IOP Conf. Series: Journal of Physics: Conf. Series **861** (2017) 012002 doi:10.1088/1742-6596/861/1/012002

# Looking for extra dimensions in compact stars

**Germán Lugones[1] and José D V Arbañil[2]**

[1]Centro de Ciências Naturais e Humanas, Universidade Federal do ABC, Avenida dos Estados 5001, 09210-580 Santo André, São Paulo, Brazil.
[2]Departamento de Ciencias, Universidad Privada del Norte, Av. Alfredo Mendiola 6062 Urb. Los Olivos, Lima, Perú.

E-mail: [1]german.lugones@ufabc.edu.br

E-mail: [2]jose.arbanil@upn.pe

**Abstract.** The properties of spherically symmetric static compact stars are studied in the Randall-Sundrum II type braneworld model assuming that the spacetime outside the star is described by a Schwarzschild metric. The integration of the stellar structure equations employing the so called causal limit equation of state (EoS) shows that the equilibrium solutions can violate the general relativistic causal limit. An analysis of the properties of hadronic and strange quark stars using standard EoSs confirm the same result: there is a branch in the mass-radius diagram that shows the typical behaviour found within the frame of General Relativity and another branch of stars that are supported against collapse by the nonlocal effects of the bulk on the brane. Stars belonging to the new branch can violate the general relativistic causal limit, may have an arbitrarily large mass, and are stable under small radial perturbations. If they exist in Nature, these objects could be hidden among the population of black hole candidates. The future observation of compact stars with masses and radii falling above the causal limit of General Relativity but below the Schwarzschild limit maybe a promising astrophysical evidence for the existence of extra dimensions.

## 1. Introduction

Braneworld models represent the universe as a three-dimensional brane where elementary particles live embedded in a higher-dimensional spacetime called the bulk, only accessed by gravity [1]. Within this framework, astrophysical and cosmological models can be constructed where the gravitational effect of extra-dimensions can be explored. In Randall-Sundrum models, ultraviolet modifications to General Relativity are introduced such that significant deviations from Einstein's theory occur at very high energies, e.g. in the early universe, in gravitational collapse and in compact objects.

In the Randall-Sundrum model, our universe is a brane embedded in one extra dimension (the bulk) which is a portion of a 5D anti-de Sitter spacetime ($\mathrm{AdS}_5$); i.e. the extra dimension is curved or warped rather than flat. At low energies, gravity has an exponentially suppressed tail into the extra dimension due to a negative bulk cosmological constant, $\Lambda_5 = -6/\ell^2$ where $\ell$ is the curvature radius of $\mathrm{AdS}_5$. The brane gravitates with self-gravity in the form of a brane tension $\lambda$, where $\lambda = 3M_p^2/(4\pi\ell^2)$ and $M_p^2 = M_5^3\ell$. On the brane, the negative $\Lambda_5$ is counterbalanced by the positive brane tension $\lambda$.

The Einstein's field equation takes the conventional form but with an effective energy-

IOP Conf. Series: Journal of Physics: Conf. Series **861** (2017) 012002 doi:10.1088/1742-6596/861/1/012002

momentum tensor $T^{\rm eff}_{\mu\nu}$, i.e., it reads:

$$G_{\mu\nu} = 8\pi G\, T^{\rm eff}_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the usual Einstein field tensor, and we consider $c = 1$.

The effective energy-momentum tensor has the form [2]

$$T^{\rm eff}_{\mu\nu} = T_{\mu\nu} + \frac{6}{\lambda}\, S_{\mu\nu} - \frac{1}{8\pi G}\, \mathcal{E}_{\mu\nu}\,, \tag{2}$$

where the first term contains the standard energy momentum tensor; e.g. for a perfect fluid we have $T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}$, where $p$ is the pressure of the fluid, $\rho$ is its energy density, $u^\mu$ is the four-velocity and $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the projection orthogonal to $u^\mu$. The second and third terms include modifications with respect to the standard 4D Einstein's field equation. The bulk correction includes a local term and a nonlocal one (second and third terms respectively) [1]. For a perfect fluid, the local contribution reads

$$S_{\mu\nu} = \frac{1}{12}\rho^2 u_\mu u_\nu + \frac{1}{12}\rho(\rho + 2p) h_{\mu\nu}. \tag{3}$$

