A novel visage of molecular resonances in $^{12}\mathrm{C} + ^{12}\mathrm{C}$

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Abstract. Resonances observed in the $^{12}\mathrm{C} + ^{12}\mathrm{C}$ collisions are studied by the molecular model. Firstly, normal-modes analysis made around the equilibrium equator-equator touching configuration gives a basic insight into dynamics of the system. The quantum numbers of the states also indicate rotational motions of the constituent $^{12}\mathrm{C}$ nuclei, which relate the normal modes to the corresponding inelastic $^{12}\mathrm{C} + ^{12}\mathrm{C}$ channels. Next, low-lying 11 states are taken up as bases, and Coriolis coupling has been diagonalized. Many excited states are obtained with intermediate widths, which is just in accord with the experimental observation. For example, the second and third excited states have characteristic feature as low-lying mutual 2^+ excitation with the spin vectors of $^{12}\mathrm{C}$ nuclei along the molecular z'-axis. Those states appear very close to the molecular ground state with the higher spin of $\Delta J = 2$, to form a bunch. Resultant excitation functions are surprisingly in good agreement with the experiments, which lights up a new physical picture of molecular structures of the high-spin $^{12}\mathrm{C} + ^{12}\mathrm{C}$ resonances.

1. Introduction

Well above the Coulomb barrier of the $^{12}\mathrm{C} + ^{12}\mathrm{C}$ system, series of resonances have been found with high spins over $10\hbar$, which exhibit prominent peaks in the elastic and inelastic 2^+ channels [1]. Band Crossing Model (BCM), based on the double resonance mechanism, has successfully explained resonance mechanism with the aligned configurations of the orbital angular momentum and the spins of the excited states of $^{12}\mathrm{C}$ nuclei [2].

We have revisited the resonances with the new molecular model [3, 4], which has been developed for the analyses of the high-spin resonances observed in $^{24}\mathrm{Mg} + ^{24}\mathrm{Mg}$ and $^{28}\mathrm{Si} + ^{28}\mathrm{Si}$. The model has an advantage to describe various geometrical configurations of two constituent deformed nuclei with the method of normal modes around the stable energy minimum under strong nucleus-nucleus interaction, which gives intuitive understanding on nuclear structure of the resonances. For the oblate-oblate systems such as $^{28}\mathrm{Si} - ^{28}\mathrm{Si}$ and $^{12}\mathrm{C} - ^{12}\mathrm{C}$, the stable configurations have a small axial asymmetry as a whole and thus exhibit a series of low-lying excited states with $K = 2$ or 4, in which spins of the constituent nuclei incline to the molecular z'-axis. Those states naturally have an interesting property of spin disalignments with the orbital angular momentum, which is contrary to the alignments in BCM. How to resolve an apparent contradiction?

In rotating systems, Coriolis coupling plays an important role in optimization of the centrifugal energy. Actually, a diagonalization of the coupling has resulted in an admixture of the $K = 1$ configuration in the molecular ground state and thus the state has aligned components similar to those of BCM [5]. Furthermore, interesting is that the $K = 2$ and $K = 4$ states are expected to be also observed through a weak coupling to the elastic channel caused by chains

of Coriolis coupling. Those nearby or overlapping states appear as a bunch of resonances or a single gross peak, which is apparently different from the interpretation of BCM. We will discuss below how the present study has revealed revolutionarily refined features of the resonances as well as consistent aspects with BCM.

