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Simulating Hadronic-to-Quark-Matter with Burn-UD: Recent work and astrophysical applications

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Abstract. We present the new developments in Burn-UD, our in-house hydrodynamic combustion code used to model the phase transition of hadronic-to-quark matter. Our two new modules add neutrino transport and the time evolution of a (u, d, s) quark star (QS). Preliminary simulations show that the inclusion of neutrino transport points towards new hydrodynamic instabilities that increase the burning speed. A higher burning speed could elicit the deflagration to detonation of a neutron star (NS) into a QS. We propose that a Quark-Nova (QN: the explosive transition of a NS to a QS) could help us explain the most energetic astronomical events to this day: superluminous supernovae (SLSNe). Our models consider a QN occurring in a massive binary, experiencing two common envelope stages and a QN occurring after the supernova explosion of a Wolf-Rayet (WO) star. Both models have been successful in explaining the double humped light curves of over half a dozen SLSNe. We also introduce SiRop our r-process simulation code and propose that a QN site has the hot temperatures and neutron densities required to make it an ideal site for the r-process.

1. Introduction

The hypothesis that quark matter (matter constituted of up, down, and strange quarks) is the most stable state of matter in the universe [1, 2], makes possible the existence of compact objects made up entirely of deconfined quark matter called (u,d,s) quark stars (QS). The core of a neutron star (NS) can reach a high enough density initiating a QCD phase transition that would allow for the formation of a QS.

The transition between hadronic matter and quark matter is an active question of research. This conversion can be thought of as a combustion, where hadronic matter acts as the fuel and quark matter as the ashes. With the conversion comes a front, and its velocity determines the type of combustion. The particular conditions of the burning and their consequences are subject of constant discussion amongst scientist and there is yet a consensus to be found.

The combustion of hadronic-to-quark matter can be characterized as either a detonation or a deflagration based on the speed of the combustion front with respect to the speed of sound. If the combustion front is faster than the speed of sound, the transition is a detonation; otherwise it is a deflagration. The transition as a weak deflagration has been studied by Olinto et al. [3], Heiselberg et al. [4], Olesen et al. [5], amongst others, as a result of a conversion induced by the diffusion of seed quark matter. Making these assumptions, Olesen et al. [5] found the time

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scale of the conversion of a NS to take place between 0.1 seconds to a few minutes. Drago et al. [6] found a fast deflagration by solving the relativistic, hydrodynamic jump conditions using a realistic equation of state (EoS) for nuclear matter.

Other studies suggest that the presence of an initial strange quark seed may not be necessary. Instead, a shock could compress hadronic matter to high densities which would trigger deconfinement into two flavored quark matter (u, d). The two-flavored quark matter can then decay into three-flavored quark matter (u, d, s) in a heat releasing process. This two-step process for hadronic-to-quark matter transition results in a detonation but depends on an initial shock that would push the hadronic matter to the required densities. Bhattacharyya et al. [7] found that the conversion from hadronic matter to two-flavored quark matter happens in the order of milliseconds whereas the conversion from two-flavored quark matter to three-flavored quark matter is in the order of hundreds of seconds. Benvenuto et al. [8, 9] found that the deconfinement of hadronic matter can lead to a fast combustion due to a shock induced detonation, making the conversion of a NS on the scale of microseconds.

While authors like Lugones et al. [10] suggest that the combustion happens as a detonation, others like Drago et al. [6] suggest that the conversion process can only happen as a deflagration. The scientific community has no consensus for the way in which the conversion happens and the result is really dependant on the assumptions made and the physics considered when building the model. A different approach to studying the combustion is to perform hydrodynamical oriented studies by solving numerically the equations driving the combustion front. Niebergal et al. [11] studied the micro-physics of the combustion front by solving the reaction-diffusion-advection equations in spherical symmetry and considering neutrino reactions. Their result gave a burning front faster by almost six orders of magnitude compared to previous estimates. Other efforts to solve the hydrodynamic equations were performed by Herzog et al. [12] by simulating the conversion of a NS into a QS. This approach found that turbulence can enhance the burning speed of the front. Manrique et al. [13] assumed an initial shock when solving numerically the special relativistic hydrodynamic equations of combustion and confirmed numerically that the assumption of an initial shock leads to a detonation.