The nonlocal contribution for static spherical symmetry reads:

$$\mathcal{E}_{\mu\nu} = -\frac{6}{8\pi G \lambda}\left[\mathcal{U} u_\mu u_\nu + \mathcal{P} r_\mu r_\nu + \frac{(\mathcal{U} - \mathcal{P})}{3} h_{\mu\nu}\right], \tag{4}$$

where $\mathcal{U}$ and $\mathcal{P}$ are respectively the nonlocal energy density and nonlocal pressure on the brane and $r_\mu$ is a unit radial vector. Notice that, as $\lambda \to \infty$, the bulk corrections vanish and General Relativity is recovered.

The braneworld generalization of the stellar structure equations for a static fluid distribution with spherical symmetry has been derived by Germani and Maartens [3], and read:

$$\begin{aligned}
\frac{dm}{dr} &= 4\pi r^2 \rho_{\rm eff}, && (5)\\
\frac{dp}{dr} &= -(\rho + p)\frac{d\phi}{dr}, && (6)\\
\frac{d\phi}{dr} &= \frac{Gm + 4\pi G r^3 \left(p_{\rm eff} + \frac{4\mathcal{P}}{(8\pi G)^2 \lambda}\right)}{r(r - 2Gm)}, && (7)\\
\frac{d\mathcal{U}}{dr} &= -(4\mathcal{U} + 2\mathcal{P})\frac{d\phi}{dr} - 2(4\pi G)^2(\rho + p)\frac{d\rho}{dr} \\
&\quad -2\frac{d\mathcal{P}}{dr} - \frac{6}{r}\mathcal{P}\,, && (8)
\end{aligned}$$

where

$$\rho^{\rm eff} = \rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{6\mathcal{U}}{(8\pi G)^2 \lambda}, \tag{9}$$

$$p^{\rm eff} = p + \frac{\rho}{2\lambda}(\rho + 2p) + \frac{2\mathcal{U}}{(8\pi G)^2 \lambda}. \tag{10}$$

To solve Eqs. (5)$-$(8) we need an equation of state $\rho = \rho(p)$ and a relation of the form $\mathcal{P} = \mathcal{P}(\mathcal{U})$ relating the nonlocal components ("dark" equation of state).

IOP Conf. Series: Journal of Physics: Conf. Series **861** (2017) 012002 doi:10.1088/1742-6596/861/1/012002

Two of the boundary conditions of the stellar structure equations on the brane are the same as for the standard General Relativistic equations. Specifically, at the center of the star ($r = 0$) the enclosed mass is zero:

$$m(r = 0) = 0, \tag{11}$$

and at the surface of the object the pressure vanishes:

$$p(R) = 0. \tag{12}$$

The remaining boundary condition is determined by the Israel-Darmois matching condition $[G_{\mu\nu} r^{\nu}]_{\Sigma} = 0$ at the surface of the object $\Sigma$, where $[f]_{\Sigma} \equiv f(R^{+}) - f(R^{-})$.

In brane-world models, the Schwarzschild solution is no longer the unique asymptotically flat vacuum exterior (for a discussion on the validity of the Schwarzschild exterior see Ref. [4] and references therein). In general, the exterior carries an imprint of nonlocal bulk graviton stresses and knowledge of the 5D Weyl tensor is needed as a minimum condition for uniqueness. Here, for simplicity, we focus on a class of models with a Schwarzschild exterior ($\mathcal{U}^{+} = \mathcal{P}^{+} = 0$). We also assume $\mathcal{P}^{-} = 0$, which is consistent with the isotropy of the physical pressure in the star. As a consequence, the interior must have nonvanishing nonlocal Weyl stresses ($\mathcal{U}^{-} \neq 0$). Therefore, the boundary condition for $\mathcal{U}$ at $r = R$ reads:

$$(4\pi G)^2 \rho^2(R) + \mathcal{U}^{-}(R) = 0. \tag{13}$$

Due to the latter boundary condition at the stellar surface, the numerical integration of the structure equations is less straightforward than for the standard structure equations in General Relativity. In the present case, a shooting method is used in order to match iteratively the boundary condition for $\mathcal{U}^{-}$ at $r = R$ (for more details see Ref. [4]).

