IOP Conf. Series: Journal of Physics: Conf. Series **863** (2017) 012005 doi:10.1088/1742-6596/863/1/012005

Alpha-Particle Condensate Structure of the Hoyle State: where do we stand?

P Schuck[1,2,3], Y Funaki[4], H Horiuchi[5], G Röpke[6], A Tohsaki[5] and T Yamada[7]

[1]Institut de Physique Nucléaire, CNRS, UMR8608, Orsay, F-91406, France
[2]Université Paris-Sud, Orsay, F-91505, France
[3]Laboratoire de Physique et de Modélisation des Milieux Condensés, CNRS et Université Joseph Fourier, UMR5493, 25 Av. des Martyrs, BP 166, F-38042 Grenoble Cedex 9, France
[4]The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama, 351-0198, Japan
[5]Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan
[6]Institut für Physik, University of Rostock, Universitätsplatz 1, 18051 Rostock, Germany
[7]Laboratory of Physics, Kanto Gakuin University, Yokohama 236-8501, Japan

Abstract. The present understanding of the structure of the Hoyle state in ^{12}C is reviewd. It is pointed out that a crucial test of any theory is the good reproduction of the experimental results for the inelastic form factor from ground to the Hoyle state. The performances of the so-called THSR wavwe function are outlined confirming the α particle condensation hypothesis proposed 15 years back in [1].

1. Introduction

One of the most amazing phenomena in quantum many-particle systems is the formation of quantum condensates. At present, the formation of condensates is of particular interest in strongly coupled fermion systems in which the crossover from Bardeen-Cooper-Schrieffer (BCS) pairing to Bose-Einstein condensation (BEC) may be investigated. Among very different quantum systems, nuclear matter is especially well suited for the study of correlation effects in a quantum liquid. In [2], the possibility of α particle (quartet) condensation in infinite matter was investigated. It was found that quartetting is possible at low densities, below about a fifth of saturation density. At higher densities, around the point where the chemical potential μ turns from negative (binding) to positive, the condensation breaks down. This is contrary to ordinary pairing which can exist for considerably positive μ values, depending only on the range of the pairing force. The reason for this strong qualitative difference between the two cases is explained in [3].

The question then arises whether in analogy to pairing, also for quartetting exist nuclei where this phenomenon is born out. In [1], we found that such a possibility very likely exists in lighter self-conjugate nuclei for excitation energies around the α disintegration threshold. In

IOP Conf. Series: Journal of Physics: Conf. Series **863** (2017) 012005 doi:10.1088/1742-6596/863/1/012005

this contribution, we want to discuss the successes and eventual failures of this idea which was proposed 15 years back.

2. The THSR approach and the Hoyle state

The Hoyle state is the first excited 0^+ state in ^{12}C at 7.65 MeV. This state is one of the most fameous states in nuclear physics because without its existence life on earth would be absent in its present form. Indeed, since ^{8}Be is unstable, the stellar production of Carbon in the universe would be lower by a huge factor without the existence of the Hoyle state. It is just at the right energy to allow for the so-called triple α reaction $\alpha+\alpha+\alpha \to ^8 \mathrm{Be}+\alpha \to ^{12}\mathrm{C}^*$ to become strongly accelerated.

For the microscopic description of the Hoyle state several approaches have been put forward in the past [4, 5, 6, 7, 8, 9, 10]. However, only the following, so-called THSR wave function (according to the authors Tohsaki, Horiuchi, Schuck, Roepke) which was proposed in [1] concentrates on the α particle condensation aspect (the spin-isospin part is not written out)

$$\Psi_{\mathrm{THSR}} \propto \mathcal{A}\psi_1\psi_2\psi_3 \equiv \mathcal{A}|B\rangle \tag{1}$$

with

$$\psi_i = e^{-((\mathbf{R}_i-\mathbf{X}_G)^2)/B^2}\phi_{\alpha_i} \tag{2}$$

and

$$\phi_{\alpha_i} = e^{-\sum_{k<l}(\mathbf{r}_{i,k}-\mathbf{r}_{i,l})^2/(8b^2)} \tag{3}$$