To fully understand the phase transition of hadronic-to-quark matter we believe that solving numerically the hydrodynamic equations of the burning front is the best approach. By following this micro-physical approach, we are capable of looking at the effects induced by neutrinos and electrons from first principles. To do so, we created Burn-UD a code that simulates the transition of hadronic-to-quark matter using micro-physics, implementing different EoS and different initial conditions (see [14] for a review). In this paper, we present the progress and changes we have done to Burn-UD. Section 2 explores the advances and new functionalities implemented in the code and the results we have obtained using our code. Section 3 presents some of the astrophysical applications of our results and lastly we present our conclusions in Section 4.

2. Developments in the code BURN-UD

Burn-UD is a hydrodynamic combustion code used to model the hadronic-to-quark matter transition. It is written in java and will be publicly available on all major platforms (Windows, Linux and OSx). Its distinct graphical user interface makes it a compelling tool for scientist to explore the micro-physics, hydrodynamical effects and EoS of the burning front. Ouyed et al. [14] offers an introduction to the software and its interface. Here, we present the two major updates of the software: a neutrino transport module and a QS evolution module.

2.1. Neutrino module

Electrons and neutrinos play a role in quark matter (u,d,s) through the following two reactions:

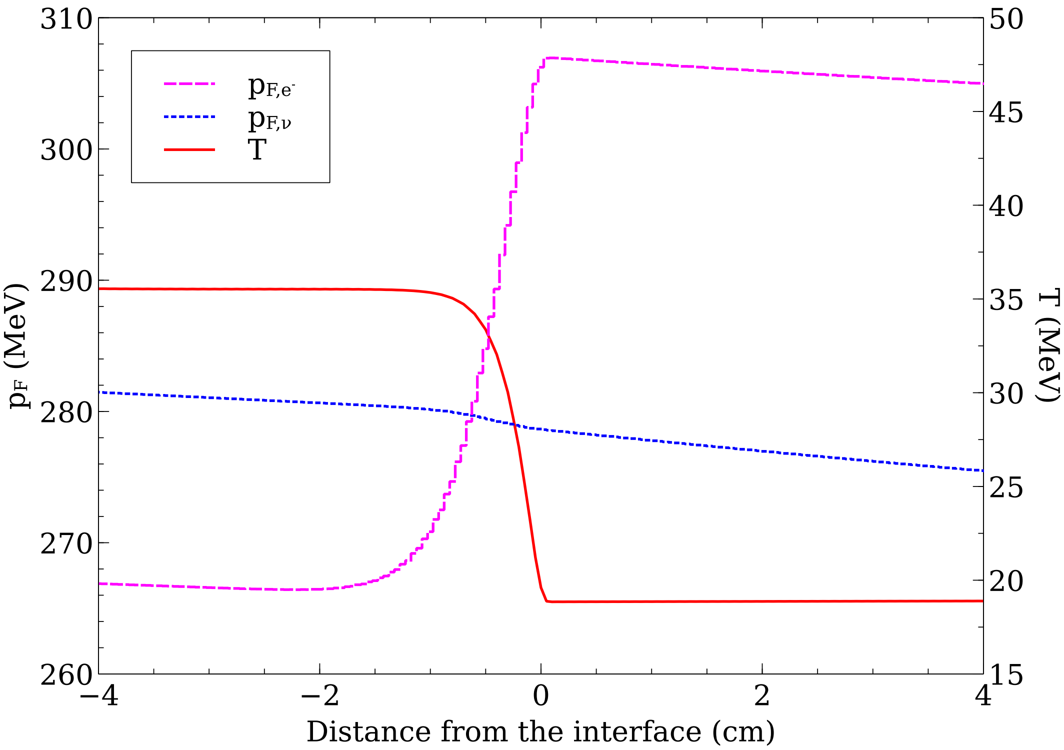

Figure 1. Electron fermi momentum ($p_{F_{e^-}}$), electron-neutrino fermi momentum ($p_{F_{\nu_e}}$), and temperature (T) as functions of distance from the interface. Burning raises the temperature and causes a strong electron pressure gradient.