2. Brief reminder of the model and the method of normal modes

Assuming a constant deformation and axial symmetry of the constituent nuclei, we have seven degrees of freedom $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2)$ to describe dynamics of the dinuclear system, as illustrated in fig. 1. The relative vector (R, θ_2, θ_1) of two ^{12}C nuclei defines the molecular z'-axis. Orientations of the symmetry axes of ^{12}C are described with Euler angles (α_i, β_i) referring to the molecular axes. α_1 and α_2 of the rotations around z'-axis are combined into normal modes; the total K-rotation defined with $\theta_3 = (\alpha_1 + \alpha_2)/2$ and the twisting degree of freedom with $\alpha = (\alpha_1 - \alpha_2)/2$.
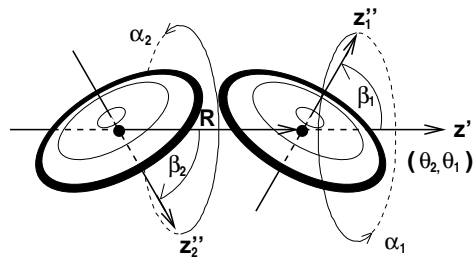
At first, the kinetic energy is given classically in terms of angular velocities, and then the quantization is done to describe the whole rotating system quantum-mechanically. The resultant kinetic energy is classified into three parts, as $\hat{T} = \hat{T}_{\text{rot}} + \hat{T}_{\text{int}} + \hat{T}_{\text{C}}$, where $\hat{T}_{\text{rot}}$ denotes the rotational energy of the whole system described with the total angular momentum operator $\hat{\boldsymbol{J}'}$, $\hat{T}_{\text{int}}$ the energy of internal motions, and finally $\hat{T}_{\text{C}}$ the Coriolis coupling.

The nucleus-nucleus potential is calculated by a folding model with nucleon-nucleon interaction DDM3Y. The potential is a function of the internal degrees of freedom $(R, \alpha, \beta_1, \beta_2)$, which specify geometrical configurations. Consistently with the coordinate system, we introduce a rotation-vibration type wave function specified with K-quantum number as basis one, $\Psi_\lambda \sim D_{MK}^J(\theta_i)\chi_K(R, \alpha, \beta_1, \beta_2)$, where χ_K describes the internal motions.

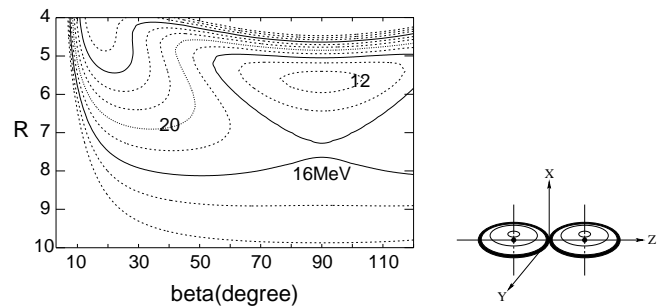In order to analyze dynamics of multidimensional internal motions, we calculate the effective potential for given J and K,

$$V_{JK}(R, \alpha, \beta_1, \beta_2) = V_{\text{interaction}}(R, \alpha, \beta_1, \beta_2) + T_{\text{rot}}'(J, K; R, \alpha, \beta_1, \beta_2), \tag{1}$$

where T_{rot}' denotes a multidimensional centrifugal potential with specified J and K, reduced from the rotational kinetic energy operator $\hat{T}_{\text{rot}}$. In fig. 2, a multidimensional energy surface is displayed for $J = 14$. A local minimum is obtained around $\beta's = 90°$, $\alpha = 0°$ and $R = 5.7$ fm. The equilibrium configuration is displayed in r.h.s. of fig. 2, which is called as *equator-equator* (E-E) configuration. The barrier height from the bottom of the local minimum is 4.8 MeV located at $R = 7.5$ fm. Strong centrifugal potential makes the configuration to be stable for $J = 10$–18, while for the smaller J, the stable configuration is found in belly-equator or belly-belly one, and for the larger J the potential pocket disappears.

Figure 1. Dinuclear molecular coordinates; seven degrees of freedom of ^{12}C + ^{12}C (oblate-oblate system) are described with $(q_i) = (\theta_1, \theta_2, \theta_3, R, \alpha, \beta_1, \beta_2)$.

Figure 2. Contour map of the effective potential energy V_{JK} with $J = 14$ and $K = 0$ for the $R - \beta(\beta_1 = \beta_2)$ degrees at $\alpha = \pi/2$. Contours are in 2 MeV.