In (1) the $\mathbf{R}_i$ are the c.o.m. coordinates of α particle 'i' and $\mathbf{X}_G$ is the total c.o.m. coordinate of ^{12}C. $\mathcal{A}$ is the antisymmetrizer of the twelve nucleon wave function with ϕ_{α_i} the intrinsic translational invariant wave function of the α-particle 'i'. The whole 12 nucleon wave function in (1) is, therefore, translationally invariant. The special Gaussian form given in Eqs. (2), (3) was chosen in [1] to ease the variational calculation. The condensate aspect lies in the fact that (1) is a (antisymmetrized) product of three times the same α-particle wave function and is, thus, analogous to a number projected BCS wave function in the case of pairing. This twelve nucleon wave function has two variational parameters, b and B. It possesses the remarkable property that for $B = b$ it is a pure harmonic oscillator Slater determinant (this aspect of (1) is explained in [11, 12]) whereas for $B \gg b$ the α's are at low density so far apart from one another that the antisymmetrizer can be dropped and, thus, (1) becomes a simple product of three α particles, all in identical 0S states, that is, a pure condensate state. The minimization of the energy with a Hamiltonian containing a nucleon-nucleon force determined earlier independently [13] allows to obtain a reasonable value for the ground state energy of ^{12}C. Variation of energy under the condition that (1) is orthogonal to the previously determined ground state allows to calculate the first excited 0^+ state, i.e., the Hoyle state. While the size of the individual α particles remains very close to their free space value ($b \simeq 1.37$ fm), the variationally determined B parameter takes on about three times this value. This entails a quite enhanced value of the rms radius of 3.83 fm of the Hoyle state with repect to the one of the ground state (2.4 fm). This gives a volume (density) of the Hoyle state about a factor 3-4 larger (smaller) than for the ground state. In such a large volume the α's have space to develope themselves what is not the case in the ground state where they overlap strongly.

Still the question may be asked: is the Hoyle state closer to a Slater determinant or to an α condensate? A precise answer is obtained from the calculation of the bosonic occupation numbers which have been obtained in three different works [14, 15, 10] with very similar results. The ones of [15] are displayed in Fig.1. We see that the distribution in the ground state is more or less equipartitioned and compatible with the SU3 shell model theory whereas the distribution

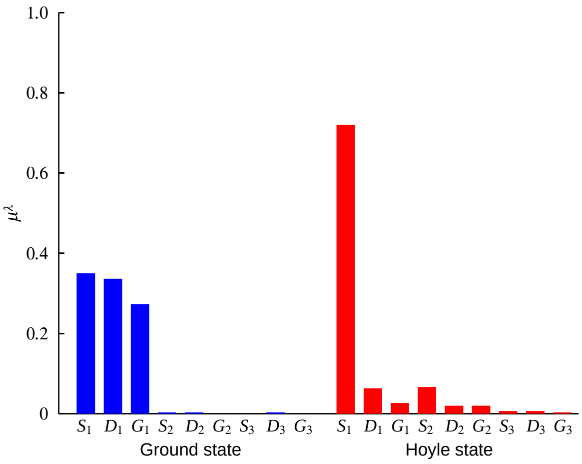

Figure 1. α particle occupation numbers in the ground state (left) and in the Hoyle state (right) [15].

of the Hoyle state has an overwhelming contribution of over 70% of the α's being in the lowest 0S state, all other contributions are down by a factor of at least ten. We, therefore, can say that the three α particles in the Hoyle state occupy with their c.o.m. motion to a large fraction the same 0S orbit meaning that, indeed, the Hoyle state can be considered to within good approximation as a condensate of 3 α particles. However, the Pauli principle is still active and antisymmetrisation scatters the α's out of the condensate about 30% of the time. It can be mentioned here that this number is very similar concerning good single particle states in odd nuclei where the fermionic occupation numbers also are in the range of 70-80%.

We mentioned that the Hoyle state has an extended volume being by a factor 3-4 larger than the one of the ground state. How to prove this? It turns out that the inelastic form factor, measured by inelastic electron scattering, is very sensitive to the size of the Hoyle state [16]. Increasing artificially the size of the Hoyle state by about 20% reduces the form factor globally by a factor of two. The fact that the THSR theory reproduces very precisely the experimental values of the inelastic form factor, see Fig.2 without any adjustable parameter can be considered as a great achievement and gives large credit to the picture that in the Hoyle state three α particles are well born out moving almost independently in their proper mean field. In the same figure we show recent Green's function Monte Carlo (GFMC) results [17] which also also reproduce the inelastic form factor very nicely. In the insert of the GFMC-panel, we see that the rather precise experimental transition radius of 5.29 ± 0.14 fm^2 given in [8] is better reproduced than with the THSR approach which yields an about 20% too large value. On the other hand, the GFMC approach gives the position of the Hoyle state about 2.5 MeV too high whereas with the THSR wave function the experimental value of 7.65 MeV is quite well reproduced with no adjustable parameter.

