

# Dynamics of the process boom machine working equipment under the real law of the hydraulic distributor electric spool control

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**Abstract.** Analytical calculation methods of dynamic processes of the self-propelled boom hydraulic machines working equipment are more preferable in comparison with numerical methods. The analytical research method of dynamic processes of the boom hydraulic machines working equipment by means of differential equations of acceleration and braking of the working equipment is proposed. The real control law of a hydraulic distributor electric spool is considered containing the linear law of the electric spool activation and stepped law of the electric spool deactivation. Dependences of dynamic processes of the working equipment on reduced mass, stiffness of hydraulic power cylinder, viscous drag coefficient, piston acceleration, pressure in hydraulic cylinders, inertia force are obtained. Definite recommendations relative to the reduction of dynamic loads, appearing during the working equipment control are considered as the research result. The nature and rate of parameter variations of the speed and piston acceleration dynamic process depend on the law of the ports opening and closure of the hydraulic distributor electric spool. Dynamic loads in the working equipment are decreased during a smooth linear activation of the hydraulic distributor electric spool.

## 1. Introduction

The improvement and complication of engineering systems stimulate the development and improvement of the differential equation solution methods. During the dynamic processes research the differential high-order equations are substituted by equivalent systems of differential equations of first and second order, as the solution methods of these equations are well-proven. Papers [1, 2, 3] are devoted to the solution methods improvement of the differential equation systems of first and second order.

In paper [1] the authors M. Abas, A. Libošvárová improve the solution of the linear differential equations systems with constant coefficients in the field of mathematical modeling of dynamic systems. New numerical solutions of nonhomogeneous linear differential equations with several variables are developed by the authors Y. Kida, T. Kida in paper [2]. In paper [3] the authors M. Greitans, E. Hermanis, A. Greitane consider linear differential equations of first and second order. Second-order equations contain constant coefficients, in paper [3] first-order equations contain time-varying coefficients.



In paper [4] the authors G. Reissig, H. Boche, P.I. Barton improve the initial conditions determination methods for the linear differential equations systems of first order. In paper [5] the authors X. Lin, K. Lu, H. Wei, Y. Bai examine the problems of solvability of time-varying linear differential equations of second order.

In paper [6] the authors M. H. Eghlidi, K. Mehrany, B. Rashidian propose the general solution of the linear differential equations by means of the differential method.

In paper [7] the authors Y. Zhang, J. Sun examine the stability of the impulse linear differential equations with delay.

In paper [8] the authors J. Zhou, Q. Wu discuss the problems of exponential stability of pulse delays in the linear differential equations.

For the mathematical description of engineering systems the differential equations of second order are used in the form of differential equations of free, damped or forced oscillations of mechanical systems. More complex dynamic processes are described by the linear differential equations of third and higher orders.

The boom bucket machines working equipment dynamics is described in this article by the differential equation of third order, which is represented by differential equation of hydraulic power cylinder [9]

$$m\ddot{s} + \mathfrak{g}\dot{s} + C\dot{s} = Kx. \quad (1)$$

Equation (1) is a differential equation of hydraulic power cylinder, where constant coefficients have a definite physical content:  $m$  – mass of the working equipment reduced to the piston  $\mathfrak{g}$  – viscous resistance coefficient;  $C$  – stiffness coefficient;  $s$  – movement of the hydraulic cylinder piston of the working equipment;  $x$  – ports opening size of the hydraulic distributor electric spool.

The right side of equation (1) may be different and represents the ports opening size of the hydraulic distributor electric spool.

The derivation of the hydraulic power cylinder differential equation is performed by V.N. Tarasov in paper [9]. According to physics the equation of third order is equivalent to the system of two equations: second-order equation (oscillatory link) and first-order equation in the form of an integrating factor.

