

Closing remarks from a theoretical perspective

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Abstract. This paper is based on a summary talk for theoretical works presented in the 11th International Conference on “Clustering Aspects of Nuclear Structure and Dynamics”, Naples (Italy), May 23-26, 2016.

1. Introduction

Among fundamental physics behind cluster phenomena, symmetry is one of the most important physics in cluster phenomena. Of course, Pauli principle, duality and saturation property are also essential features in nuclear systems consisting of two species of Fermions. Symmetries in coordinate space of nuclei are rotational and parity symmetries. As well known, spontaneous breaking and restoration of these symmetries provide specific features of energy spectra. The symmetry in isospin-spin space is another important symmetry. For example, an alpha cluster is the lightest scalar particle in the isospin-spin space. The Pauli principle, duality, and saturation property also play important roles in formation and dynamics of clusters. In particular, the Pauli blocking strongly influences on cluster formation and cluster motion. It also gives a major contribution to density dependence of clustering. Moreover, the duality and saturation property are responsible for coexistence of mean-field and cluster states.

2. Algebraic approaches

The symmetry breaking and restoration in the coordinate space are essential to understand low-lying spectra of light $Z = N$ nuclei. An uncorrelated state has the rotational symmetry. However in realistic system, clusters are formed because of many-body correlations and the system has periodic density as a standing wave on the surface. As a result, the rotational symmetry is broken into a discrete point-group symmetry. Once clusters are formed, inter-cluster motion can be activated. With decrease of density, the symmetry is restored and the system goes to a spherical cluster gas in a low density limit.

Respecting symmetries, algebraic approaches are developing to describe energy spectra and transitions in light nuclei (see Refs. [1, 2, 3] and references therein). Iachello and Bijker discussed symmetries in 2α , 3α , and 4α dynamics, and successfully described energy spectra and transitions of the ground and excited bands in ${}^8\text{Be}$, ${}^{12}\text{C}$, and ${}^{16}\text{O}$ with Z_2 , D_{3h} , T_d symmetries and vibration modes built on them. Theoretical spectra correspond well to the recently observed experimental data of ${}^{12}\text{C}$ as presented in many talks in this conference. It means that the realistic ${}^{12}\text{C}$ has such the discrete symmetry as a leading component. Cseh and Lévai have applied algebraic approaches to $Z = N$ sd -shell nuclei such as ${}^{28}\text{Si}$, and neutron-rich nuclei [2, 3].



The algebraic approach has been also adopted in *ab initio* calculation to reveal the hidden symmetry as discussed by Draayer [4]. In the symplectic no-core shell model, the large scale full model space is decomposed into physically relevant subspaces. It has been shown that the leading symplectic symmetry accounts for the dominant components of low-lying states of ${}^6\text{Li}$, ${}^{12}\text{C}$, ${}^{16}\text{O}$, ${}^{20}\text{Ne}$, and so on. This success indicates again that the cluster, namely, the symmetry is not a model assumption but it is a fundamental degree of freedom and actually exists in realistic nuclear systems.

3. Localization and non localization

As mentioned previously, because of many-body correlations, clusters are formed at the nuclear surface. In a compact state with normal density, clusters are localized at the nuclear surface because the Pauli blocking effect is rather strong in the inner region. With decrease of density, clusters become free from the Pauli blocking effect and can move freely like a gas at low density. This is nothing but non-localization of clusters. Thus, the cluster feature changes depending on the density from the localized cluster at normal density and the non-localized cluster at low density. It should be stressed that there exist two kinds of clustering, the localized and non-localized clusters.

Horiuchi and his collaborators proposed a new idea of the container picture which unifies these two kinds of clustering [5]. They pointed out that clusters are localized at the nuclear surface because of the Pauli blocking effect, whereas clusters far from a core are non-localized by a dynamical effect. In other words, the non-localization of clusters is a natural consequence of the symmetry restoration at low density, namely, the quantum fluctuation of the cluster center of mass motion. With the container picture, Horiuchi and Funaki described the localization and the non-localization of clusters in the ground and excited states of ${}^{12}\text{C}$, ${}^{16}\text{O}$, ${}^{20}\text{Ne}$, ${}^{10}\text{Be}$, and so on. Funaki discussed rotation of cluster gas states starting from the Hoyle state in ${}^{12}\text{C}$ using the THSR wave functions. More general discussions of the THSR wave functions and α condensation were given by Schuck. Suhara presented how the α -breaking component affects 3α dynamics in ${}^{12}\text{C}$ [6]. A new description of cluster gas states in medium-mass nuclei was proposed by Imai [7].