$$u + e \leftrightarrow s + \nu_e \tag{1}$$

$$u + e \leftrightarrow d + \nu_e. \tag{2}$$

The above two reactions lead to a neutrino distribution function that moves across the burning interface. The neutrino transport module implements neutrino transport in a flux-limited, diffusion scheme. It uses the mean free paths of quark matter and couples the transport consistently to the momentum and entropy evolution equations:

$$\frac{\partial n_{\nu_e}}{\partial t} + \nabla \cdot (\mathbf{v}\nu_e) + \nabla \cdot (\nabla D_{\nu_e} n_{\nu_e}) = \Gamma_1 + \Gamma_2 \tag{3}$$

$$\frac{\partial Q}{\partial t} = \frac{\partial U_{\nu_e}}{\partial t}. \tag{4}$$

There, ν_e is the electron neutrino number density, $\mathbf{v}$ is the fluid velocity, D_{ν_e} is the diffusion coefficient, Q is the heat density, and U_{ν_e} is the internal energy density. Γ_1 and Γ_2 are the reaction rates due to equation (1) and equation (2) respectively.

We find that the addition of neutrino transport gives rise to various hydrodynamic effects, related to the pressure gradients induced by the lepton physics, such as electron capture and neutrino transport.

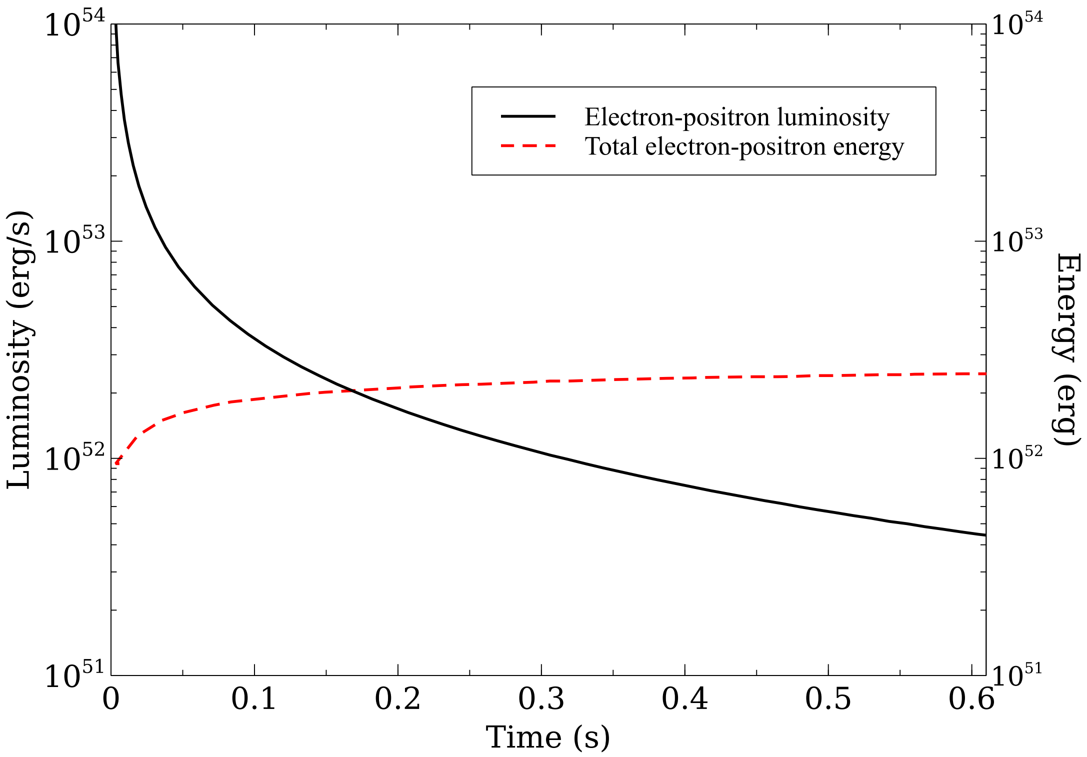

Figure 2. The luminosity of electron-positron pairs deposited through neutrino-antineutrino annihilation. The total energy deposited (the integrated luminosity) is shown. This is a few times $> 10^{52}$ ergs of kinetic energy.