## 2. The causal limit in braneworld models

The equation of state of neutron star matter can be determined with confidence up to $\sim 2\rho_{sat}$, being $\rho_{sat} \approx 151\,\mathrm{MeV/fm^3}$ the nuclear saturation density. For larger densities, the determination of the EoS depends strongly on the knowledge of strong interactions in a regime that cannot be reached experimentally. As a consequence, there is a large amount of high-density EoSs in the literature that incorporate several aspects that may play a crucial role at the inner core of the star, such as three-body forces, bosonic condensates, hyperonic degrees of freedom and quark matter [5, 6].

An important aspect of neutron stars within the frame of General Relativity, is that there exists a maximum gravitational mass above which there are no stable stellar configurations. The maximum mass exists no matter what the EoS, but its determination depends on a deep comprehension of the EoS up to several times $\rho_{sat}$. However, using the so called *causal limit EoS*, it is possible to circumvent the uncertainties related to the properties of high-density matter and obtain upper bounds to the maximum allowed mass of a neutron star [5, 6]. The causal limit EoS can be constructed by using a detailed EoS at density ranges where they can be safely regarded as accurate and imposing generic constraints at densities exceeding some fiducial density, e.g., subluminal sound velocity and thermodynamic stability (see e.g. [5, 6, 7]). Here, we adopt the well established Baym, Pethick, and Sutherland (BPS) EoS [8] at densities below a fiducial density $\rho_t$, and a causal equation of state (i.e. sound velocity = speed of light) $p = \rho - a$ above $\rho_t$ [6, 7]. Since both EoSs are matched at an energy density $\rho_t$ and a pressure $p_t$, the constant $a$ in the high density EoS is given by $a = \rho_t - p_t$, where $\rho_t$ and $p_t$ also fulfill the BPS EoS.

Varying the value of $a$ and integrating the stellar structure equations we obtain the results shown in Fig. 1. Within General Relativity, the causal limit EoS allows obtaining a straight line in the mass-radius diagram that connects the maximum mass stars for different values of

IOP Conf. Series: Journal of Physics: Conf. Series **861** (2017) 012002 doi:10.1088/1742-6596/861/1/012002