What is the answer to ${}^{12}\text{C}$? Our main concern is the cluster structure in the Hoyle band. Is it a localized 3α clusters with a triangle shape or not? My answer is as follows. We have two different answers. If we look at the inner part of the Hoyle state we find the localized 3α as a dominant component because of the strong Pauli blocking effect. However, if we look at the outer tail region of the Hoyle state, we see a non-localized clusters like a gas at a low density. This is nothing but one of the multifacet aspects of ${}^{12}\text{C}$.

4. Cluster formation

In the cluster formation, the Pauli blocking, i.e., Fermi surface plays an important role. As already mentioned, the localized clusters are formed at nuclear surface or inside nuclei at normal density, whereas clusters are non-localized at low density. It should be emphasized that the localized clustering can be described within mean-field approaches because it is caused by many-body correlation at the Fermi surface. However, the non-localized clustering is beyond the mean-field picture because there is no Fermi surface.

Khan discussed the transition between quantum liquid and crystal in nuclear systems [8]. Here, the crystal is a kind of localized clustering. He showed clusterization in $Z=N$ -even nuclei and neutron-rich Be isotopes with a relativistic mean-field calculation. Sambataro and Lasseri investigated the 4-body correlation with a quarteting model, in which correlation between two pairs is taken into account [9]. The isospin symmetry restoration in correlating two pairs plays a significant role to form quarteting α -type correlation. Horiuchi *et al.* achieved full 5-body

calculation of $^{12}\text{C}+4N$ and showed the α cluster formation and spectroscopic amplitudes in excited states of ^{16}O .

In heavy nuclei, the α -cluster formation has been a long-standing problem discussed in relation to α -decay lifetimes. Two kinds of clustering contribute to the α -decay width; the α -cluster formation at the surface (the localized cluster) and the α -decay dynamics (the non-localized cluster). A theoretical problem is how to combine these two kinds of clustering. To this end, hybrid approaches have been adopted to study α decay and α spectroscopic factor in heavy nuclei. In particular, the theoretical description of α decays from ^{212}Po is one of the revival hot topics as discussed by Roepke and Lovas [10, 11]. It was pointed out that the effect of Fermi surface (Pauli blocking), i.e., the antisymmetrization effect should be carefully taken into account to calculate the α -decay life time.

Two-nucleon correlation is another type of many-body correlation in nuclear systems. The proton-neutron pair formation in $Z=N=\text{odd}$ nuclei such as ^{10}B and ^{18}F has been studied by Morita and Masui [12, 13]. These works may give a hint to deuteron-type and quarteting condensations in nuclear systems.

Let me emphasize recent remarkable progress of *ab initio* calculations, which gave a great impact to cluster physics. In these years, many efforts have been made to describe the cluster structure of excited states of ^{12}C with *ab initio* calculations. The recent results of *ab initio* Monte Carlo shell model [14] and those of symplectic no-core shell model [4] were presented by Otsuka and Draayer, respectively. It was proved that clusters are evidently built in many nucleon dynamics from realistic nuclear force in excited states of ^{12}C and also in neutron-rich Be isotopes. Now we are ready to go to further fundamental questions: What is the roles of the tensor and 3-body forces in nuclear clustering? Answers will come soon in *ab initio*-type calculations. Myo proposed a new method of semi *ab initio* approaches called tensor optimized AMD [15].

5. Cluster phenomena in isospin asymmetric systems and heavy systems

Cluster physics is being expanding widely toward the isospin asymmetric and heavy-mass regions in nuclear chart. In such systems, sub systems are not simple rigid clusters but they are themselves complex objects. A key problem is smooth connection between one-center and two-center limits. A one-center system changes gradually into a two-center system through a largely deformed state, a strong-coupling two-center state, and to a weak-coupling two-center state. In the weak-coupling limit, the essential degree of freedom is the relative motion between clusters, which is well decoupled from internal degrees of freedom (DOF) of clusters. As two clusters approach to each other, the internal DOF (excitations) of clusters become more important and two systems merge into a strong-coupling state. A theoretical problem is how to access to the transitional region, in which DOF are not clearly separated.

