IOP Conf. Series: Journal of Physics: Conf. Series **863** (2017) 012017 doi:10.1088/1742-6596/863/1/012017

Semimicroscopic algebraic description of α clustering in Ne isotopes

Géza Lévai

Institute for Nuclear Research, Hungarian Academy of Sciences (Atomki), Debrecen, Pf. 51, Hungary 4001

E-mail: levai@atomki.mta.hu

Abstract. The α-cluster states of ^{20}Ne, ^{19}Ne and ^{18}Ne have been analyzed as members of a supermultiplet predicted by the $U(4|12)$ cluster supersymmetry scheme. The analysis of $B(E2)$ data and one-nucleon transfer reactions seem to support the validity of this scheme. The α-cluster states of the ^{22}Ne nucleus have been analyzed independently, and were compared with DAMD and GCM calculations. Based on these results, a similar study of ^{21}Ne seems worthwhile.

1. Introduction

Nuclei in the neighborhood of ^{20}Ne are known to possess marked α cluster bands. Among them, some Ne isotopes are in the forefront of investigations: α-clustering in ^{19}Ne is known to play an important role nuclear astrophysical processes in stars [1], while Ne isotopes with $A > 20$ serve as examples for systems with molecular orbitals [2].

The semimicroscopic algebraic cluster model (SACM) [3] has been applied to many p- and sd-shell nuclei assuming core+α or other types of clustering (see e.g. the references of [4]). This model makes heavy use of the SU(3) algebra, which appears in the desciption of both the relative motion and that of the internal cluster structure, where it accounts for the orbital structure in terms of Elliott's SU(3) model. The semimicroscopic nature of the SACM is manifested by the fact that it combines a microscopic model space with effective interactions. Due to the former one the model gives rigorous account of the Pauli exclusion principle between the nucleons, while the latter one allows a relatively simple parametrization of the physical operators.

The systematic application of the SACM to neighbouring core+α-type cluster systems revealed correlations in the model parameters with the mass number [5]. This finding later inspired the extension of the formalism of the SACM to a supersymmetry scheme that allows the unified description of the α-cluster states of neighbouring nuclei [6]. Supersymmetry links bosonic and fermionic degrees of freedom, which appear naturally in the SACM as the excitation of the relative motion, and that of the internal structure of non-closed-shell clusters, respectively. In this scheme the α-cluster states of ^{20}Ne (0 fermion), ^{19}F, ^{19}Ne (1 fermion) and ^{18}O, ^{18}F, ^{18}Ne (2 fermions) belong to the same U(4|12) supermultiplet.

More recently the SACM was appied to describe α-clustering in the ^{22}Ne nucleus [4]. Besides the interpretation of the experimental energy levels and E2 transitions, the results were also compared with the predictions of two other models, the generator-coordinate method (GCM) and the deformed-basis antisymmetrized molecular dynamics (DAMD) approach.

IOP Conf. Series: Journal of Physics: Conf. Series **863** (2017) 012017 doi:10.1088/1742-6596/863/1/012017

Table 1. α-cluster bands in A=20, 19, and 18 nuclei up to two major shell excitations.

^{16}O+α $T=0$	(^{15}N, ^{15}O)+α $T=\frac{1}{2}$	(^{14}C, ^{14}N, ^{14}O)+α $T=1$ and $T=0$ (^{14}N+α)
8(8,0)	7(6,0)	6(4,0)
9(9,0)	8(8,1), 8(7,0)	7(6,1), 7(5,0)
10(10,0)	9(9,1), 9(8,0)	8(8,2), 8(7,1), 8(6,0)

2. ^{20}Ne, ^{19}Ne and ^{18}Ne: members of a cluster supermultiplet

The algebra chain characterizing the U(4|12) supersymmetry scheme is

$$\begin{aligned} &\mathrm{U}(4|12) \supset \mathrm{U_B}(4) \otimes \mathrm{U_F}(12) \supset \mathrm{SU_B}(3) \otimes \mathrm{SU_F}(3) \otimes \mathrm{U_F^{ST}}(4), \\ &\supset \mathrm{SU}(3) \otimes \mathrm{SU_F^S}(2) \otimes \mathrm{SU_F^T}(2) \supset \mathrm{SO}(3) \otimes \mathrm{SU_F^S}(2) \otimes \mathrm{SU_F^T}(2) \supset \mathrm{SU}(2) \otimes \mathrm{U_F^T}(1) \end{aligned} \quad (1)$$

where B and F refer to *bosonic* and *fermionic* structures, while S and T indicate spin and isospin characterizing the core cluster. The algebra chain (1) contains that of the SACM [3], which starts with $\mathrm{U_B}(4) \otimes \mathrm{SU_F}(3) \otimes \mathrm{U_F^{ST}}(4)$ and breaks down to the same subalgebras.

The $\mathrm{U_B}(4)$ bosonic and $\mathrm{U_F}(12)$ fermionic algebra are generated by the SU(3) coupled, particle number conserving operators $B^{(\lambda,\mu)L}_{M_L}(l,l') = [b^{\dagger(l,0)} \times \tilde{b}^{(0,l')}]^{(\lambda,\mu)L}_{M_L}$, and $A^{(\lambda,\mu)LST}_{M_L M_S M_T} = [a^{\dagger(0,1)\frac{1}{2}\frac{1}{2}} \times \tilde{a}^{(1,0)\frac{1}{2}\frac{1}{2}}]^{(\lambda,\mu)LST}_{M_L M_S M_T}$, and can be embedded into the U(4|12) superalgebra by adding the operators

$$D^{(\lambda,\mu)Lst}_{M_L m_s m_t}(l') = [a^{\dagger(0,1)st} \times \tilde{b}^{(0,l')}]^{(\lambda,\mu)Lst}_{M_L m_s m_t} \quad (2)$$

and their inverse, which conserve only the *total* particle number. Here $l, l' = 0$ and 1 refer to σ (monopole) and π (dipole) bosons, while the labels in the $A^{(\lambda,\mu)LST}_{M_L M_S M_T}$ operator characterize the 12 single-particle levels on the p shell in LS coupling [6].

