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Geometric symmetries in light nuclei

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Abstract. The algebraic cluster model is is applied to study cluster states in the nuclei ^{12}C and ^{16}O. The observed level sequences can be understood in terms of the underlying discrete symmetry that characterizes the geometrical configuration of the α-particles, *i.e.* an equilateral triangle for ^{12}C, and a regular tetrahedron for ^{16}O. The structure of rotational bands provides a fingerprint of the underlying geometrical configuration of α-particles.

1. Introduction

Ever since the early days of nuclear physics the structure of ^{12}C has been extensively investigated both experimentally and theoretically [1, 2, 3, 4]. In recent years, the measurement of new rotational excitations of both the ground state [5, 6, 7] and the Hoyle state [8, 9, 10, 11] has generated a lot of renewed interest to understand the structure of ^{12}C and that of α cluster nuclei in general. Especially the (collective) nature of the 0^+ Hoyle state at 7.65 MeV which is of crucial importance in stellar nucleosynthesis to explain the observed abundance of ^{12}C, has presented a challenge to nuclear structure calculations, such as α-cluster models [12], antisymmetrized molecular dynamics [13], fermionic molecular dynamics [14], BEC-like cluster model [15], (no-core) shell models [16, 17], *ab initio* calculations based on lattice effective field theory [18, 19], and the algebraic cluster model [7, 20, 21].

In this contribution, I discuss some properties of the α-cluster nuclei ^{12}C and ^{16}O in the framework of the algebraic cluster model.

2. Algebraic Cluster Model

The Algebraic Cluster Model (ACM) describes the relative motion of the n-body clusters in terms of a spectrum generating algebra of $U(\nu+1)$ where $\nu = 3(n-1)$ represents the number of relative spatial degrees of freedom. For the two-body problem the ACM reduces to the $U(4)$ vibron model [22], for three-body clusters to the $U(7)$ model [20, 23] and for four-body clusters to the $U(10)$ model [21, 24]. In the application to α-cluster nuclei the Hamiltonian has to be invariant under the permuation group S_n for the n identical α particles. Since one does not consider the excitations of the α particles themselves, the allowed cluster states have to be symmetric under the permutation group.

The potential energy surface corresponding to the S_n invariant ACM Hamiltonian gives rise to several possible equilibrium shapes. In addition to the harmonic oscillator (or $U(3n-3)$ limit) and the deformed oscillator (or $SO(3n-2)$ limit), there are other solutions which are of special interest for the applications to α-cluster nuclei. These cases correspond to a geometrical

Table 1. Algebraic Cluster Model for two-, three- and four-body clusters

	2α	3α	4α
ACM	$U(4)$	$U(7)$	$U(10)$
Point group	$\mathcal{C}_2$	$\mathcal{D}_{3h}$	$\mathcal{T}_d$
Geometry	Linear	Triangle	Tetrahedron
G.s. band	0^+	0^+	0^+
	2^+	2^+	
		3^-	3^-
	4^+	$4^\pm$	4^+
		5^-	
	6^+	$6^{\pm+}$	$6^\pm$

configuration of α particles located at the vertices of an equilateral triangle for ^{12}C and of a regular tetrahedron for ^{16}O. Even though they do not correspond to dynamical symmetries of the ACM Hamiltonian, one can still obtain approximate solutions for the rotation-vibration spectrum

$$E = \begin{cases} \omega_1(v_1+\frac{1}{2})+\omega_2(v_2+1)+\kappa\, L(L+1) & \text{for } n=3 \\ \omega_1(v_1+\frac{1}{2})+\omega_2(v_2+1)+\omega_3(v_3+\frac{3}{2})+\kappa\, L(L+1) & \text{for } n=4 \end{cases}$$

The rotational structure of the ground-state band depends on the point group symmetry of the geometrical configuration of the α particles and is summarized in Table 1.

The triangular configuration with three α particles has point group symmetry $\mathcal{D}_{3h}$ [20]. Since $\mathcal{D}_{3h} \sim \mathcal{D}_3 \times P$, the transformation properties under $\mathcal{D}_{3h}$ are labeled by parity P and the representations of $\mathcal{D}_3$ which is isomorphic to the permutation group S_3. The corresponding rotation-vibration spectrum is that of an oblate top: v_1 represents the vibrational quantum number for a symmetric stretching A vibration, v_2 denotes a doubly degenerate E vibration. The rotational band structure of ^{12}C is shown in the left panel of Fig. 1.

The tetrahedral group $\mathcal{T}_d$ is isomorphic to the permutation group S_4. In this case, there are three fundamental vibrations: v_1 represents the vibrational quantum number for a symmetric stretching A vibration, v_2 denotes a doubly degenerate E vibration, and v_3 a three-fold degenerate F vibration. The right panel of Fig. 1 shows the rotational band structure of ^{16}O.

3. Electromagnetic transitions

For transitions along the ground state band the transition form factors are given in terms of a product of a spherical Bessel function and an exponential factor arising from a Gaussian distribution of the electric charges, $\mathcal{F}(0^+ \to L^P; q) = c_L\, j_L(q\beta)\, \mathrm{e}^{-q^2/4\alpha}$ [20]. The charge radius can be obtained from the slope of the elastic form factor in the origin $\langle r^2\rangle^{1/2} = \sqrt{\beta^2 + 3/2\alpha}$. The transition form factors depend on the parameters α and β which can be determined from the first minimum in the elastic form factor and the charge radius.