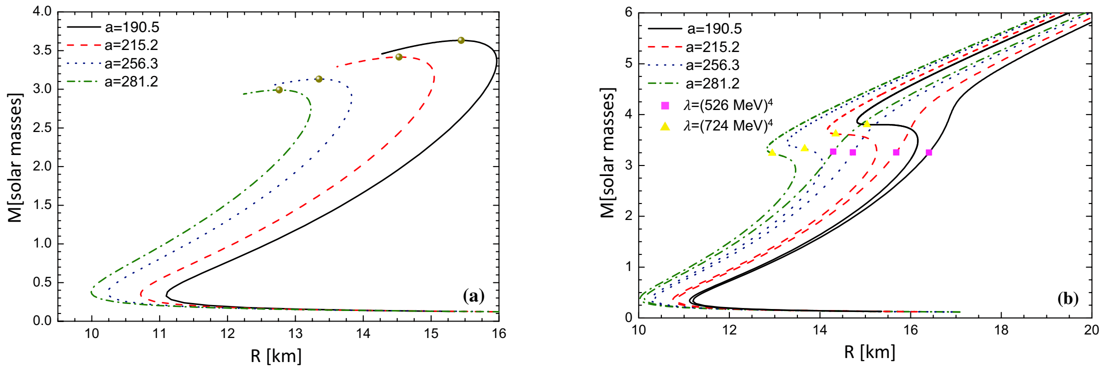


**Figure 1.** Mass-radius relationship for the causal limit EoS matched continuously with the BPS EoS in (a) General Relativity and (b) braneworld models. Both EoSs are matched at different fiducial densities that lead to different values of $a = \rho_f - p_f$. In the case of General Relativity, the dots over the curves indicate the maximum masses. For braneworld stars, the values of $\lambda$ lead to $M_5 = (\frac{4}{3}\pi\lambda M_p^2)^{1/6} \sim 2000\,\mathrm{TeV}$; i.e. larger than 10 TeV, in compatibility with LHC.

$a$. Above such line, known as *causal limit*, stellar configurations are not allowed. The forbidden region is given by [6, 5, 9, 10]:

$$M > 0.345R, \tag{14}$$

or, equivalently:

$$\left(\frac{M}{M_\odot}\right) > 0.234 \left(\frac{R}{\mathrm{km}}\right). \tag{15}$$

In the case of braneworld stars, we find that for small masses, the curves show the typical behavior found within the frame of General Relativity. Specifically, very small mass stars have very large radii, and as the mass increases above a few tenths of solar masses the radii fall within a range of few kilometers around $\sim$ 10 km. Nevertheless, for large mass objects, local high-energy effects as well as nonlocal corrections lead to significant deviations with respect to General Relativity. At around $1.5 - 2\,M_\odot$ the $M(R)$ curves bend anticlockwise as in the general relativistic case. However, instead of reaching a maximum mass as in General Relativity, the curves bend once more (clockwise) for larger masses and thereafter they increase roughly linearly (see Fig. 1).

A striking feature of this behavior is that once the $M - R$ curves bend clockwise they may fall above the causal limit obtained within General Relativity (c.f. Eqs. (15) $-$ (14)). It can also be checked that as the masses and radii increase, the curves tend asymptotically to the Schwarzschild limit $M = 2R$.

Since the $M - R$ curves for the causal EoS approach asymptotically the line $M = 2R$, but do not go beyond it, the Schwarzschild limit $M = 2R$ is a good representation of the causal limit in the braneworld model. In other words, the equilibrium solutions found in the braneworld can violate the limit of causality for General Relativity (Eqs. (15) $-$ (14)) and, for sufficiently large mass, can occupy the region between the straight lines shown in Fig. 2.

## 3. Hadronic and strange stars in braneworld models

Now, we consider two representative equations of state for hadronic and strange quark stars. To describe hadronic matter, we use a relativistic mean-field model which is widely used in stellar

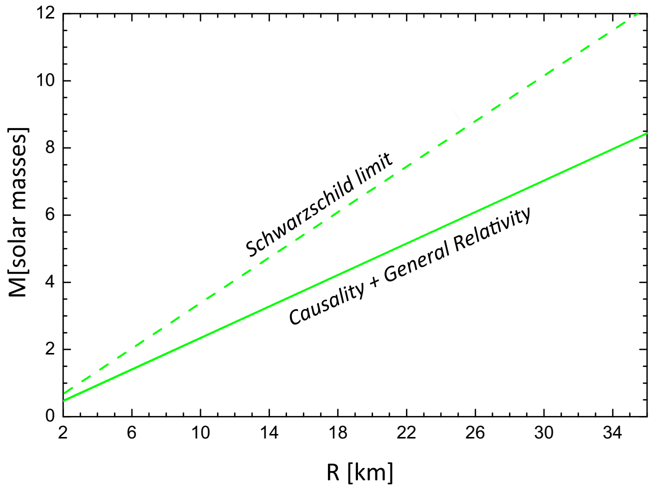


**Figure 2.** The causal limit for General Relativity and the Schwarzschild limit $M = 2R$. In the braneworld model, static stellar configurations fulfilling a causal EoS (sound velocity < speed of light) can occupy the region between both straight lines.

