

α -cluster breaking effects on cluster structures in ^{12}C

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Abstract. To clarify the effects of α -cluster breaking on 3α cluster structures in ^{12}C , we investigate ^{12}C using a hybrid model that combines the Brink-Bloch cluster model with the $p_{3/2}$ subshell closure wave function. We have found that α -cluster breaking significantly changes the cluster structures of 0^+ states through orthogonality to lower states. As a result of the structure changes of 0^+ states, the band assignment for the 2_2^+ state is changed. The 0_3^+ state is likely to be the band-head of the 2_2^+ state instead of the 0_2^+ state because the present calculation reproduces experimental data of the low-lying states well with α -cluster breaking. We have found that this band can be considered a linear chain-like band from the analysis of the intrinsic structures.

1. Introduction

In light nuclei, it is known that cluster formation occurs in low-lying states and competes with the shell-model structure. ^{12}C is a typical example of this competition [1, 2, 3, 4, 5, 6]. Because the spin-orbit force usually tends to hinder the cluster formation, the ground state of ^{12}C is not the pure 3α cluster state but is a mixture of 3α cluster structure and jj -coupling shell model structure of the $p_{3/2}$ subshell closure. On the other hand, in excited states of ^{12}C near the threshold energy, developed 3α cluster structures have been discovered. It is important to understand the magnitude of this competition and how much it affects the structures of the ground and excited states of ^{12}C .

In 3α cluster models, the ground and low-lying states have a compact triangle structure, whereas excited states near and above the 3α cluster threshold energy have well-developed cluster structures [7, 8, 9, 10, 11]. For example, the 0_2^+ state, which is well-known as the Hoyle state, is considered to be an α condensate state in which weakly interacting three α clusters occupy an identical lowest orbit of a mean-field potential [9]. Another interesting cluster structure was predicted in the 0_3^+ state, which is considered to be a vibration mode of acute and obtuse triangle configurations [7].

The antisymmetrized molecular dynamics (AMD) and the fermionic molecular dynamics (FMD) models have been successful in describing the nature of ^{12}C from the ground state to the higher excited states [1, 2, 3, 4, 5, 6]. In these models, the ground and low-lying states are a mixture of cluster and jj -coupling shell model configurations. For states near the threshold, these models reproduce the cluster nature, but the detailed cluster structures obtained with these models are somewhat different from those of cluster model results. For example, the 0_3^+ state is considered to be a chain-like structure having an obtuse triangle configuration of three α clusters instead of the vibrational state predicted by the cluster model.



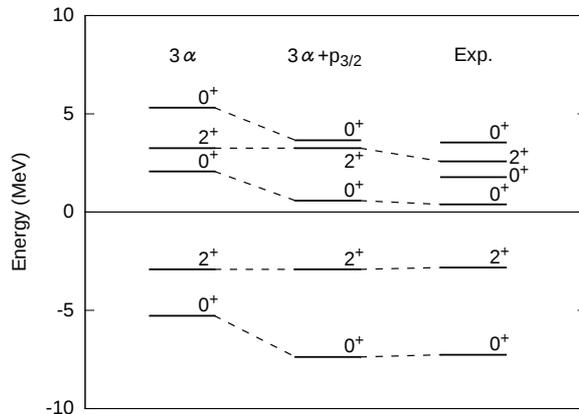


Figure 1. Comparison of calculated energy levels of 0^+ and 2^+ states with experimental ones.

Recently, several interesting experimental data on ^{12}C for excited states near the threshold energy have been reported. A new state, the 2_2^+ state at 9.84 MeV, was discovered [12, 13, 14]. Itoh *et al.* found that the broad 0^+ state at 10 MeV consists of two 0^+ states [15]. A band structure including these new states has been an open problem. In particular, the assignment of the band-head state of the newly measured 2_2^+ state is now controversial.

2. Formalism

In this study, we investigate ^{12}C with a simple model to clarify α -cluster breaking effects on 3α cluster structures. In our model, we superpose the Brink-Bloch (BB) cluster model wave functions and the $p_{3/2}$ subshell closure wave function to incorporate the mixing of cluster breaking components in cluster structures [16]. This model is regarded as an extension of the generator coordinate method of a cluster model. The difference from traditional cluster models is the mixing of the $p_{3/2}$ subshell closure wave function, which is the lowest configuration of the jj -coupling shell model. The results obtained with this model are consistent with the AMD and FMD results even though it is much simpler than those models. We used the same interaction as that used in a previous study of ^{12}C using AMD [17].