Couplings among various molecular configurations are taken into account by the method of normal modes around the equilibrium at $\beta_i = 90°$, which gives rise to the molecular modes of excitation. We expand the effective potential $V_{JK}(R, \alpha, \beta_1, \beta_2)$ into a quadratic form to describe normal modes. Combining the kinetic energy operator $\hat{T}_{\text{int}}$, we obtain hamiltonian H_0, which is almost separated into each internal degree of freedom. This means the internal degrees of freedom $(R, \alpha, \Delta\beta_1, \Delta\beta_2)$, where $\Delta\beta_i$ denote $\beta_i - \pi/2$, are the normal modes themselves. Thus we use basis wave functions, $\Psi_{JMK}(\theta_i; R, \alpha, \beta_1, \beta_2) \sim D^J_{MK}(\theta_i) f_n(R) \phi_\nu(\alpha) \varphi_{n_1}(\Delta\beta_1) \varphi_{n_2}(\Delta\beta_2)$, where f_n, φ_{n_1} and φ_{n_2} are H.O. functions, while ϕ_ν is a rotational function $e^{i\nu\alpha}$ as the confinement is very weak in α-degree due to the shallow potential pocket. The energy of the basis state is given as $E_{JK}(n, \nu, n_1, n_2) = E_0(R_e) + \{J(J+1) - K^2 - 1\}\hbar^2/2\mu R_e^2 + \{K^2 - 2\}\hbar^2/4I + \nu^2\hbar^2/4I + (n + 1/2)\hbar\omega_R + (n_1 + n_2 + 1)\hbar\omega_\beta$, where $\hbar\omega_\beta \sim 5$ MeV. In fig. 3, the spectrum of the normal modes with $J = 12$ is displayed, classified with quantum numbers $K = 0$–4. The axially asymmetric shape of the whole composite system displayed in r.h.s. of fig. 2 allows states with $K = 2m$, m being integer, in low excitation. The system has states with $K = 2m+1$ for the $1\hbar\omega_\beta$ excitation.
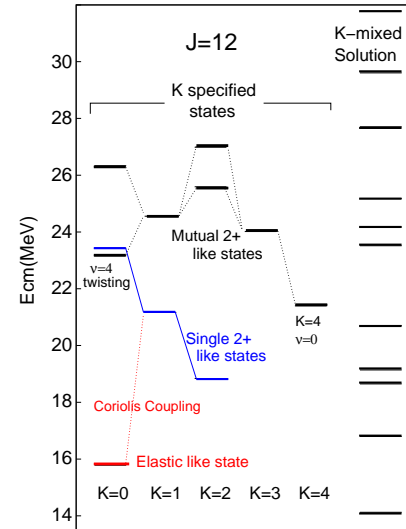
3. Coriolis coupling and characteristic features of solutions with K-mixing

With the normal-mode hamiltonian, the total hamiltonian is written as

$$H = H_0 + \hat{T}_C + (higher\ order). \qquad (2)$$

Generally in the rotating system, we have Coriolis coupling $\hat{T}_C$, which can be described with a simple product $-\boldsymbol{J}' \cdot (\boldsymbol{S}'_1 + \boldsymbol{S}'_2)/\mu R^2$ of the total angular momentum operator $\boldsymbol{J}'$ and the spins of ^{12}C $\boldsymbol{S}'_i$. Note that they refer to the molecular axes as designated with prime. In the model, it is given with many terms; $\boldsymbol{J}'$ consists of differential operators of (θ_i), which are combined into lowering and raising operators of K, $J'_\pm = J'_x \pm iJ'_y$, respectively. As for the spins $\boldsymbol{S}'_i$, they are described usually with differential operators of Euler angles β_i, i.e., given with those of the internal variables $(\alpha, \beta_1, \beta_2)$. We consider the Coriolis terms as residual coupling, which causes K-mixing with $\Delta K = 1$. The mixing induces aligned components. Actually, in the molecular ground state, a $(K = 1, \nu = 1)$ mixed component appears to bring strong alignments in the single 2^+ channel [5].

Figure 3. Energy spectra of molecular normal modes of ^{12}C + ^{12}C for $J = 12$, specified with $K = 0$ to 4 from the l.h.s. Diagonalization of the Coriolis coupling gives a new spectrum shown in the rightmost column.