3. A brief account of the situation in ^{16}O

The situation in ^{16}O is quite a bit more complicated than in ^{12}C. The fact is that between the 4α threshold and the ground state, there are several 0^+ states which can be interpreted as α+^{12}C cluster configurations. In Fig. 3, we show the result of a calculation with the so-called Orthogonal Condition Model (OCM) method [18].

We see that there is a very nice one to one correspondence between the first six calculated 0^+ states and experiment. In regard of the complexity of the situation the agreement between

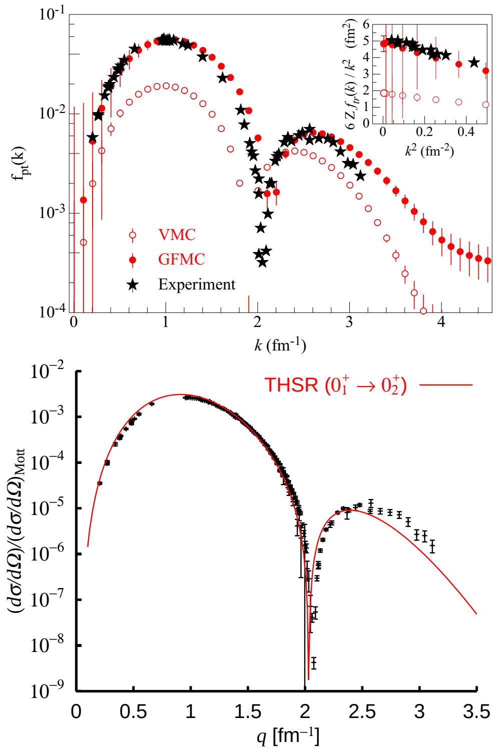

Figure 2. Inelastic form factors from GFMC [17], upper panel, and THSR [16], lower panel. The THSR result cannot be distinguished from the one of [5] on the scale of the figure meaning that also the approach of [5] implicitly contains the α condensation aspect (this, by the way, is also the case with the approach in [6]).

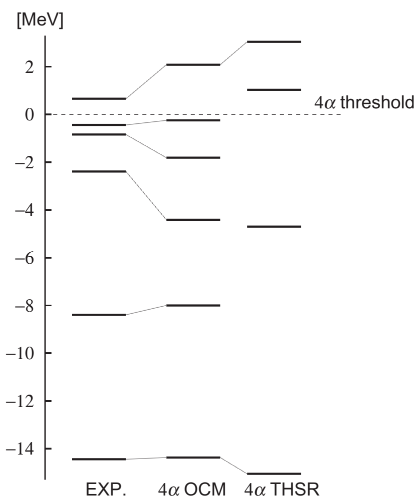

Figure 3. Spectrum of 0^+ states in ^{16}O from the OCM and THSR approaches [18, 1].

both can be considered as very satisfactory. Only the highest state was identified with the 4α condensate state. The four other excited 0^+ states are α+^{12}C configurations. For example the 5-th 0^+ state is interpreted as an α orbiting in a higher nodal S-wave around the ground state of ^{12}C. The 4-th 0^+ state contains an α orbiting in a P-wave around the first 1^- state in ^{12}C. In the 3-rd 0^+ state the α is in a D-wave coupled to the 2_1^+ state of ^{12}C and in the 2-nd 0^+ state the α is in a 0S-wave and the ^{12}C in its ground state. The single parameter THSR calculation can only reproduce correctly the ground state and the α condensate state (0_6^+). By construction it cannot describe α+ ^{12}C configurations. So, the two intermediate states give some sort of average picture of the four α plus ^{12}C configurations. One would have to employ a more general ansatz like in ([19] to cope with the situation. Work in this direction is in progress. The 0_6^+ state is theoretically identified as the α-condensate state from the overlap squared $|\langle 0_6^+|\alpha +^{12} \mathrm{C}(0_i^+)\rangle|^2$ [20].