3. Results

In Fig. 1, we show the energy levels of 0^+ and 2^+ states calculated with the present model and those obtained with the 3α -cluster model calculation. The former and the latter correspond to calculations with and without α -cluster breaking, i.e., the mixing of the $p_{3/2}$ subshell closure wave function caused by the spin-orbit force, which we call “ $3\alpha+p_{3/2}$ ” and “ 3α ”, respectively. Energies are measured from the 3α threshold energy. The theoretical and experimental threshold energies are -82.9 MeV and -84.9 MeV, respectively.

The results for “ $3\alpha+p_{3/2}$ ” agree with experimental results very well except for the absence of a 0^+ state observed around 10 MeV. In particular, the level spacing between 0_1^+ and 2_1^+ states of “ $3\alpha+p_{3/2}$ ” agrees well with the experimental one, which is largely underestimated in “ 3α ”. This large spacing comes from the energy gain of the spin-orbit force in the 0_1^+ state with the mixing of the $p_{3/2}$ subshell closure wave function. Any microscopic α cluster model fails to reproduce this large level spacing, and therefore, this is an evidence for α -cluster breaking. The “ $3\alpha+p_{3/2}$ ” result is qualitatively consistent with the AMD and FMD results for ^{12}C [18]. This implies that the major difference between the AMD and FMD results and 3α cluster model calculations is the α -cluster breaking effect, which can be qualitatively simulated by the mixing of only one

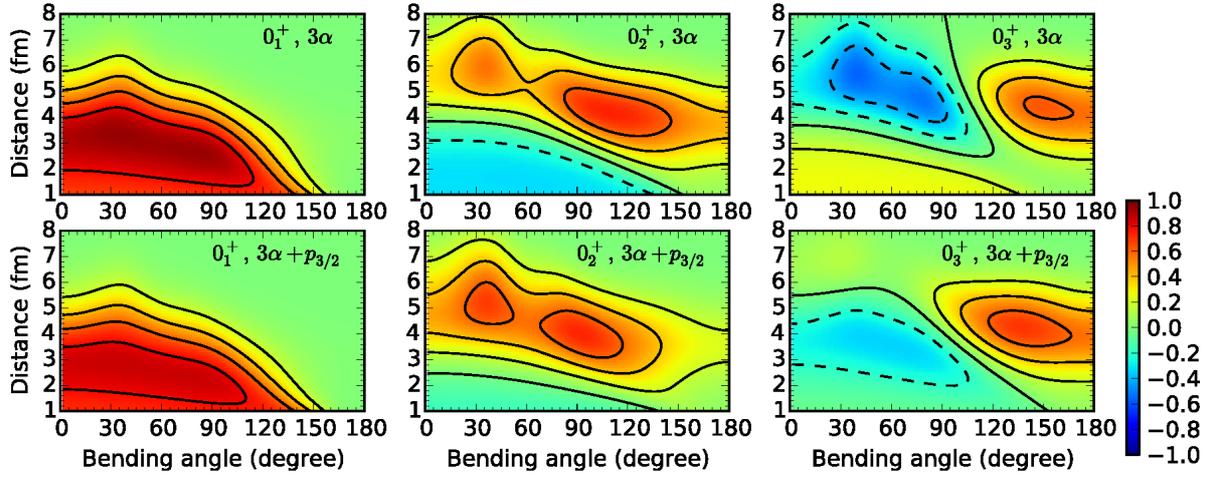


Figure 2. Overlap surfaces between 0^+ states and single BB wave functions.

configuration of the $p_{3/2}$ subshell closure to the 3α cluster model space in the present model.

In Fig. 2, we show overlap surfaces between the 0^+ states and the single BB wave functions. For the BB wave function, the isosceles triangle structure is assumed. These overlap surfaces show the cluster motion projected onto the 3α cluster configuration space for 0^+ states.