The molecular quantum numbers $(J, K; n, \nu, n_1, n_2)$ indicate rotational motions of two constituent ^{12}C nuclei by $\{n_1, \mu_1 = (K + \nu)/2\}$ and $\{n_2, \mu_2 = (K - \nu)/2\}$, respectively. μ_i indicates the quantum of the rotation along the molecular z'-axis, while n_i corresponds to the rotations perpendicular to the z'-axis. Thus a set of numbers $I_i = n_i + |\mu_i|$ is related to the spins of ^{12}C nuclei, like a channel assignment. This gives us tokens of the basis wave functions corresponding to inelastic channels. We take 11 bases for the single 2^+ and mutual 2^+ channels; states with $n_1 + n_2 = 3$ and 4 are omitted as the energies are high. In fig. 3, single 2^+ and mutual 2^+ assignments are depicted with different colors and connected lines. In the rightmost column of fig. 3, levels of the K-mixed solutions caused by the Coriolis coupling are displayed for $J = 12$. In table 1, for those resultant states, probabilities in each channel are analyzed. Surprisingly, characteristic feature of dominant mutual 2^+ components with $L = J - 2$ is found

Table 1. Properties of resonance states with $J = 12$ indicated by probability distributions.

Resonance states	elastic	single 2^+	mutual 2^+
ground state[1]	36%	52% (52% $L = J - 2$)	10%
1st exc. state	31%	33% (25% $L = J$)	34% (18% $L = J - 4$, 16% $L = J - 2$)
2nd exc. state[2]	7%	15%	74% (62% $I = 4, L = J - 2$)
3rd exc. state[3]	4%	9%	82% (69% $I = 2, L = J - 2$)

[1] $K = \nu = 0$ lowest energy configuration (55%) and $K = \nu = 1$ (26%) mixed state.
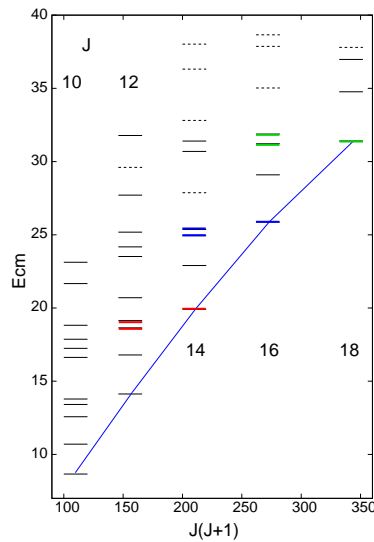
[2] $K = 4$ (43%) and $K = 3$ (28%) mixed state.

[3] $K = 0$ twisting state (30%), butterfly state (14%) and ($K = 1, \nu = 3$) state (40%).

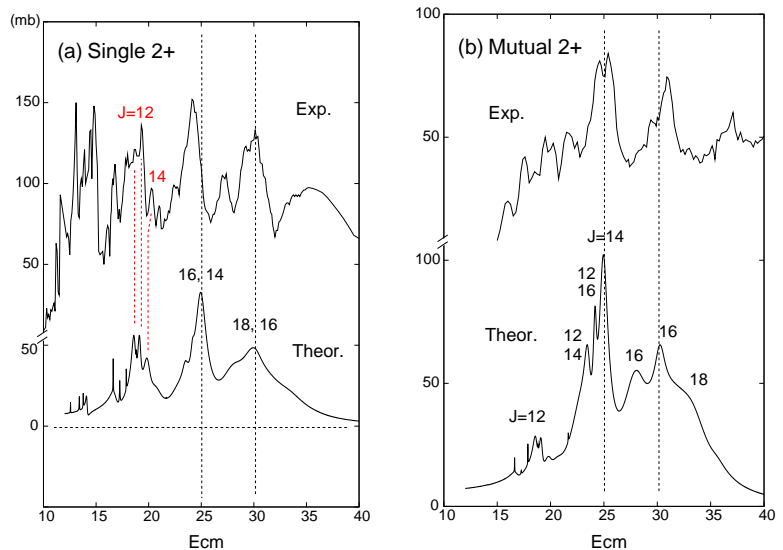in the second and third excited states, which appear just below $E_{cm} = 20$ MeV. Those states are inferred to be originally $K = 4$ state and $K = 0$ twisting state with relatively low excitation energies due to zero-point oscillation of the β-vibration at the E-E configuration, which have the property of spin disalignments $L = J$, because the ^{12}C spins are along the molecular z'-axis. However they both affected with $K = 3$ or $K = 1$ mixing, which brings spin alignments. Such a mechanism is similar to that of BCM for the molecular ground state [5].