structure calculations within General Relativity. We adopt the following standard Lagrangian for matter composed by nucleons, hyperons and electrons [5, 6],

$$\begin{aligned}\mathcal{L}_H &= \sum_B \bar{\psi}_B[\gamma_\mu(i\partial^\mu - g_{\omega B}\omega^\mu - \frac{1}{2}g_{\rho B}\vec{\tau}.\vec{\rho}^\mu) \\ &- (m_B - g_{\sigma B}\sigma)]\psi_B + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{3}bm_n(g_\sigma\sigma)^3 \\ &- \frac{1}{4}c(g_\sigma\sigma)^4 - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{\rho}_{\mu\nu}.\vec{\rho}^{\mu\nu} \\ &+ \frac{1}{2}m_\rho^2\vec{\rho}_\mu.\vec{\rho}^\mu + \sum_L \bar{\psi}_L[i\gamma_\mu\partial^\mu - m_L]\psi_L. \end{aligned} \tag{16}$$

Leptons L are treated as non-interacting and baryons $B$ are coupled to the scalar meson $\sigma$, the isoscalar-vector meson $\omega_\mu$ and the isovector-vector meson $\rho_\mu$. The five constants in the model are fitted to the bulk properties of nuclear matter; here we use the parametrization GM1 [11]. The explicit form of the EoS obtained from the above Lagrangian as well as the coupling constants used for the GM1 parametrization can be found in [12, 13, 14, 15] and references therein. At low densities we use the Baym, Pethick and Sutherland model [8].

For quark matter we use the MIT bag model with zero strong coupling constant and massless quarks. The equation of state adopts the simple form $\rho = 3p + 4B$, where $B$ is the bag constant. Witten [16] conjectured that, at zero pressure and temperature, three flavor quark matter may have an energy per baryon smaller that ordinary nuclei. This would make strange quark matter the true ground state of strongly interacting matter and would lead to the existence of strange quark stars, i.e. stellar objects completely composed by strange quark matter [17, 18]. Within the MIT bag model for massless quarks and zero strong coupling constant, the Witten hypothesis is verified if the bag constant is in the range $57\,\mathrm{MeV/fm^3} < B < 94\,\mathrm{MeV/fm^3}$. In this paper we consider $B = 60\,\mathrm{MeV/fm^3}$.

In Fig. 3 we show the mass-radius relationship for strange quark stars and for hadronic stars. We also include the causal limit in General Relativity and the Schwarzschild limit $M = 2R$. For small masses ($< 1.5 - 2M_\odot$), the curves show the typical behavior found within the frame

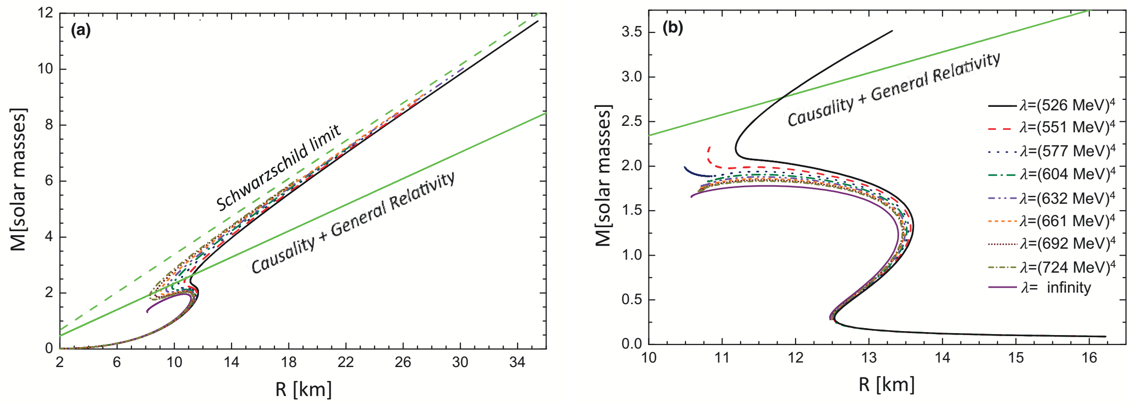


**Figure 3.** Mass-radius relationship for several values of the brane tension $\lambda$: (a) strange quark stars and (b) hadronic stars.