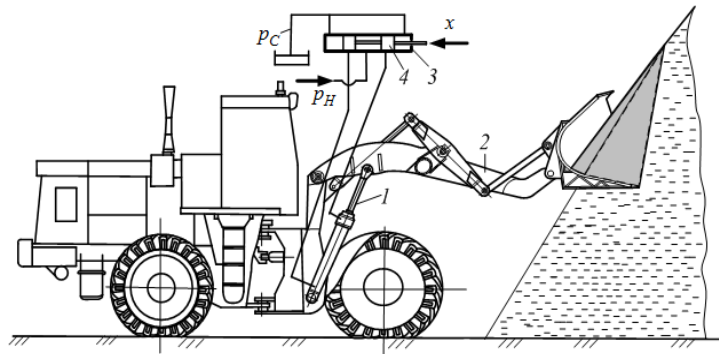
The numerical solution of differential equation (1) is performed in paper [10] using the software package MATLAB, where third-order differential equation is represented in the form of the differential equations system of first order

$$\dot{S}_1 = S_2; \quad \dot{S}_2 = S_3; \quad \dot{S}_3 = -2nS_3 - \omega^2 x_2 + d(t).$$

The numerical solution has the indicated peculiarities, that's why this article proposes the direct analytical solution of differential equation (1). The direct analytical solution of the hydraulic power cylinder differential equation is considered as third-order differential equation with the right side in the form of the ports opening linear size of the hydraulic distributor electric spool.

Process boom bucket machines possess multifunctional working equipment for usage in mining industry construction, agriculture and other spheres.

Figure 1 shows as an example a front-end hydraulic loader, which hydraulic power cylinders of the working equipment together with the hydraulic distributor and hydraulic pump receive masses reduced to the piston, possess elastic characteristics and damping properties.



**Figure 1.** Front-end loader: 1 – hydraulic power control cylinders; 2 – working equipment; 3 – hydraulic distributor of working equipment; 4 – electric spool;  $p_H$ ,  $p_C$  – working fluid pressure during delivery and discharge to the tank, respectively.

Modern process bucket machines are in demand and form size ranges which include small-size and capacity machines, heavy machines and super heavy models of machines.

Small-size and medium-size machines are sufficiently studied and well-trying to a greater extent.

## 2. Task definition

The article sets the task to develop the analytical solution of third-order differential equation for the dynamics research of the hydraulic machines working equipment transient processes.

Regarding high-capacity machines and super models the dynamic characteristics and performance features of dynamic parameters among which the most important ones are represented by inertia masses of the working equipment reduced to the hydraulic cylinder piston, stiffness of the hydraulic cylinder, viscous resistance, pressure in the hydraulic system etc. are not sufficiently studied.

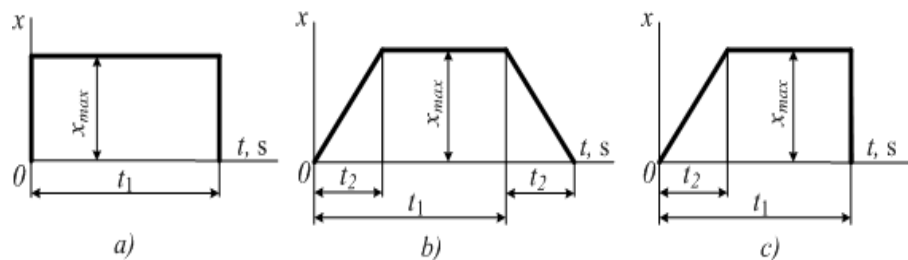
The dignity of modern hydraulic power cylinders lies in a quick acceleration of the working equipment during the hydraulic distributor electric spool activation and rigid fixation of working equipment positions in the technological process.

## 3. Theory

The linear differential equation of third order contains the right side in the form of the coordinate  $x$ , characterizing the ports opening size of the hydraulic distributor electric spool [9, 10, 11]. At present the solution of the differential equations is usually performed by numerical methods [10]. But the proposed analytical solution is simpler and more effective.

The dynamic transient processes of the parameters variation: movements of the piston  $s$ , piston speed  $\dot{s}$ , piston acceleration  $\ddot{s}$  depend on the hydraulic distributor electric spool activation modes  $x=f(t)$ .

Figure 2 shows the basic kinds of the distributor control laws: stepped (step-wise); linear; real laws of the hydraulic distributor electric spool control.



**Figure 2.** Control laws of the hydraulic distributor electric spool: a) stepped law  $t = t_1$ ; b) linear law  $t = t_2$ ; c) real law.

The linear diagram of the hydraulic distributor electric spool control (Figure 2,b) is as follows: electric spool activation mode is as follows

$$x = x_{\max} \frac{t}{t_2} \quad \text{at } 0 \leq t \leq t_2; \quad (2)$$

electric spool activated status mode

$$x = x_{\max} \quad \text{at } t_2 < t \leq t_1; \quad (3)$$

electric spool deactivation mode

$$x = x_{\max} \left( 1 - \frac{t}{t_2} \right) \quad \text{at } t_1 \leq t \leq (t_1 + t_2); \quad (4)$$

locked status mode of the hydraulic power cylinder chambers

$$x = 0 \quad \text{at } t > (t_1 + t_2). \quad (5)$$

Figure 2,c shows that the linear real activation mode of the electric spool is complemented with the real stepped deactivation of the hydraulic distributor electric spool.

