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Analytical study of the cord path character in rubber-cord flexible elements

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Abstract. The mathematical model of the cord path character in rubber-cord flexible elements is investigated. The applicability boundaries of known classical expressions for the determining of the cord path character are analysed. It is shown that the nominal initial cord angle on the assembly drum exists and is uniquely determined for the given assembly drum radius, the element carcass radius as well as the ply inclination to the meridian at the element surface equator. Sinus multiplication formulas which relate the initial cord angle on the drum, fiber tilt to the manufactured element meridian and the fiber tilt at an arbitrary point of manufactured flexible element carcass are obtained. The dependence of the technological constant on the initial ply angle is analysed. The coefficient of dependence between the ply angle and the effective radius in manufactured flexible element are obtained.

1. Introduction

Rubber-cord flexible elements (RCE) are widely used due to their elastic properties, which allow considerable mechanisms movement under the influence of external loads [1].

The RCE carcass consists of several layers or plies of rubberized cord forming a fiber network where the plies are superimposed at different angles. Usually, the carcass is a surface of revolution, working under the distributed loads and loads, caused by the axial, radial and other forces in its embedding.

This model case is well described in terms of pneumatic tires theory with its basic equations [2–5] which can be used to:

–determine the fiber tilt angle β at any point of the manufactured element if the angle $\beta_k$at the element surface equator (see fig. 1) is known:

$$\sin\beta = \frac{R}{R_\kappa}\sin\beta_\kappa \tag{1}$$

– determine the slope angle of the normal at an arbitrary point of the element surface

$$\sin\varphi = \frac{(R - R_o^2)\cos\beta}{(R_k^2 - R_o^2)\cos\beta_k} \tag{2}$$

In the expressions (1) and (2), R_o is the effective radius; R_k is the radius of the element equator; R is the radius value at an arbitrary point of the element surface

Expressions (1–2) are obtained by using the formula

$$\sin\beta_\kappa = \frac{\sin\alpha}{R_{SB}(1+\delta)}R \tag{3}$$

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In expression (3)

$$\chi = \frac{\sin\alpha}{R_{SB}(1+\delta)} \tag{4}$$

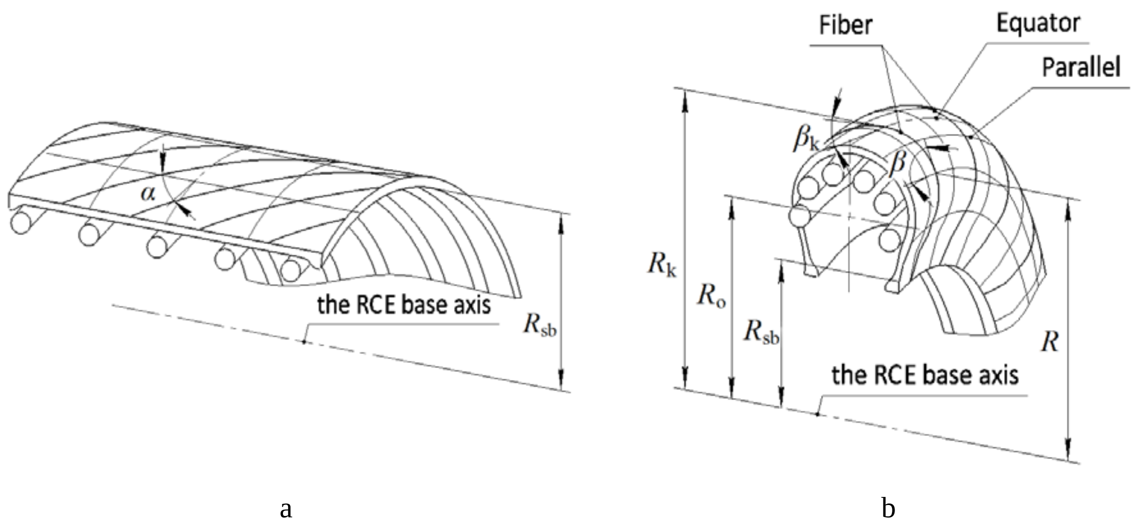

Figure 1. The RCE carcass geometry: on the assembly drum (a) and in manufactured RCE (b)

– technological constant (TC), determined for the fixed inclination α to the meridian of the assembly drum with R_{sb} radius and the cord stretch δ.

Taking into account (4), the law of the fibers arrangement in the fiber network (on the surface of the element) is obtained:

$$\sin\beta = \chi R. \tag{5}$$

The above equations have their own applicability boundaries, which are primarily related to the element and the assembly drum dimensions and the fiber inclination on the assembly drum.