of General Relativity. Specifically, very small mass hadronic stars have very large radii, while strange stars follow roughly $M \propto R^3$. For large mass objects, local high-energy effects as well as nonlocal corrections lead to the same deviations with respect to General Relativity that were found in the previous section for the causal EoS. Instead of reaching a maximum mass as in general relativity, the $M(R)$ curves have a new branch that approaches asymptotically the Schwarzschild limit $M = 2R$. In some cases there is a local maximum in the $M(R)$ curves.

## 4. Stability of stellar configurations

In the previous sections we showed that in braneworld models there is a new branch of stellar configurations that is not present within General Relativity. Since only compact objects in stable equilibrium are acceptable from the astrophysical point of view, we will check here the stability of these models. A well known static criterion that is widely used in the literature states that a necessary condition for a model to be stable is that its mass $M$ increases with growing central density, i.e.

$$\frac{dM}{d\rho_c} > 0. \tag{17}$$

The latter is a necessary but not sufficient condition.

Here, we employ a sufficient criterion which enables one to determine the precise number of unstable normal radial modes using the $M(R)$ curve [5, 19]. According to such criterion, at each critical point of the $M(R)$ curve (local maxima or minima) one and only one normal radial mode changes its stability, from stable to unstable or vice versa. There are no changes of stability associated with radial pulsations at other points of the $M(R)$ curves. Moreover, one mode becomes unstable (stable) if and only if the $M(R)$ curve bends counterclockwise (clockwise) at the critical point.

In our previous results we found two qualitatively different types of $M(R)$ curves. One type presents one local maximum and one local minimum in $M(R)$ (and in $M(\rho_c)$ as well). The other one has no critical points. These two types are represented separately in Fig. 4, where we show the $M(R)$ and $M(\rho_c)$ curves for strange quark stars for two different values of $\lambda$ (for simplicity we do not show hadronic stars because the stability analysis is completely equivalent, as we shall see below).

In order to analyze the stellar stability using the above criterion, we assume that the low mass branch (up to $< 1.5 - 2M_\odot$) of the $M(R)$ curves is stable for all radial modes, as it is in the general relativistic case. For the curves with two critical points (panels (c) and (d) of Fig. 4),