The transition probabilities $B(EL)$ along the ground state band can be extracted from the form factors in the long wavelength limit

$$B(EL; 0^+ \to L^P) = \frac{(Ze)^2}{4\pi}\, c_L^2\, \beta^{2L} \ ,$$

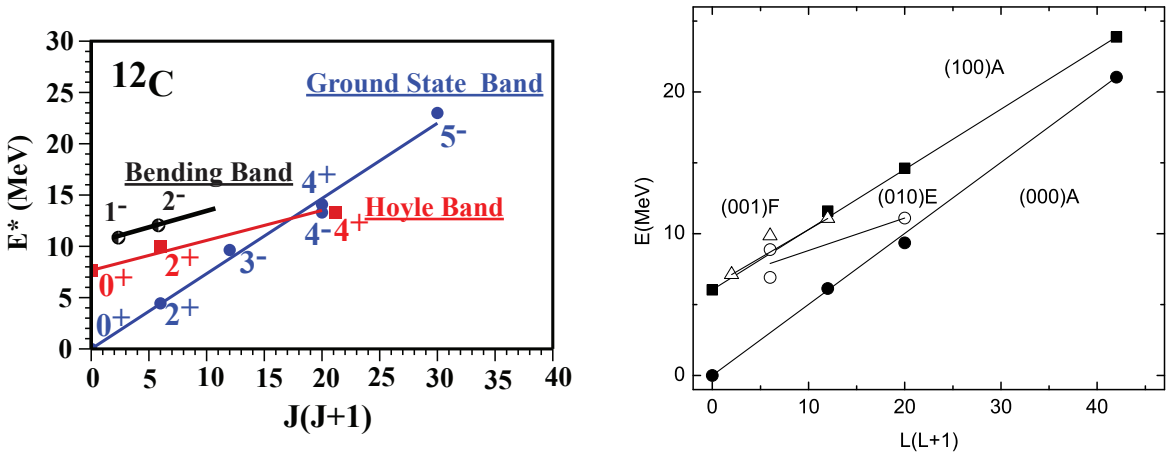

Figure 1. (Color online) Rotational band structure of the ground-state band, the Hoyle band (or A vibration) and the bending vibration (or E vibration) in ^{12}C (left) [7], and the ground-state band (closed circles), the A vibration (closed squares), the E vibration (open circles) and the F vibration (open triangles) in ^{16}O (right) [21].

Table 2. $B(EL)$ values in ^{12}C (top) and ^{16}O (bottom).

^{12}C	Th	Exp		Ref
$B(E2; 2_1^+ \to 0_1^+)$	8.4	7.6 ± 0.4	$e^2\text{fm}^4$	[25, 26, 27]
$B(E3; 3_1^- \to 0_1^+)$	44	103 ± 17	$e^2\text{fm}^6$	[25, 26, 27]
$B(E4; 4_1^+ \to 0_1^+)$	73		$e^2\text{fm}^8$	[25, 26, 27]
$M(E0; 0_2^+ \to 0_1^+)$	0.4	5.5 ± 0.2	fm^2	[25, 26, 27]
^{16}O	Th	Exp		Ref
$B(E3; 3_1^- \to 0_1^+)$	215	205 ± 10	$e^2\text{fm}^6$	[28]
$B(E4; 4_1^+ \to 0_1^+)$	425	378 ± 133	$e^2\text{fm}^8$	[28]
$B(E6; 6_1^+ \to 0_1^+)$	9626		$e^2\text{fm}^{12}$	[28]
$M(E0; 0_2^+ \to 0_1^+)$	0.54	3.55 ± 0.21	fm^2	[28]

with

$$c_L^2 = \begin{cases} \frac{2L+1}{3}\left[1 + 2P_L(-\frac{1}{2})\right] & \text{for } n = 3 \\ \frac{2L+1}{4}\left[1 + 3P_L(-\frac{1}{3})\right] & \text{for } n = 4 \end{cases}$$

The good agreement for the $B(EL)$ values for the ground band in Table 2 shows that both in ^{12}C and in ^{16}O the positive and negative parity states merge into a single rotational band. Moreover, the large values of $B(EL; L_1^P \to 0_1^+)$ indicate a collectivity which is not predicted for simple shell model states. The large deviation for the $E0$ between the first excited 0^+ (Hoyle) state and the ground state indicates that the 0_2^+ state cannot be interpreted as a simple vibrational excitation of a rigid triangular (^{12}C) or tetrahedral (^{16}O) configuration, but rather corresponds to a more floppy configuration with large rotation-vibration couplings. A more

detailed study of the electromagnetic properties of α-cluster nuclei in the ACM for non-rigid configurations is in progress.

4. Summary and conclusions

In this contribution, the cluster states in ^{12}C and ^{16}O were interpreted in the framework of the ACM as arising from the rotations and vibrations of a triangular and tetrahedral configuration of α particles, respectively. In both cases, the ground state band consist of positive and negative parity states which coalesce to form a single rotational band. This interpretation is validated by the observance of strong $B(EL)$ values. The rotational sequences can be considered as the fingerprints of the underlying geometric configuration (or point-group symmetry) of α particles.

For the Hoyle band in ^{12}C there are several interpretations for the geometrical configuration of the three α particles. In order to determine whether the geometrical configuration of the α-particles for the Hoyle band is linear, bent or triangular, the measurement of a possible 3^- Hoyle state is crucial, since its presence would indicate a triangular configuration, just as for the ground state band.

Finally, the results presented here for ^{12}C and ^{16}O emphasize the occurrence of α-cluster states in light nuclei with $\mathcal{D}_{3h}$ and $\mathcal{T}_d$ point group symmetries, respectively.

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