These four considered statuses of the hydraulic distributor electric spool characterize sequential cases of the working equipment dynamic transient processes appearance and their attenuation. By means of replacing  $\ddot{s} = \ddot{V}$ ,  $\dot{s} = \dot{V}$ ,  $s = V$  differential equation (1) of third order is reduced to differential equation of second order with the right side [9, 11]

$$\ddot{V} + 2n\dot{V} + \omega^2 V = \frac{K}{m} x, \quad (6)$$

where  $2n = \mathfrak{G}/m$ ;  $n$  – resistance coefficient;  $\omega$  – natural vibration frequency,  $\omega = \sqrt{C/m}$ .

The dynamic study of the process machines working equipment under the linear law of the electric spool activation is fulfilled using differential equation

$$\ddot{V} + 2n\dot{V} + \omega^2 V = \frac{K}{m} x_{\max} \frac{t}{t_2}, \quad (7)$$

where  $K = \frac{CV_{\text{cont}2}}{x_{\max}}$ ;  $n = \beta\omega$ .

The solution of differential equation (7) is considered as the sum of two solutions  $V = V_1 + V_2$ , where  $V_1$  – solution of the differential equation (7) without the right side;  $V_2$  – particular solution of general equation (7),  $V_2 = C_3 + C_4 t$ .

The analytical solution of differential equation (7) in case of the linear activation of the electric spool is as follows

$$V = e^{-nt} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t) + C_3 + C_4 t; \quad (8)$$

$$\dot{V} = e^{-nt} [(C_2 \omega_1 - C_1 n) \cos \omega_1 t - (C_2 n + C_1 \omega_1) \sin \omega_1 t] + C_4, \quad (9)$$

where  $\omega_1$  – circular frequency of damping oscillations,  $\omega_1 = \sqrt{\omega^2 - n^2}$ .

The initial conditions of differential equation (7) are as follows: at  $t=0$   $V_0 = 0$ ;  $\dot{V}_0 = 0$ . Constants of integration in obtained expressions (8), (9) are determined proceeding from the initial conditions as per expressions:

$$C_1 = 2 \frac{\beta V_{con}}{\omega t_2}; C_2 = \frac{V_{con}}{\omega_1 t_2} (2\beta^2 - 1); C_3 = -\frac{2\beta V_{con}}{\omega t_2}; C_4 = \frac{V_{con}}{t_2}, \quad (10)$$

where  $\beta$  – damping coefficient of the hydraulic power cylinder,  $\beta = n/\omega$ ;  $V_{con}$  – steady-state value of the piston speed after the oscillating process damping.

The electric spool activation operation is performed during the period of time  $t = t_2$ .

During this period the speed and acceleration are varied according to formulas (8), (9) during the time period of the electric spool ports opening.

After the electric spool activation the right side of the differential equation becomes constant and is as follows

$$\ddot{V} + 2n\dot{V} + \omega^2 V = \frac{K}{m} x_{max}. \quad (11)$$

The initial conditions in differential equation (11), characterizing the previous process conditions are as follows: at  $t=0$   $V = V_0$ ;  $\dot{V} = \dot{V}_0$ . The process of the working equipment movement to a new position is performed as a damping oscillation process of the speed transition to a steady-state value with completely open ports of the electric spool during the period of time  $t = t_1 - t_2$  of the working equipment movement.

After obtaining a required movement of the working equipment the electric spool linear deactivation operation is performed using the differential equation with the right side

$$\ddot{V} + 2n\dot{V} + \omega^2 V = \frac{Kx_{max}}{m} \left(1 - \frac{t}{t_2}\right). \quad (12)$$

The constant values of differential equation (12) are as follows

$$\omega^2 = \frac{C}{m}; K = \frac{CV_{con}}{x_{max}}; n = \beta\omega.$$

The initial conditions of equation (12) are as follows: at  $t=0$   $V = V_0$ ;  $\dot{V} = \dot{V}_0$ . We take the electric spool deactivation time as  $t = t_2$ .