First we compare the structure of the 0_1^+ state. For “ 3α ”, the wave function has a large overlap around the compact 3α cluster configuration and also contains components of developed cluster configurations with larger inter-cluster distances. For “ $3\alpha + p_{3/2}$ ”, the behavior of the overlap surface is similar to that for “ 3α ”. However, the absolute amplitude decreases and 3α cluster components become small because of the significant mixing of the $p_{3/2}$ subshell closure component in the 0_1^+ state.

Next we discuss α -cluster breaking effects on the structure of the 0_2^+ state, which reflects the structure change of the 0_1^+ state because of the orthogonal condition to lower states. For “ 3α ”, the overlap is distributed over a wide area of well-developed cluster structures, indicating that the 0_2^+ state is described by the superposition of various triangle configurations of three α clusters. For “ $3\alpha + p_{3/2}$ ”, the overlap is also distributed over a wide area of well-developed cluster structures and is qualitatively similar to the distribution for “ 3α ”. However, quantitatively, the overlap shifts to smaller bending angles and the components for acute triangles are slightly enhanced. Moreover, the maximum peaks also shift toward shorter distances region compared to those for “ 3α ”. These results imply that the spatial development of the cluster structure is reduced in the 0_2^+ state with α -cluster breaking.

In the structure of the 0_3^+ state, a qualitative difference is observed between the results with and without α -cluster breaking. For “ 3α ”, the phase of the overlap changes as θ increases along the line for $d \sim 5$ fm. This indicates that the 0_3^+ state is the vibration mode of acute and obtuse triangle configurations. For “ $3\alpha + p_{3/2}$ ”, the behavior differs from the vibration mode. The amplitude of the negative minimum for the acute triangle configuration is considerably hindered, and as a result, the 0_3^+ state is dominated by the obtuse triangle configuration. Hence, α -cluster breaking produces significant effects not only in the ground state but also in the cluster configurations of excited states. This is surprising because the naive expectation is that α -cluster breaking can affect only ground state properties.

The mixing of the α -cluster breaking in 0^+ states affects the $E2$ transition strengths between 0^+ and 2^+ states. The calculated $E2$ transition strengths from 2^+ states to 0^+ states are shown in Table 1. The $E2$ transition strengths from the 2_2^+ state are important for determining the

Table 1. $E2$ transition strengths from 2^+ states to 0^+ states. The unit is $e^2\text{fm}^4$.

Transition	3α	$3\alpha+p_{3/2}$	Exp.
$2_1^+ \rightarrow 0_1^+$	10.8	7.4	7.6 ± 0.4
$2_1^+ \rightarrow 0_2^+$	1.4	5.1	2.6 ± 0.4
$2_1^+ \rightarrow 0_3^+$	0.4	0.2	
$2_2^+ \rightarrow 0_1^+$	4.0	1.1	$1.57^{+0.14}_{-0.11}$
$2_2^+ \rightarrow 0_2^+$	183	76.5	
$2_2^+ \rightarrow 0_3^+$	64.4	166	

band assignment for this state, which has been attracting significant interest. Its $E2$ transition strengths to the $0_{2,3}^+$ states are remarkably large. Interestingly, the behavior of the magnitude is reversed by α -cluster breaking. Without α -cluster breaking, the $2_2^+ \rightarrow 0_2^+$ transition strength is almost three times larger than the $2_2^+ \rightarrow 0_3^+$ one. However, with α -cluster breaking, the $2_2^+ \rightarrow 0_3^+$ transition strength is larger by a factor of two than the $2_2^+ \rightarrow 0_2^+$ one. The strongest $E2$ transition to the 0_3^+ state with α -cluster breaking, in which the reproduction of experimental results is good [18], indicates that the 0_3^+ state is likely to be the band-head of the 2_2^+ instead of the 0_2^+ state. The present results indicate that the band structure of higher excited states is affected by α -cluster breaking caused by the spin-orbit force.

4. Summary

To clarify α -cluster breaking effects on 3α cluster structures in ^{12}C , we investigated ^{12}C with a hybrid model composed of the BB cluster model and the $p_{3/2}$ subshell closure wave function. We have found that α -cluster breaking significantly changes the cluster structures of 0^+ states through orthogonality to lower states. As a result, the band assignment for the 2_2^+ state is changed. As a future work, we will investigate the effects of tensor correlation on cluster structures with a new method to include the tensor correlation in AMD [19].

Acknowledgments

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