2. The application range of the tire geometry basic equations

The first step is to determine the pneumatic tires theory basic assumptions which are limiting its usage [6, 7].

Let us consider the RCE condition on the assembly drum. The conditional cord stretch δ is taken equal to zero. We will also assume that the element manufacturing process is not causing the fibers elongation.

The change in technological constant χ is due to a change in its argument α, determined on a finite interval [0, π/2]:

$$\chi(\alpha) = \frac{\sin\alpha}{R_{SB}} \tag{6}$$

If R_{sb} is measured in meters, the base unit of χ will be [1/m]. The assembly drum radius R_{sb} is constant, therefore, the minimum and maximum values of χ (α) are attainable at the end points of the α variation interval. The minimum value is reached when α_o=0, in this case $\chi(0) = 0$. The maximum value is reached for $\alpha = \pi / 2$:

$$\sin\frac{\pi}{2} = 1. \tag{7}$$

Wherein χ (α) is the assembly drum radius reverse value:

$$\chi\left(\frac{\pi}{2}\right) = \frac{1}{R_{SB}}. \tag{8}$$

Thus, the graph of χ (α) monotonically increasing from zero to the assembly drum radius R_{sb} reverse value:

$$0 \le \chi(\alpha) \le \frac{1}{R_{SB}} < 1 \tag{9}$$

Reverse values of χ (α) will have the base unit [m] and can be represented as

$$R_p(\alpha) = \frac{1}{\chi(\alpha)} = \frac{R_{SB}}{\sin\alpha}, \tag{10}$$

where $R_p(\alpha)$ is the calculated reverse value of the χ (α) from expression (10) . It should be noted that

$$R_p(\alpha)\chi(\alpha) = 1. \tag{11}$$

Maximum of $R_p(\alpha)$ grows without bound while α approaches zero; minimum value is achievable when

$$\alpha = \frac{\pi}{2},\ \sin\alpha = 1. \tag{12}$$

Thus, the $R_p(\alpha)$ reaches a minimum value equal to the assembly drum radius R_{sb}:

$$[R_p(\alpha) \to \infty\ when\ \alpha \to 0] \ge R_p(\alpha) \ge \left[R_p\left(\frac{\pi}{2}\right) = R_{SB}\right]. \tag{13}$$

Under given inclination to the meridian at the element surface equator β_k there is the following dependence between the calculated value R_p and the RCE equator radius R_k:

$$R_к(\alpha) = R_p(\alpha)\sin\beta_к. \tag{14}$$

3. Nominal angle of the cut on the assembly drum

Under given angle β_k, there is an initial fiber inclination on the assembly drum α_H for which the following relation comprising the sin β_k, R_k and R_{sb} radius exists:

$$\sin\beta_к = \frac{R(\alpha_н)}{R_{SB}}\sin\alpha_н. \tag{15}$$

Angle value α_H satisfying the equation (15) for given R_{sb}, R_k and sin β_k will be called *the nominal initial fiber inclination* on the assembly drum or simply *the initial angle*.

If put the equator radius $R_k(\alpha_H)$ (corresponding to theinitial angle α_H) in correspondence to the following value of calculated radius $R_p(\alpha_d)$:

$$R_p(\alpha_d) = R_K(\alpha_н), \tag{16}$$

then, with an account to (10), the following equation will take a place:

$$R_p(\alpha_d) = \frac{R_{SB}}{\sin\alpha_d}. \tag{17}$$

By equating (16) to (17), we obtain

$$\sin\alpha_d = \frac{R_{SB}}{R_K(\alpha_н)}, \tag{18}$$

$$R_{SB} = R_K(\alpha_н)\sin\alpha_{\mathrm{d}} \tag{18.1}$$

and finally

$$\alpha_d = \arcsin\frac{R_{SB}}{R_K(\alpha_н)}. \tag{18.2}$$

The obtained equation (18.2) allows the calculation of the fiber inclination α_d under given values of the assembly drum radius R_{sb} (or diameter D_{sb}) and radius R_K (or diameter D_K) of the RCE carcass.

