  $2\alpha$   $4^+$  and  $2^+$  cluster states [3]. In that instance the states are separated by close to 8 MeV and hence the transition probability is strongly amplified by the  $E^5$  dependence of the  $B(E2)$  reduced transition amplitude. Hence, the competition with the  $\alpha$ -decay is enhanced and a measurement is possible. The measurement demonstrates the collective nature of the excitations of  $^8\text{Be}$ . This raises an interesting question [4]. The excited states of  $^8\text{Be}$  are associated with resonances with large widths ( $2^+$  1.5 MeV,  $4^+$   $\sim$ 3.5 MeV) which imply lifetimes of  $10^{-22}$  seconds. This is the timescale for a nucleon to cross a nucleus and it is anticipated that the time scale for collective correlations and rotations to develop is significantly longer. It is intriguing to then understand why the  $0^+$ ,  $2^+$  and  $4^+$  states have a sequence of energies which display a rotational behaviour  $E(4^+)/E(2^+)=3.7$ . Is the key feature, the underlying cluster structure and the resulting symmetry rather than rotational characteristics and the states are symmetry-linked rather than rotationally-linked?

Understanding such issues may be improved by examining the structure of  $^{12}\text{C}$ .

Here the dynamical symmetries of  $3\alpha$ -system correspond to a spinning top with a triangular point symmetry. The rotational properties of these states are given by

$$E_{J,K} = \frac{\hbar^2 J(J+1)}{2\mathcal{I}_{Be}} - \frac{\hbar^2 K^2}{4\mathcal{I}_{Be}} \quad (1)$$

where  $\mathcal{I}_{Be}$  is the moment of inertia corresponding to two touching  $\alpha$ -particles, which can be determined from the  $^8\text{Be}$  ground-state rotational band [5]. One would expect that based on this structure there should be a number of rotational bands with different values of  $K$ . For  $K^\pi=0^+$ , the rotations will be around an axis which lies in the plane of the three  $\alpha$ -particles, generating a series of states  $0^+$ ,  $2^+$ ,  $4^+$  .... These correspond to the rotation of a  $^8\text{Be}$  nucleus - the rotation axis

passing through the centre of the third  $\alpha$ -particle. The next set of rotations correspond to the rotation around an axis perpendicular to the plane of the triangle, with each  $\alpha$ -particle having one unit on angular momentum - giving  $L = 3 \times 1\hbar$ ;  $K^\pi=3^-$ . Rotations around this axis and that parallel to the plane combine to give a series of states  $3^-, 4^-, 5^- \dots$ . Such an arrangement possesses a  $D_{3h}$  point group symmetry. The experimental quest to observe this structure has resulted in the observation of the  $4^-$  and  $5^-$  members of the  $K^\pi = 3^-$  rotational band [6], which has been taken as confirmation of the  $D_{3h}$  structure. However, as observed in the present conference, the arrangements of the experimentally observed states into rotational-like bands is not unique and equally it is possible to construct a sequence that has  $SU(3)$  symmetry [7]. Experimental measurements of the electromagnetic transition rates is likely to be important in resolving this open question.

Similarly, the  $4\alpha$  system should be described by the tetrahedral symmetry group;  $T_d$ . Here the properties are those of a spherical top, with equal moments of inertia. If one assumes the separation of the  $\alpha$ -particles is that which is associated with the  $^8\text{Be}$  ground state,  $\mathcal{I}_{Be}$ , then the rotational energies are given by