2.2. Quark star evolution module

We also added a preliminary QS evolution module where the number densities and chemical potentials of quarks, electrons, and neutrinos are evolved at each time step for a whole compact star. The evolution is done by enforcing beta-equilibrium at each time-step while simultaneously solving the lepton and energy transport equations:

$$\frac{\partial E_{\nu_e}}{\partial t} + \frac{\partial}{\partial r}\frac{1}{r^2}\left(r^2 \frac{\partial}{\partial r} D_{\nu_e} E_{\nu_e}\right) = S_1 \tag{5}$$

$$\frac{\partial E_{\nu_\mu}}{\partial t} + \frac{\partial}{\partial r}\frac{1}{r^2}\left(r^2 \frac{\partial}{\partial r} D_{\nu_\mu} E_{\nu_\mu}\right) = S_1 \tag{6}$$

$$\frac{\partial n_{\nu_e}}{\partial t} + \frac{\partial}{\partial r}\frac{1}{r^2}\left(r^2 \frac{\partial}{\partial r} D_{\nu_e} n_{\nu_e}\right) = S_3 \tag{7}$$

where E_i is the energy density of particle i and S_i are source terms.

It also calculates the e^+e^- luminosity caused by the process $\nu\overline{\nu} \rightarrow e^+e^-$ above the surface of the QS. This module will incorporate general relativistic effects in the transport equations in its upcoming versions.

2.3. Results with new updates.

2.3.1. Electron-gradient induced instability We find that the coupling of reactions (1), (2), and neutrino transport to the hydrodynamic equations gives rise to important hydrodynamic effects.

One of them is an electron-gradient induced instability. As equation (1) and equation (2) lead to electron capture behind the interface, and leptons deplete in the upstream region due to neutrino diffusion, a strong electron gradient develops that leads to a pressure gradient (see Fig. 1) that induces fluid velocities parallel to the upstream direction. In the case of trapped neutrinos, the front momentarily halts until the momentum and heat deposited by neutrinos revives it. However, in the case untrapped neutrinos, the electron pressure gradient is unopposed, halting the burning front in the time-scale of the simulation.

We speculate that in a multidimensional situation, uneven slowing down and halting of the interface due to the electron pressure gradient could wrinkle the interface, leading to turbulence. Niebergal et al. [11] discovered a similar instability (deleptonization instability) due to neutrino cooling as heat leaks from the upstream region due to escaping neutrinos and the thermal pressure behind the front decreases, which could lead to halting. Both the electron-gradient induced instability and deleptonization instability would exacerbate each other because both are caused by the depletion of leptons from the upstream region.

2.3.2. Mass ejection due to $\nu\overline{\nu}$ annihilation We used the QS evolution module (see section 2.2) and imposed an initial thermal energy distribution due to the release of binding energy through a hadron-quark phase transition. This is calculated by imposing hadronic EoS in the compact star, and then switching to an MIT Bag Model EoS, while conserving energy density. We find an intense e^-e^+ pair luminosity burst of $> 10^{53}$ erg/s for the duration of 0.5 s (see dashed curve in Fig. 2) due to $\nu\overline{\nu}$ annihilation near the surface of the QS, which suggests relativistic mass ejection of the hadronic matter overlaying on the surface of the QS. We find total integrated e^-e^+ pair luminosity of about 5×10^{52} ergs. This suggests a strong candidate for an engine of some of the most energetic events in astrophysics, such as superluminous supernovae, *hypernovae*, or gamma ray bursts.