The solution of differential equation (12) is as follows

$$V = e^{-nt} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t) + C_3 + C_4 t; \quad (13)$$

$$\dot{V} = e^{-nt} [(C_2 \omega_1 - C_1 n) \cos \omega_1 t - (C_2 n + C_1 \omega_1) \sin \omega_1 t] + C_4. \quad (14)$$

The obtained constants of integration correspond to the adopted initial conditions

$$C_1 = V_0 - V_{con} (1 + 2 \frac{\beta}{t_2 \omega}); C_2 = \frac{\dot{V}_0 + C_1 n - C_4}{\omega_1}; C_3 = V_{con} (1 + 2 \frac{\beta}{t_2 \omega}); C_4 = -\frac{V_{con}}{t_2}. \quad (15)$$

The working equipment transient process with the electric spool linear deactivation according to equations (13), (14) is performed during the time period  $t = t_2$ .

At the moment of the complete closure of the electric spool ports the process of the working equipment oscillations damping is started which is simulated by the differential equation with zero right side:

$$\ddot{V} + 2n\dot{V} + \omega^2 V = 0. \quad (16)$$

The solution of differential equation (16) with the deactivated electric spool is as follows

$$V = e^{-nt} (C_1 \cos \omega_1 t + C_2 \sin \omega_1 t); \quad (17)$$

$$\dot{V} = e^{-nt} [(C_2 \omega_1 - C_1 n) \cos \omega_1 t - (C_2 n + C_1 \omega_1) \sin \omega_1 t], \quad (18)$$

where  $C_1, C_2$  – constants of integration which are as follows

$$C_1 = V_0; \quad C_2 = \frac{\dot{V}_0 + nV_0}{\omega_1}. \quad (19)$$

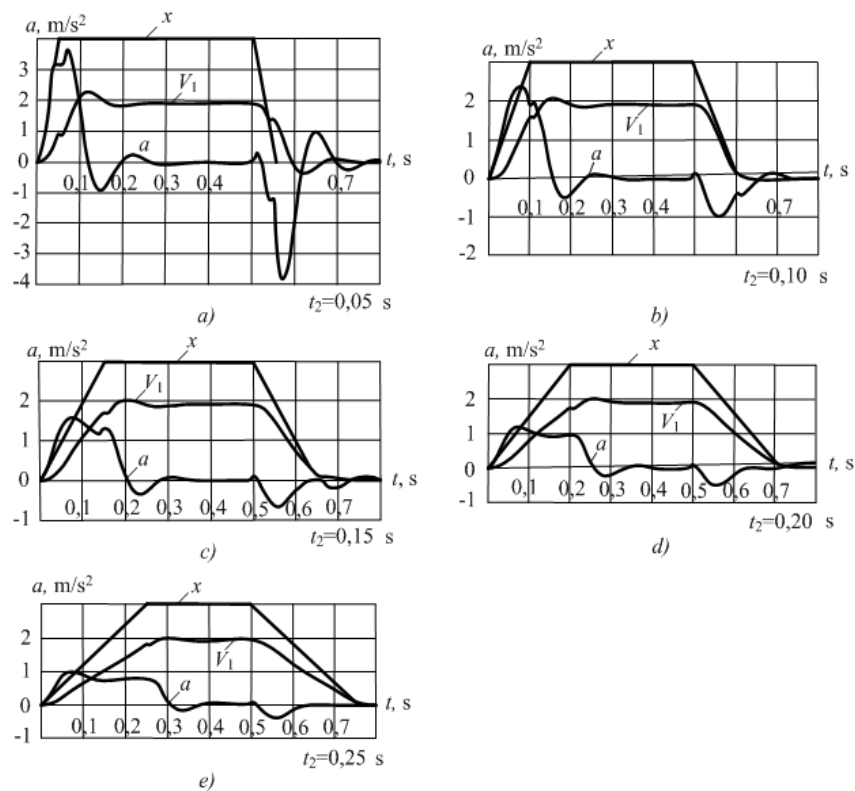
Analytical solutions of differentials equations of the working equipment movement are recorded for the first time and they have allowed to fulfill the researches of the hydraulic drive parameters influence on the dynamics of transient processes with the linear control of the hydraulic distributor electric spool.

#### 4. Research results

The nature and rate of parameter variations of the speed and piston acceleration dynamic process depend on the law of the ports opening and closure of the hydraulic distributor electric spool.