$$E_J = \hbar^2 \frac{J(J+1)}{4\mathcal{I}_{Be}} \quad (2)$$

The rotations of the tetrahedral structure corresponds to the equivalent rotation of two  $^8\text{Be}$  nuclei around their symmetry axis and hence the  $4\mathcal{I}_{Be}$  in the denominator. The symmetry then dictates that all values of  $J$  are permitted except  $J=1, 2$  and  $5$ ; states with  $J=0, 4$  and  $8$  have even parity and  $J=3, 7$  and  $11$  have negative parity. A key feature of this structure would be degenerate  $6^+$  and  $6^-$  states. A similar conclusion can be found in the recent work of Bijker and Iachello [8]. The experimentally observed states at 6.130 MeV,  $3^-$ ; 10.356,  $4^+$  and 21.052 MeV  $6^+$  have been linked in this latter work to the collective excitations of the tetrahedral structure. These same calculations predicted states at 6.132, 10.220 and 21.462 MeV and electromagnetic transition strengths  $B(E3)$  and  $B(E4)$  of 181 and 338  $\text{e}^2\text{fm}^{2L}$  compared with experimental values of 205(10) and 378(133)  $\text{e}^2\text{fm}^{2L}$ . The comparison between experiment and theory is compelling.

An alternative theoretical approach is provided by the Alpha Cluster Model (ACM) calculations [9]. These calculations identify a number of cluster structures, including a tetrahedral arrangement of the four  $\alpha$ -particles in the ground-state. In addition, a planar arrangement of  $\alpha$ -particles is found for the first excited  $0^+$  state. These structures gives rise to rotational bands. The main difference between the ACM and Algebraic Cluster Model (ACM') of Ref. [8] is evident in the assignment of the 10.356 MeV  $4^+$  state. The ACM assigns it to the planar rotational structure, whereas the ACM' links it to the tetrahedral ground-state. What is clear from measurements of the  $\alpha$ -decay branching ratios for decay to the  $^{12}\text{C}$  ground state and first excited states is that the states in the ACM planar band, above the alpha-decay threshold, all have very similar decay properties - they predominantly decay to the  $^{12}\text{C}$  ground state [10]. This similar structure conflicts with the tetrahedral interpretation and indicates a collective excitation built around a  $^{12}\text{C}+\alpha$  cluster structure where the total angular momentum of the state is generated by the orbital motion of the  $\alpha$ -particle around the  $^{12}\text{C}$  core. To arrive at a better understanding of the cluster symmetries of  $^{16}\text{O}$  further electromagnetic transition strengths need to be determined. These include states above the  $\alpha$ -decay threshold, where small branching ratios ( $< 10^{-5}$ ) make such studies very challenging.

A further outstanding puzzle is the question over the existence of the  $4\alpha$ -particle chain state. This dates back to some ground-breaking measurements performed by Chevallier [11] of the  $^{12}\text{C}(^4\text{He}, ^8\text{Be})^8\text{Be}$  reaction which showed a number of resonances that decayed into two  $^8\text{Be}$  nuclei. Given the structure of the final state and that the resonances appeared to lie on a  $J(J+1)$  trajectory with a moment of inertia consistent with a  $4\alpha$  linear arrangement, a chain state was conjectured. These data have stood without proper test for many decades. The theoretical

study using a cranked Hartree-Fock approach [12] indicated that, unlike the experimental data, the chain-like structure should *not* appear at low spins, but is rather stabilised by the rotations and hence would exist at higher spins. These are spins beyond when the experimental data indicated sharp resonances. A subsequent experimental study, repeating the measurement of the same reaction used by Chevallier came to a different conclusion regarding the nature,  $J^\pi$ , of the resonances [13]. As such, there remains no definitive evidence for the existence of chain-like states in  $^{16}\text{O}$ . A study at higher spins would help in this exploration.

A significant contribution to the Cluster Conference continues to be Condensate, or gas-like,  $\alpha$ -particle like states [14]. In  $^{12}\text{C}$  these are associated with the Hoyle-state and in  $^{16}\text{O}$  the 15.2 MeV  $0^+$  state. The relationship between the Hoyle-state and the  $2^+$  excitation at 10 MeV has yet to be definitively resolved as has the existence of the proposed  $4^+$  state close to 13 MeV. The theoretical behaviour in terms of the predicted energies of these  $2^+$  and  $4^+$  states does not match that observed experimentally and this remains a puzzle. Similarly, it has not been demonstrated that the 15.2 MeV state in  $^{16}\text{O}$  does indeed have a  $4\alpha$  structure. As such there remains much work to be done in the characterisation of the states in both these nuclei above the  $\alpha$ -decay thresholds.