The physical causes of this e^-e^+ pair luminosity burst are the extreme surface temperatures T $\sim$ 20 MeV of a nascent QS and its compactness. Large temperatures lead to large thermal ν and $\overline{\nu}$ number densities which would annihilate near the neutrino-sphere. This filters the neutrino energy by transforming it into electron-positron energy, the latter which is strongly coupled with matter. Most of the e^-e^+ pair luminosity is deposited as kinetic energy in the overlaying hadronic matter. Furthermore, the relative insensitivity of the quark matter EoS to temperature implies that the radius of QS is relatively small ($\sim$ 10 km) even at high temperatures which leads to general relativistic effects that enhance $\nu\overline{\nu}$ annihilation [15]. These results are studied and discussed in details in [16, 17].

3. Astrophysical applications

A complementary approach to these simulations is to look for astronomical signatures that help us probe the nature of this process. Particularly, we believe that by looking closely at the most luminous events in the sky, our understanding of stellar evolution, and the production of elements we can complement the state of the art simulations and theoretical work performed.

The Quark-Nova (QN) model assumes the explosive transition of a NS into a QS [18]. The explosion can eject $\sim 10^{-3}$ $M_\odot$ of the outer layers of the NS at relativistic speeds with an average Lorentz factor of $\Gamma_{QN} \sim 10$ [11, 19]. The ejecta could be visible by interacting with surrounding material or by the decay of unstable r-process isotopes. Ouyed et al. [20, 21] offer an in-depth review to the QN model and their observational signatures in astronomical data. In this section we present the latest updates of the QN model in superluminous supernova events (SLSNe).

3.1. Superluminous Supernova Events (SLSNe)

SLSNe are explosions seen in space with extreme luminosities that extend hundreds of days. Although they seem to resemble regular supernovae (SNe), their extreme luminosity and spectral

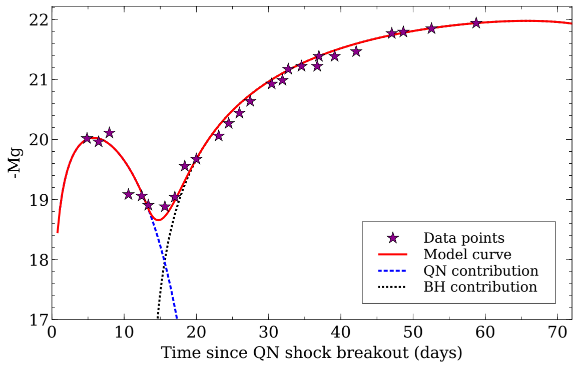

Figure 3. QN model fit to the g-band light curve of LSQ14bdq. The stars are the observations from Nicholl et al. [35], the black line is the BH accretion and the blue line is the QNe. The first model was employed because the SLSNe is not characterized by a strong oxygen absorption line.

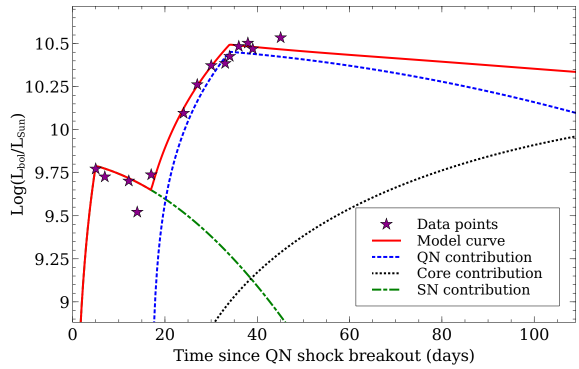

Figure 4. QN model fit to the bolometric light curve of SN2006oz. The stars are the observations from Leloudas et al. [36]. The green curve is the light curve of a WO type star SN. The blue curve is the envelope's contribution after being shocked by the QN while the black curve is the core's contribution after being heated by the QN. This event has an oxygen absorption line in its spectrum making it an ideal candidate for this model.

features point towards a yet unknown mechanism that has puzzled astrophysicists since their discovery a little over a decade ago. For some, the decay of ^{56}Ni can be the mechanism behind some of the events [22, 23, 24]. For others, the core collapse of massive stars, the thermonuclear explosion of white dwarfs or a combination of both on a binary arrangement that interacts with a circumstellar material could explain SLSNe [25, 26, 27, 28]. Another alternative is powering by a black hole (BH) experiencing accretion from surrounding material [29] or by a highly magnetic NS rotating with a millisecond period (magnetar) releasing rotational energy into a dense shell [30, 31].