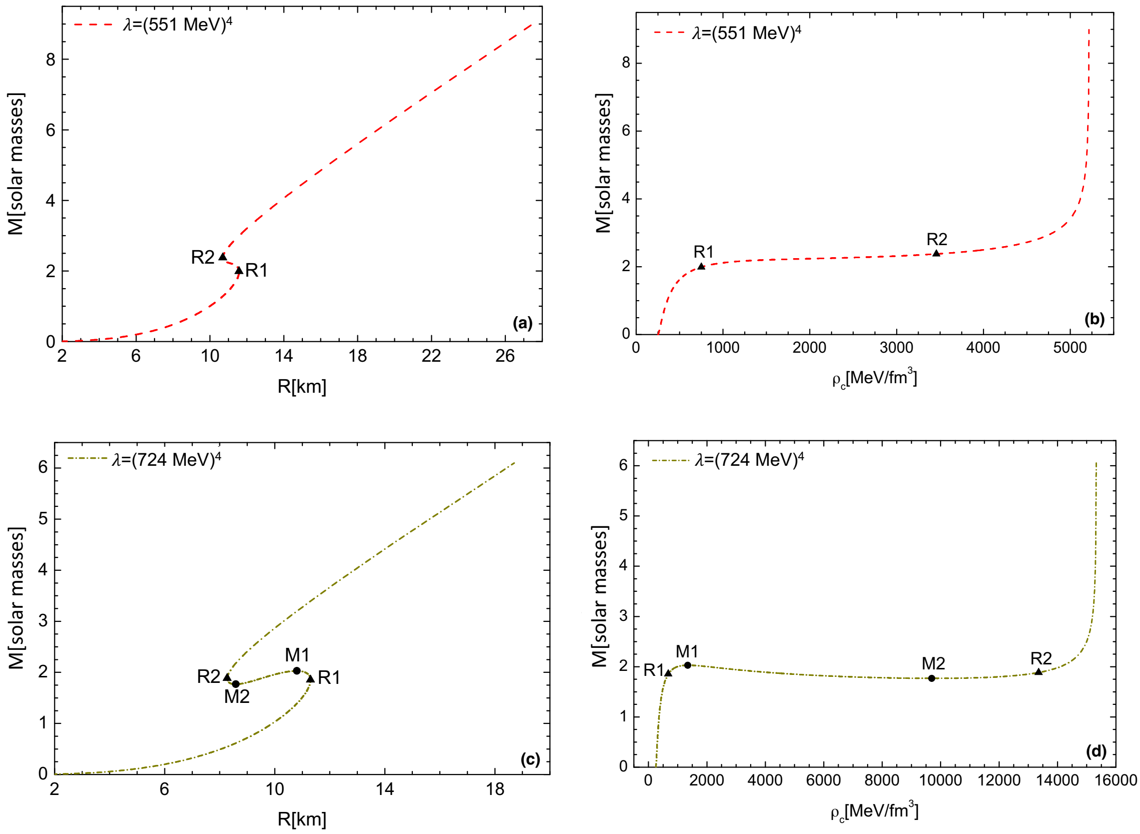


**Figure 4.** Analysis of stellar stability in braneworld models based on the critical points in the $M$ versus $R$ diagram. The curves represent strange quark stars but the stability analysis for hadronic stars is completely equivalent. Figures (a) and (b) correspond to a value of the brane tension, $\lambda = (551\,\mathrm{MeV})^4$, that results in no critical points. Figures (c) and (d) correspond to a different value of the brane tension, $\lambda = (724\,\mathrm{MeV})^4$, that results in a local maximum and a local minimum in both the $M(R)$ and $M(\rho_c)$ curves.

the $M(R)$ curve bends counterclockwise at the local maximum and the fundamental oscillation mode becomes unstable. However, at the local minimum the fundamental mode becomes stable again because the curve bends clockwise there. Beyond the local minimum there are no more critical points and all the radial modes remain stable. In the case without critical points (panels (a) and (b) of Fig. 4), the whole sequence remains stable for all radial modes provided that the low mass configurations are stable. Thus, we can conclude that the branches that approach asymptotically to the Schwarzschild limit are always stable under small radial perturbations, and therefore, stellar configurations of arbitrarily large mass would be allowed within braneworld models.

The previous analysis concerns the stability of non-rotating stars. However, if the star rotates, there is a handful of perturbations to be considered for a full stability analysis. For example, rapidly rotating neutron stars can be unstable to the gravitational-wave-driven Chandrasekhar, Friedman and Schutz (CFS) mechanism [20, 21]. These instabilities deserve further study within braneworld models.

## 5. Conclusions

In this work we have studied the structure of spherically symmetric static compact stars in the Randall-Sundrum II type braneworld model assuming that the spacetime outside the star is described by a Schwarzschild metric. We obtain the low mass branch of compact star configurations already known from general relativistic calculations, but we also find a new branch that approaches asymptotically to the Schwarzschild limit which is always stable under small radial perturbations. As a consequence, stellar configurations of arbitrarily large mass, supported against collapse by the nonlocal effects of the bulk on the brane, are allowed within braneworld models. It is worth emphasizing that black holes are still possible within the here studied braneworld models. Moreover, the stellar configurations that asymptotically approach to the Schwarzschild limit are expected to be stable under small perturbations, but not necessarily under large ones. Therefore, a very large mass braneworld compact star could collapse into a black hole if strongly perturbed in a catastrophic astrophysical event, e.g. in a binary stellar merging.