### 3. Molecular states

The story behind molecular structures was told by von Oertzen at the present conference (see the proceedings for the detail). The experimentally observed behaviour of the beryllium isotopes is remarkable in terms of how it may be characterised in terms of the covalent exchange of one or more valence neutrons. This behaviour is also captured in the no core shell model [15] and AMD calculations [16]. The extension of these ideas to the 3-centre systems where neutrons are delocalised beyond two-cores was also explored. Here the experimental situation is not so clear. Measurements presented at the conference indicated the original interpretation of molecular bands [16] may need some revision. The conference heard about measurements of  $^{10}\text{Be}+\alpha$  resonant scattering [17] populating resonances in  $^{14}\text{C}$ . Together with other recent measurements of the same reaction [18, 19] it is possible that a consistent understanding of the structure of molecular structures will emerge in these more complex systems.

For both the two-centre and three-centre molecular systems it is important to go beyond the present description of their molecular behaviour in terms of experimental states matching rotational-like character. The Coriolis-decoupling of the  $1/2^+$  band in  $^9\text{Be}$  provides some confidence of the underlying structure, but precision measurements are now required to really make progress in addressing questions such as “Do these states really have a molecular structure, or are they equally well described by a mean-field type approach?” Measurements such as those performed by for the  $A = 10$  analogues  $^{10}\text{C}$ ,  $^{10}\text{B}^*$  and  $^{10}\text{Be}$  are called for [20].

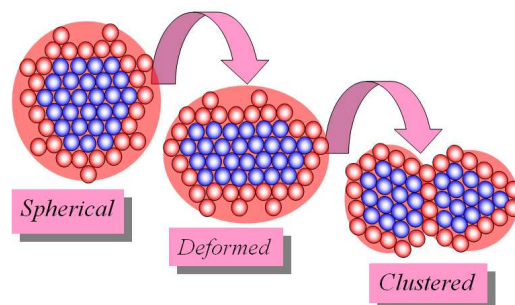
### 4. The drip-lines

A long-standing prediction, of which the Conference was reminded [21], was that made at the Rab Cluster Conference. This was the prediction that at the neutron drip-line the large neutron excess and the energetic advantage of maximising the  $n - p$  interaction is accommodated by clusterising the core, which is shown as a cartoon in Fig. 2.

Such a prediction remains to be verified, or otherwise. Nevertheless, the recent measurements of the potential  $4n$ -decay of  $^8\text{He}$ , which were presented [22], appear to show evidence for  $4n$ -correlations does indicate that the drip-line may yet hold some surprises. The Conference is keen to learn more from the ongoing programme at RIKEN [23].

### 5. Summary and Acknowledgements

The contribution to the Conference from the experimental community was rich and varied. This continues to drive an understanding of the structure of light nuclei and the role of cluster



**Figure 2.** A schematic of the evolution of cluster structure in the ground-states of drip-line nuclei. The blue spheres represent nucleons associated with nuclear matter of *normal* isospin, whereas the red spheres are the excess valence neutrons. The transition from spherical through deformed to clustered permits a more even distribution of the valence neutrons.

correlations. However, to make further and substantial progress there is a strong need for comprehensive and precision measurements. Comprehensive in the sense that for key states all of their characteristics are measured and with a *precision* which allows the testing of models of the structure and interaction. In order to make progress there is a strong need for greater coordination in the experimental and indeed theory communities, to define the key objectives for future experimental measurements, much as is done in communities such as particle physics. This should be a key objective on the journey to Brazil.

The Conference would like to acknowledge the professional efforts of the local organising committee and further to recognise the contribution of Paulo Gomez, whose research into nuclear reactions at low energies often took advantage of the clustered structure of light nuclei. He will be greatly missed.

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