When thinking about SLSNe, it is evident that a regular SN cannot provide the energy required to power them, so the idea of an even more energetic explosion seems appealing. The QN offers the energy required to explain these high luminosity events (see section 2). Currently we have two compelling scenarios that could explain SLSNe. Ouyed and his collaborators have published a series of papers on SLSNe and they have used the model to fit more than half a dozen SLSNe [21, 32, 33, 34]. Here we present an overview of the two models we have developed for double humped SLSNe, their achievements and limitations.

For SLSNe-I (hydrogen-poor), a QN occurring in a massive binary undergoing two common envelopes (CE) phases can account for the double-humped light curve. Two massive stars in a binary configuration can undergo their regular stellar evolution leading to the more massive going SN. The resulting NS can accrete matter until it reaches deconfinement densities. At this point the NS will go QN and the first hump in the light curve is explained. Later stages of the system allow for the formation of a BH accreting matter over a subtended period of time effectively powering a long lasting second hump in the light curve. Ouyed et al. [32] offer an in-depth explanation and step-by-step of the model.

The QN binary model's strength resides in the ejection of the first CE (H rich) and in being able to explain light curves that show multiple rebrightening epochs (i.e. bumps in the light curve). By ejecting the first CE, this model can explain the lack of H in the observed spectrum of SLSN-I or its later appearance. The rebrightening at different epochs naturally arises from the interactions between the QN and CO core and BH and CO disc; these elements behave as mass reservoirs that can be exploited and used at later times. This model has successfully explained the light curves of half a dozen SLSNe. However, one of its limitations is explaining events with strong oxygen absorption on the early stages of their light curve (see ASSASN-15lh [37] and its late UV rebrightening [38]).

Our second model is the basis for SLSNe with early strong oxygen absorption. The presence of a QN after the supernova explosion of a stripped oxygen sequence Wolf-Rayet star can explain the double hump nature of these events and the large presence of oxygen in the early spectra. First, the WO type star goes SN and the ejecta expands homologously. After the SNe, a two-component configuration is created by the extended low density envelope and the dense core. Later, a QN going off in this configuration will deposit energy in the extended envelope of the SN resulting in the first hump of the light curve. Lastly, the heated core, initially energized by the QN shock, acts as a *hot plate* that re-brightens the light curve after it is revealed by the receding photosphere of the shocked envelope giving result to the second hump in the light curve. This model has been successful in explaining the light curve, photospheric radius and effective temperature of ASASSN-15lh [39]. Furthermore, we believe that this model could yield similar SLSNe with different spectra if it were to happen inside other types of WR stars (e.g. WN and WC).

Both models have been successful in reproducing the light-curves of different SLSNe. Figure 3 and figure 4 are two of the multiple fits we have performed for SLSNe (more fits are available at http://quarknova.ca/LCGallery.html). The full understanding of hadronic-to-quark matter will allow us to make more robust models and eventually determine if one scenario is more likely to happen than the other. Furthermore, fitting more observations can help us delimit the energetics of this transition.

3.2. Stellar nucleosynthesis

The origin of elements heavier than iron is thought to be slow and rapid neutron capture processes, s-process and r-process respectively [40, 41]. While the s-process occurs at low neutron densities, the r-process is the opposite operating at much higher neutron densities. Simulating the conditions of r-process sites and matching our simulations to chemical abundances provides knowledge about element formation and allows us to look for astronomical signatures of proton rich, hot nucleosynthesis environments.