Let me first mention about cluster features in neutron-rich nuclei. von Oertzen discussed cluster structure of neutron-rich nuclei with the molecular orbital model, which is successful to describe energy spectra of neutron-rich Be and Ne isotopes [16]. When valence neutrons occupy the longitudinal σ -orbital sticking two clusters, the state has a strong-coupling cluster structure with a large deformation and constructs rotational bands. On the other hand, when valence neutrons occupy atomic orbitals instead of molecular orbitals, the state has a weak-coupling cluster structure and corresponds to a cluster resonance, which usually appears in an energy region higher than molecular orbital states. Further rich cluster phenomena are expected in neutron-rich nuclei. For instance, a three-center structure with a linear-chain configuration was predicted in neutron-rich C by von Oertzen, Suhara, and Baba [16, 17, 18]. Valence neutrons play an important role to stabilize the linear chain structure. Experimental measurements of band members of the linear chain structure were reported by Yamaguchi [19].

With increase of the mass number, systems become more complex, and internal excitations

give significant effects to resonances and low-energy reactions via coexistence of different cluster channels, neck formation, multi-center phenomena, breakup process in reactions, and so on. In *sd*-shell and medium-mass nuclei, different cluster channels degenerate in excited states near threshold energies. For instance, α -cluster states and strong-coupling two-center cluster states appear almost in the same energy region near threshold energies. Because of duality, those cluster states correspond to superdeformed states. In highly excited states, molecular resonances have been observed. Usually observed molecular resonances are weak-coupling cluster resonances near or above the Coulomb barrier. A key problem is how to access to the transitional region near the threshold much below the Coulomb barrier. In a two-center cluster picture, key problems are internal excitations, channel coupling, cluster dissociations. Many attempts from the theoretical and experimental sides have been made to reveal cluster states in the transitional region. The coexistence of different cluster channels such as α -cluster states, 2α -cluster states, and ^{12}C -cluster states in ^{24}Mg and ^{28}Si have been studied theoretically and experimentally as discussed by Kimura, Royer, Chiba, and Kravvaris (Refs. [20, 21, 22] and references therein). Rotational bands of the superdeformation and its relation to O+O cluster cores with valence neutrons in ^{34}S were discussed by Taniguchi based on AMD calculation [23] and Afanasjev with Cranking RMF calculation [24].

Uegaki investigated C+C resonances constructed by two oblatelly deformed clusters and compared them with Si+Si cluster resonances [25]. In talks by Spieker, Kimura and Chiba, it was addressed that isovector dipole, isoscalar dipole, and monopole transitions are useful probes for nuclear clustering [21, 26, 27, 28].

Fusion and fission phenomena are contributed by further complex dynamics in the transitional region between one-center and two-center systems. Itkis gave an excellent review on fission of superheavy nuclei induced by light- and medium-mass nuclei and discussed clustering effects [29]. Carjan investigated spontaneous fission of very heavy nuclei by Cassinian oval models, and showed important roles of the compact fission mode, ^{132}Sn -like fragments, and octupole DOF in competition of (super)symmetric and asymmetric fission [30]. Dynamics of three fragments fission such as collinear cluster tri-partition and clustered chain-like pre-scission has been discussed by Pyatkov and Kamanin [31].

6. Advances in nuclear reactions

An extreme case of internal excitation of a cluster is breakup process in nuclear reaction. breakup of a cluster play an important role in low-energy reaction of weakly bound nuclei, which has been often discussed with astrophysical interests. I should remark recent developments of theoretical approaches for 3-body decay such as Hyperspherical approaches, 3-body CSM, and 4-body CDCC, in which 3-body continuum states are properly taken into account. These approaches enable us to directly connect theoretical calculations with experimental data with reliable reaction theories. Using these approaches, dynamics of 2,3-body breakup of weakly-bound systems and its effect to reaction cross sections have been intensively investigated. Descouvemont presented significant effects of breakup in ^6He scattering on heavy targets with the microscopic 4-body CDCC [32]. Watanabe *et al.* investigated breakup effects in ^6Li scattering with 4-body CDCC[33]. 3-body Coulomb breakup of ^{22}C at the neutron-drip line has been studied with 4-body Coulomb corrected eikonal model by Pinilla *et al.* [34]. Two-body breakup and incomplete fusion of weakly bound nuclei and ^{16}O have been attracting a great interest as discussed in Hussein's review talk and Samarin's talk [35, 36].