4. Discussion and Conclusions

In this contribution, we concentrated on the α particle condensation aspect of the Hoyle state introduced with the THSR wave function 15 years back [1]. This approach reproduces all known experimental results of the Hoyle state without any adjustable parameter and, thus, gives credit to the condensation scenario. This, despite of the fact that its direct experimental verification is difficult. Indeed, while pairing induces clear signs of superfluidity in rotating nuclei, no analogous effects have been detected so far from quartetting. However, several experiments are under way or planned concerning the Hoyle state and analogous states in ^{16}O or even heavier nuclei, what may shed further light on the situation in the near future [21, 22]. A major issue in this respect is the understanding not only of the Hoyle state but of excited states thereof. The 0_3^+ and 0_4^+ have been identified experimentally recently and have been interpreted as α gas states with one α in a higher nodal S-state and a linear chain state, respectively, see [19] and references therein for discussions. Also the structure of the second 2^+ state is strongly debated. It is considered either as a member of a rotational band with the Hoyle state as band head [23] or more as a nodal excitation of one of the α's into a D-wave [19]. Further experimental and theoretical studies are necessary to elucidate the situation. With respect to the excited Hoyle states an interesting paper has appeared recently [24] where the authors explain with a single adjustable parameter very well the Hoyle spectrum on grounds that the Hoyle state is a Bose condensate with broken U1 symmetry (particle number). However, also this approach is not well tested and needs further work.

References

[1] Tohsaki A, Horiuchi H, Schuck P and Röpke G 2001 *Phys. Rev. Lett.* **87** 192501
[2] Röpke G, Schnell A, Schuck P and Noziéres P 1998 *Phys. Rev. Lett.* **80** 3177
[3] Sogo T, Lazauskas R, Röpke G and Schuck P 2009 *Phys. Rev.* C **79** 051301
[4] Horiuchi H 1974 *Progr. Theor. Phys.* **51** 1266; idem 1975 **53** 447
[5] Kamimura M 1981 *Nucl. Phys.* A **351** 456
[6] Uegaki E, Okabe S, Abe Y and Tanaka H 1977 *Progr. Theor. Phys.* **57** 1262; *idem* 1978 **59** 1031
[7] Kanada-En'yo Y 1998 *Phys. Rev. Lett.* **81** 5291; 2007 *Progr. Theor. Phys.* **117** 655
[8] Chernykh M, Feldmeier H, von Neumann-Cosel P and Richter A 2007 *Phys. Rev. Lett.* **98** 032501
[9] Lazauskas R and Dufour M 2011 *Phys. Rev. Lett.* **84** 064318
[10] Ishikawa S 2014 *Phys. Rev.* C **90** 061604(R) and private communication.
[11] Bayman B F and Bohr A 1958 *Nucl. Phys.* **9** 596
[12] Yamada T, Funaki Y, Horiuchi H, Ikeda K and Tohsaki A 2008 *Prog. Theor. Phys.* **120** 1139
[13] Tohsaki A *Phys. Rev.* C **49** 1814
[14] Matsumara M and Suzuki Y *Nucl. Phys.* A **739** 238
[15] Yamada T and Schuck P 2005 *Eur. Phys. J.* A **26** 185
[16] Funaki Y, Tohsaki A, Horiuchi H, Schuck P and Röpke G *Eur. Phys. J.* A **28** 259
[17] Carlson J, Gandolfi S, Pederiva F, Pieper S C, Schiavilla R, Schmidt K E and Wiringa R B 2015 *Rev. Mod. Phys.* **87** 1067
[18] Funaki Y, Yamada T, Horiuchi H, Röpke G and Schuck P 2008 *Phys. Rev. Lett.* **101** 082502
[19] Funaki Y *Phys. Rev.* C **92** 021302
[20] Yamada Y, Funaki Y, Horiuchi H, Röpke G and Schuck P 2011 arXiv:1103.3940; "Cluster in Nuclei (Lecture Notes in Physics)-Vol.2-", ed. by C. Beck, (Springer -Verlag, Berlin)
[21] Natowitz J, private communication.
[22] Freer M, private communication.
[23] Marin-Lambarri D J, Bijker R, Freer M, Gai M, Kokalova Tz, Parker D J and Wheldon C 2014 *Phys. Rev. Lett.* **113** 012502
[24] Nakamura Y, Takahashi J, Yamanaka Y and Ohkubo S 2016 *Phys. Rev.* C **94** 014314