We have developed SiRop (Simulation of Rapid neutron capture) a code for the simulation of r-process under the same principles of Burn-UD, a friendly user interface, high optimization for running simulations and available for the public in all platforms. SiRop allows the user to study the chemical abundances of different astronomical sites (see [42] for a full review of SiRop and its manual). We have used SiRop (and r-Java, a previous version of the code) to simulate the conditions of a QN site (see [20, 43, 44]). Our findings show that the high temperature and proton rich ejecta of a QN make it an ideal site for the r-process (see Fig. 5). By continuing our study of the QN and the r-process we are looking at the elemental abundances of stars and matching our simulations to astronomical observations. Not only are we understanding the role of r-process sites but we are also finding signatures that help us establish constraints on the energetics of the transition of hadronic-to-quark matter and the QN events.

4. Conclusion

We believe that the study of hadronic-to-quark matter transition, its causes and consequences is vital for both theoretical physics and astronomy. The use of computational tools to perform high level hydrodynamical simulations of the burning front of a NS to a QS is important to

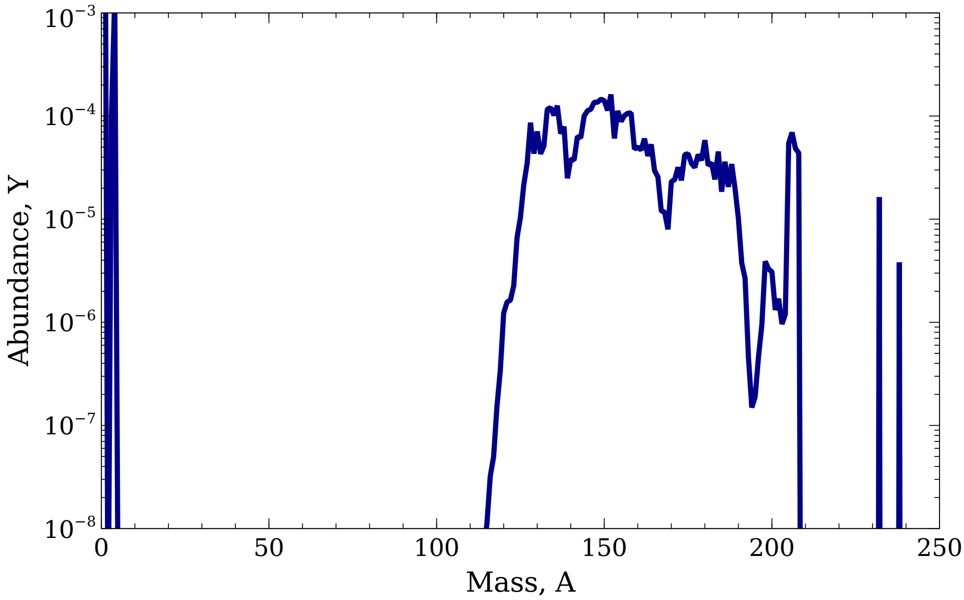

Figure 5. Results of an r-process simulation using SiRop from a QN which has ejected the NS crust (i.e. the QN ejecta). The parameters used are a QN ejecta of 10^{-3} $M_\odot$, an initial electron fraction $Y_e = 0.1$ and a 1% of total energy of the QN transformed into kinetic energy of the ejecta.

help bring consensus to the scientific community about the nature of hadronic matter and the conditions that allow it to undergo a QCD phase transition.

Burn-UD, our specialized in house software[1], offers the tools for scientists to study the time-dependent phase transition of hadronic-to-quark matter. The two new modules, neutrino reactions and QS evolution, have pointed at an electron-gradient induced instability and mass ejection due to $\nu\overline{\nu}$ annihilation. This mechanism seems to be possible in addition to others like core-collapse [45] and photon fireball [46]. The joint result of these mechanisms make us believe that the transition of hadronic-to-quark mater can result in an explosion that releases about $\sim 10^{52}$ ergs.

Such an energetic event will undoubtedly have consequences in astronomy. We believe that the explosive transition of a NS to a QS, a QN, is the mechanism powering the double humped light curve of SLSNe. Our model assumes the presence of a QN inside a massive binary system or in a WO type SN remnant and has successfully reproduced the light curves of over half a dozen SLSNe. Furthermore we have simulated the conditions of a QN site and the hot temperatures along with the rich neutron density expose the QN as a preferred site for the r-process.

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