The basis states associated with the chain (1) are labelled by the same quantum numbers as in the SACM [3], plus the fermion number N_F and the total particle number $\mathcal{N} = N_\mathrm{B} + N_\mathrm{F}$ [6]:

$$|\mathcal{N} N_\mathrm{B} n_\pi, (\lambda_\mathrm{F}, \mu_\mathrm{F}); (\lambda,\mu) K_L L S J M_J T M_T\rangle. \quad (3)$$

One important implication of the Pauli blocking between the nucleons of the clusters is that n_π, and thus N_B and $\mathcal{N}$ are restricted from below.

The predictions of the cluster SUSY scheme involving the six nuclei displayed in Table 1 were tested in two ways: comparing the calculated and experimental values for electric quadrupole transition intensities for the six nuclei, and checking the one-nucleon transfer intensities in six known reactions between five of them.

The comparison of the experimental and theoretical B(E2) values was done by adjusting the three model parameters to the most precisely measured transitions in two different ways. First the fitting procedure was carried out for each nucleus separately, then a global fit was done, involving the available electric quadrupole transition data for *all* six nuclei.

There were altogether 52 experimental B(E2) values with well defined mean values and errors, and a few more with lower or upper limits. There were 5, 5 an 2 experimental $B(E2)$ data for ^{20}Ne (in two bands assigned to $(\lambda,\mu) = (8,0)$ and $(9,0)$), ^{19}Ne (with $(6,0)$ and $(8,1)$) and ^{18}Ne (with $(4,0)$). It was found that the parameters determined for the six nuclei individually did not differ much from the parameters extracted from the fit to the experimental data of *all six* nuclei, indicating the approximate validity of the SUSY scheme.

Table 2. α-cluster bands in ^{22}Ne and ^{21}Ne up to two major shell excitations.

n_π	(λ,μ) for ^{22}Ne					(λ,μ) for ^{21}Ne		
8			(8,2)	(6,3)	(4,4)		(8,1)	(6,2)
9		(11,1)	(9,2)	(7,3)	(5,4)	(11,0)	(9,1)	(7,2)
10	(14,0)	(12,1)	(10,2)	(8,3)	(6,4)	(12,0)	(10,1)	(8,2)

In order to calculate the C^2S spectroscopic factors of the one-nucleon transfer reactions, the matrix elements of the SUSY generators (2) were evaluated. Since these operators are the generators of the SUSY algebra, they predict strict selection rules between the states of neighbouring nuclei. Analyzing the six reactions it was found that the average of the experimental C^2S spectroscopic factors for the allowed 28 transitions was typically about an order of magnitude larger than for the 20 forbidden transitions. This finding also seems to support the validity of the U(4|12) supersymmetry scheme [6].

3. ^{22}Ne and ^{21}Ne: candidates for molecular orbital structures

In some $N > Z$ systems the neutrons can be pictured as moving on molecular orbitals around the core+α configuration [2], which, for Neon isotopes is $^{16}\mathrm{O} + \alpha$. This development inspired us to apply the SACM to the ^{22}Ne nucleus [4]. The SACM α-cluster model space of ^{22}Ne and ^{21}Ne is displayed in Table 2.

The low-lying positive-parity part of the experimental ^{22}Ne spectrum is dominated by the 0^+_1 and 2^+ bands, while the 0^+_2 band with large prolate deformation starts at 6.23 MeV. In the SACM these bands correspond to $n_\pi(\lambda,\mu)\kappa$ =8(8,2)0, 8(8,2)2 and 10(14,0)0. For negative parity the 1^-, 0^-_1 and 2^- bands are known at low energy, which correspond to $n_\pi(\lambda,\mu)\kappa$ =9(11,1)1, 9(9,2)0 and 9(9,2)2. The energy spectrum was fitted with the SACM Hamiltonian containing five parameters, and good agreement was found for the positive-parity spectrum. The calculated $E2$ transitions agreed well with the handful of known experimental data . The model predicted further bands, but the available experimental data was insufficient for assigning experimental states to the model ones. However, these predictions generally compared well with the results of two microscopic models, the deformed-basis antisymmetrized molecular dynamics (DAMD) [7] approach and the generator-coordinate method (GCM) [8]. The low-lying bands were described by the three models in a qualitatively similar way.

Preliminary studies indicate that the experimental $K^\pi = 3/2^+$ and $1/2^-$ bands of ^{21}Ne may be assigned to the $(\lambda,\mu) = (8,1)$ and (11,0) SU(3) multiplets.

Acknowledgments
Work supported by the Hungarian Scientific Research Fund – OTKA, grant No. K112962.

References

[1] Otani R, Kageyama R, Iwasaki M, Kudo M, Tomita M and Ito M 2014 *Phys. Rev. C* **90** 034316
[2] von Oertzen W and Milin M 2014 *Springer Lecture Notes in Physics* **875** 147
[3] Cseh J and Lévai G 1994 *Ann. Phys. (N.Y.)* **230** 165
[4] Lévai G 2013 *Phys. Rev. C* **88** 014328
[5] Lévai G and Cseh J 1996 *Phys. Lett. B* **381** 1
[6] Lévai G, Cseh J and Van Isacker P 2007 *J. Phys. G: Nucl. Part. Phys.* **34** 1729
[7] Kimura M 2007 *Phys. Rev. C* **75** 034312
[8] Descouvemont P 1988 *Phys. Rev. C* **38** 2397