Figure 3 shows the transient processes research results of the hydraulic working equipment of a front-end loader with the loading capacity of 3 tons under the linear control law of the hydraulic distributor electric spool by means of varying the activation and deactivation time of the electric spool  $t_2 = \text{var}$ .

Figure 3,a shows that with the electric spool activation time  $t_2 = 0.05\text{s}$  the transient processes of the acceleration  $a$  and speed  $V_1$  possess the oscillatory nature with overshoot.



**Figure 3.** Transient processes of the hydraulic cylinder piston during the variation of the electric spool activation time for the loader with the loading capacity of 3 tons.

## 5. Discussion of results

The electric spool activation time increase leads to the reduction of oscillability and parameters overshoot. At a maximum value of the electric spool activation time  $t_2 = 0.25\text{s}$  the acceleration  $a$  reduced to  $a = 0.8\text{ m/s}^2$ , at the same time the speed overshoot  $V_1$  practically disappeared (Figure 3,e).

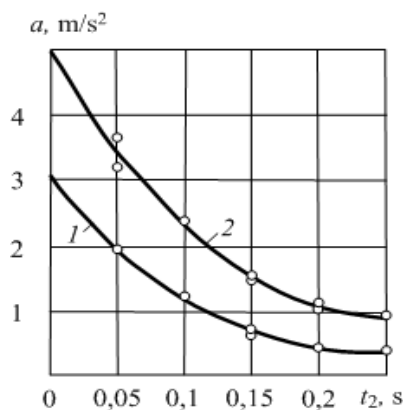
That's why the electric spool activation time increase, i.e. a smooth nonstepped adjustment is the reserve of the transient processes dynamics adjustment smoothness increase of the bucket machines working equipment with hydraulic power cylinders.

That's why an important conclusion arises that the electric spool activation time increase, i.e. a smooth nonstepped adjustment is the reserve of the transient processes dynamics adjustment smoothness increase of the bucket machines working equipment with hydraulic power cylinders. Another important conclusion proceeding from the submitted results of the research represents the conclusion that with the electric spool activation time  $t_2 > 0.2\text{s}$  the adjustment time  $t_p$  practically coincides with the electric spool linear activation time  $t_p = t_2$ .

This means that in case when  $t_p = t_2$ , after the electric spool activation the acceleration  $a \rightarrow 0$ , and the speed  $V_1$  acquires the steady-state value  $V_1 = V_{con}$  practically without oscillations.

Figure 4 shows dependences of the acceleration  $a$  of the reduced mass on the opening time of the electric spool ports  $t_2$  for the boom machine with the lifting capacity of  $Q = 3\text{ t}$ .

Curve 1 corresponds to the piston speed  $V_1 = 0.1\text{ m/s}$ , curve 2 – to the piston speed  $V_1 = 0.2\text{ m/s}$ .



**Figure 4.** Dependence of acceleration on the electric spool activation time.

The inertia force at the piston is determined by the formula

$$\Phi_d = ma.$$

The dynamic additional pressure excited in hydraulic cylinders by the force  $\Phi_d$ , is determined by the formula

$$p_d = \frac{\Phi_d}{n_c(\pi D_c^2/4)},$$

where  $n_c$  – number of hydraulic power cylinders,  $n_c = 2$ ;  $D_c$  – diameter of hydraulic power cylinder,  $D_c = 0.125\text{ m}$ .

The indicated pressure  $p_d$  is a pulse pressure, which operates during the time period  $t = 0.1\text{s}$  (see Figure 3,a).

For the loader with the lifting capacity of  $Q = 3\text{ t}$ , reduced mass of which is equal to  $m = 80270\text{ kg}$ ,



acceleration  $a_{\max}=3 \text{ m/s}^2$  (see Figure 4), inertia force and pressure have the real values  $\Phi_d=240810 \text{ N}$ ,  $p_d=9.811 \text{ MPa}$ .

## 6. Summary and conclusion

1. The dynamics of the modern boom machines working equipment, simulated by third-order linear differential equation with the right side, depends on the electric spool ports opening size and nature.
2. It's proposed to describe the dynamics of the boom bucket machines working equipment using the analytical method by means of the differential equation depression of the working equipment hydraulic power cylinders.
3. Control processes of the working equipment acceleration and deceleration dynamics are adjusted by the right side of the differential equation. The damped oscillating dynamic process of the working equipment is simulated by the differential equation with zero right side.

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