4. Molecular spectrum and excitation functions

We have found the stable E-E configuration for high spins from $J = 10$ to $J = 18$, and the corresponding molecular bands are shown in fig. 4. Of course, there are many possible molecular bands for lower spins, to be observed around the Coulomb barrier region. We have obtained many molecular excited states for the same spin, the number of which are comparable with the

Figure 4. Molecular band structure for $J = 10$–18. The ground band is connected for eye-guide. The levels with widths larger than 4 MeV are displayed by dashed lines.

Figure 5. Theoretical excitation functions: (a) for the single 2^+ channel, and (b) for the mutual 2^+ channel. The levels of $J = 10$–18 ($\Gamma \leq 4$MeV) are included in. Spins J are indicated for some resonance peaks. At many peaks, different J contributions are overlapping.

experimental observations. Surprisingly, widths of those states are not large due to very small amounts of the elastic components. That is consistent with that the elastic components are almost exhausted in the molecular ground state and the first excited state. In fig. 5, theoretical excitation functions for the single 2^+ and mutual 2^+ channels are compared with the experiments [1]. Note that the calculated resonance energies are not precisely correct for higher spins such as $J = 16$ and 18, as harmonic approximation is adopted for the radial motions in shallow potential pockets. So the level energies of those spins are shifted by -1 MeV for $J = 16$, and -1.5 MeV for $J = 18$, respectively.

Ledoux et al. assigned spins of three resonance states around $E_{cm} = 20$ MeV as $J = 12$ for lower two states $(18.4, 19.3$ MeV) and $J = 14$ for the 20.3 MeV state [6]. This spin assignment is very important, because spins of all three resonances are suggested to be $J = 12$ by Cormier [1]. In the present results, the molecular ground state with $J = 14$ appears at 19.9 MeV in good agreement with the Ledoux's data. With $J = 12$, the ground state is obtained at 14 MeV, while the second and third excited states appear around 19 MeV (see fig. 4). Thus three close-by resonances obtained around $E_{cm} = 19$ MeV are in good correspondence with three intermediate peaks on the gross peak observed in the single 2^+ channel. Note that a few highly excited states with $J = 10$ also overlap in the region $E_{cm} = 17$–20 MeV of the gross peak, which is expected to correspond with the observation of $J = 10$ resonances [7]. As for another gross peak just below 15 MeV, three $J = 10$ states and the $J = 12$ molecular ground state appear to overlap, which is similar situation around $E_{cm} = 19$ MeV. However shape of the gross peak is not seen in fig. 5(a), as the resonance peaks are too narrow due to lack of $^{20}\text{Ne} + \alpha$ and $^{16}\text{O} + {}^8\text{Be}$ decay contributions.

In the region $E_{cm} \geq 25$ MeV displayed in Fig. 5(b), peaks of the mutual 2^+ excitation are prominent as well as those of the single 2^+. Around $E_{cm} = 25$ and 30 MeV, bunches of peaks are well reproduced. There are contributions from different J in the bunches again. Such a structure seems to be due to the excitation energy of 2_1^+ state of ^{12}C very close to the centrifugal energy difference for $\Delta L = 2$.

5. Conclusions

High-spin resonances of $^{12}\text{C} + {}^{12}\text{C}$ has been studied by the dinuclear molecular model. Normal-modes analysis made around the E-E configuration provides good perspectives both for the dynamics of the system and for the rotational motions of ^{12}C nuclei corresponding to the channels. Intrinsic states favored by the $^{12}\text{C} + {}^{12}\text{C}$ interaction, especially, excitations with non-aligned configurations due to rotations around the molecular z'-axis play an important role. K-mixing by the role of the Coriolis coupling changes those states into spin-aligned characteristics, as well as the molecular ground state, consistent with BCM.

Those *multi-excited molecular states* weakly couple to the elastic channel and thus are to be observed as resonances with intermediate widths. As a result, characteristic features of excitation functions in the single 2^+ and mutual 2^+ channels, i.e., the intermediate structures on the top of the prominent gross peaks have been very well reproduced over a wide energy range. This gives rise to a novel refined view to the known molecular resonances. Experiments to explore nature of those resonance structures are strongly called for.

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