The low-energy resonances and reactions have been intensively studied theoretically and experimentally in particular with astrophysical interests. Many theoretical and experimental works concerning this subject have been presented in this conference: $2N$ emission [37], $2p$ capture [38], photodisintegration of ^9Be [39], radiative α capture, α -decay, scattering and quasi-molecule [40]. $2N$ decays, in particular, $2p$ decays from proton-rich nuclei beyond the drip line

are attracting a great interest in physics of unstable nuclei. Grigorenko investigated dynamics of $2p$ decays from resonances with 3-body calculations while paying attention to true and sequential decays [37]. Hove calculated reaction rate of $2p$ capture at rapid-proton process waiting points in $Z \sim 70$ region and showed dominant sequential and direct $2p$ capture at high and low temperature, respectively [38].

7. Clustering in heavy ion collisions and nuclear matter

Clustering in finite temperature medium is a key to understand fragmentation in heavy ion collision (HIC). Emitting particles reflect nuclear matter information such as the EOS and symmetry energy through formation and dissolution of clusters in dynamical process of HIC. Recently, isospin asymmetric collision data have been used to obtain asymmetric matter information with molecular dynamics (MD) and statistical approaches [41, 42]. For example, Papa found that dipole degree of freedom relates to the symmetry energy through isospin equilibration process in HIC by investigating interaction dependence of average dipole in the Constrained MD calculation [41]. In discussions of phase diagram of nuclear matter, Moretto showed the liquid-vapor line obtained from HIC fragments assuming saturated vapor in a cluster gas [43]. Clustering in finite temperature medium also gives contribution to stellar matter as discussed by Typel [44]. He demonstrated model dependence on mass fraction in neutron star matter.

8. How to probe clustering

How one can probe clustering from observed data? Ito proposed that sizes of excited states can be determined by inelastic scattering [45]. He obtained enhanced spatial size of $^{12}\text{C}(2_2^+)$ from scattering data. Fukui discussed α -transfer reactions as a probe for α -cluster probability at surface [46].

9. Summary

In summary, I give my perspectives to the following questions: What we learn so far? What are the keys? Where we are going? Key words for theoretical study in cluster physics could be: N -body calculations with continuum. Microscopic treatment of nucleonic degrees of freedom. Calculations based on realistic nuclear force. Challenges to heavier systems and heavier clusters. Density, isospin, temperature dependences of clustering. Experimental probes for clustering. Our research subject is now being expanding widely toward the large mass number, isospin asymmetry, high excitation energy regions. By changing the excitation energy and the isospin asymmetry, we can access to low-density systems. In such new areas, we will see rich physics and encounter exotic phenomena related to cluster physics, because clustering is an essential DOF, in particular, at sub normal density. One might wonder a question “to be clustering or not to be clustering”. However, it may not be the problem in a sense because cluster aspects arise everywhere in nuclear systems because of duality and symmetry. In general, different aspects in two limits can be smoothly connected with each other as seen in the connection between shell-model and cluster structures, the connection between one-center and two-center systems, and that between light-mass and heavy-mass regions, low energy and high energy regions, and so on. In many cases, the most essential and difficult problems exist in the transitional region between two limits. What we should do is to approach to the transitional region from both limits and find analogies and differences between two limits. Then we obtain a unified understanding of nuclear phenomena. Of course, we should not forget about other connections, the one between realistic and effective nuclear forces and that between theoretical and experimental studies. I would like to ask a question to myself. Is “Cluster physics” progressing in nuclear physics. The word “cluster” is useful but sometimes confusing. I would say, in the progress of nuclear physics,

cluster concept is separating from and merging with other concepts again and again following decoupling and coupling of DOF in nuclear many-body systems.

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