From the astrophysical point of view, the existence of more than 20 compact objects, whose masses are in the range $M = 5 - 30 M_\odot$, has been confirmed via dynamical observations. These objects are usually assumed to be stellar mass black holes but, strictly speaking, they are only black hole candidates. They are located in X-ray binary systems, where the X-rays are produced by gas that flows from the companion star on to the compact object via an accretion disk. The strongest argument for identifying them as black holes is that they are sufficiently massive and compact, and observations could not be matched with any object in stable equilibrium other than a black hole. However, this by itself does not prove that these objects are true black holes, defined as objects with event horizons [22]. At least some black hole candidates could, in principle, be exotic objects made of some kind of unusual matter that enables them to have a surface (no horizon), despite their extreme compactness. The hypothetic brane world stars discussed in this work fall in this category, and at least in principle, they could be hidden among the population of black hole candidates. While a number of tests have been devised to check whether black hole candidates have a surface, at present all evidence is still indirect.

As explained above, the very existence of the new branch of large mass objects can be tested through the observation of masses and radii of compact stars falling above the causal limit of General Relativity but below the Schwarzschild limit. If found, such objects could be an astrophysical manifestation of the existence of extra dimensions.

## Acknowledgments

G L acknowledges the Brazilian agencies FAPESP and CNPq for financial support.

## References

[1] Maartens R and Koyama K 2010 *Living Rev. Relativity* **13** 5
[2] Shiromizu T, Maeda K-I and Sasaki M 2000 *Phys. Rev. D* **62** 024012
[3] Germani C and Maartens R 2001 *Phys. Rev. D* **64** 124010
[4] Lugones G and Arbañil J D V 2017 *Phys. Rev. D* **95** 064022
[5] Haensel P, Potekhin A Y and Yakovlev D G 2007 *Neutron Stars 1: Equation of State and Structure* Springer Verlag, New York
[6] Glendenning N K 2000 *Compact Stars: Nuclear Physics, Particle Physics and General Relativity* 2nd ed. Springer Verlag, New York
[7] Rhoades C E and Ruffini R 1974 *Phys. Rev. Lett.* **32** 324
[8] Baym G, Pethick C and Sutherland P 1971 *Astrophys. J.* **299** 170
[9] Lattimer J M and Prakash M 2004 *Science* **304** 536
[10] Lattimer J M 2012 *Annu. Rev. Nucl. Part. Sci* **62** 485
[11] Glendenning N K and Moszkowski S A *Phys. Rev. Lett.* **67** 2414
[12] Lugones G, do Carmo T A S, Grunfeld A G and Scoccola N N 2010 *Phys. Rev. D* **81** 85012
[13] do Carmo T A S, Lugones G and Grunfeld A G 2013 *J. Phys. G: Nucl. Part. Phys.* **40** 035201

IOP Conf. Series: Journal of Physics: Conf. Series **861** (2017) 012002 doi:10.1088/1742-6596/861/1/012002

[14] Lugones G, Grunfeld A G, Scoccola N N and Villavicencio C 2009 *Phys. Rev. D* **80**
[15] Lugones G and Grunfeld A G *Phys. Rev. D* 2011 **84** 85003
[16] Witten E 1984 *Phys. Rev. D* **30** 272
[17] Lugones G *Eur. Phys. J. A* 2016 **52** 53
[18] Benvenuto O G and Lugones G 1998 *Int.J.Mod.Phys. D* **7** 29
[19] Harrison B K, Thorne K S, Wakano M and Wheeler J A 1965 *Gravitation Theory and Gravitational Collapse*, University of Chicago Press, Chicago
[20] Chandrasekhar S 1970 *Astrophys. J.* **161** 561
[21] Friedman J L and Schutz B F 1978 *Astrophys. J.* **222** 281
[22] Narayan R and McClintock J E 2013 arXiv:1312.6698 [astro-ph.HE]