4. The interconnection between the angles α_H, α_D and β_K

With the obtained expression (18.2), the RCE carcass geometry can be fully described by its fiber angles. Converting (15), using expression (18), we obtain

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$$\sin\beta_{\kappa} = \frac{\sin\alpha_{\text{и}}}{\sin\alpha_{\text{d}}} \tag{19}$$

or

$$\sin\alpha_{\text{и}} = \sin\alpha_{d}\sin\beta_{\kappa}. \tag{20}$$

The expression (21) can be represented as following:

$$\alpha_{\text{и}} = \arcsin(\sin\alpha_{d}\sin\beta_{\kappa}). \tag{21}$$

In expression (21), the fiber inclination to the meridian on the assembly drum is less than 1. Consequently, the angle exists and is uniquely determined for a given radius of the assembly drum R_{sb}, the radius of the equator R_{k}, and the value β_{k} of the fiber inclination to the meridian of the manufactured RCE.

The expression (20) will be called **the first sinus multiplication** formula which determines the sine of the fiber nominal angle on the assembly drum.

All obtained expressions were considered for a constant value of fiber inclination to the manufactured RCE meridian and different values of fiber inclination on the assembly drum. It also allows to formulate a number of new equations-consequences, which are given below.

Implication 1. Similarly to formula (20), expression

$$\sin\alpha_{m} = \sin\alpha_{d}\sin\alpha_{\text{и}} \tag{22}$$

will be **the second sinus multiplication** formula. It determines the smallest value α_{m} of the fiber inclination on the assembly drum.

The expression (22) can be represented as following:

$$\alpha_{m} = \arcsin(\sin\alpha_{d}\sin\alpha_{\text{и}}). \tag{23}$$

Converting (22), using expression (20), we obtain

$$\sin\alpha_{m} = \sin^{2}\alpha_{d}\sin\beta_{\kappa}. \tag{24}$$

General case is the formula

$$\sin\alpha_{m_j} = \sin^{j+1}\alpha_{d}\sin\beta_{\kappa} \tag{25}$$

where j is integer.

Implication 2. The third sinus multiplication formula has the following forms:

$$\sin\alpha_{g} = \sin\alpha_{\text{н}}\sin\beta_{\text{к}}, \tag{26}$$

$$\sin\alpha_{g} = \sin\alpha_{\text{d}}\sin^{2}\beta_{\text{к}}. \tag{27}$$

The position of the considered point α_{g} with respect to the point α_{m} on the interval [0, π / 2] is defined by the following equations:

$$((\alpha = \beta_{\kappa}) > \alpha_{\text{d}}) \rightarrow (\alpha_{\text{g}} > \alpha_{\text{m}}), \tag{28}$$

$$((\alpha = \beta_{\kappa}) < \alpha_{\text{d}}) \rightarrow (\alpha_{\text{g}} < \alpha_{\text{m}}). \tag{29}$$

Implication 3. The interval [0, π / 2] contains a point where

$$\alpha = \beta_{\kappa}, \tag{30}$$

and the following relation occurs:

$$\frac{\sin\beta_{\kappa}}{R_{\kappa}(\alpha)} = \left(\frac{\sin\alpha}{R_{SB}} = \frac{\sin\beta_{\kappa}}{R_{SB}}\right). \tag{31}$$

Taking into account (30), theequation (31) can be represented as following:

$$R_{SB} = R_{K}(\alpha) = R_{\kappa}(\beta_{\kappa}). \tag{32}$$

5. Change of technological constant

Consideration of the technological constant as the angle α function leads to a visual table 1 of the geometric parameters at the considered points corresponding to certain values of the angle.

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In the table 1 the geometric values of the RCE equator radius and assembly drum radius in $R_k(\alpha)$ function correspond to the specified initial conditions, while the fiber inclination to the RCE surface equator and initial fiber inclination on the assembly drum are interchanged relatively to named radiuses.

Table 1. The RCEgeometric parameters under $\alpha_\beta > \alpha_d$

α	**0**	α_m	α_g	α_H	α_d	α_β	$\pi/2$
$\sin\alpha$	0	$\sin\alpha_m$	$\sin\alpha_g$	$\sin\alpha_H$	$\sin\alpha_d$	$\sin\alpha_\beta$	1
$\chi(\alpha)=\dfrac{\sin\alpha}{R_{SB}}$	0	$\chi(\alpha_m)$	$\chi(\alpha_g)$	$\chi(\alpha_H)$	$\chi(\alpha_d)$	$\chi(\alpha_\beta)$	$1/R_{sb}$
$R_к(\alpha)=\dfrac{\sin\beta_к}{\chi(\alpha)}$	∞	$\dfrac{R_к(\alpha_н)}{\sin\alpha_d}$	$\dfrac{R_{SB}}{\sin\alpha_н}$	$R_k(\alpha_H)$	$R_k(\alpha_H)\sin\beta_k$	R_{sb}	$R_{sb}\sin\beta_k =$ $R_k(\alpha_H)\sin\alpha_H$

To match the angles β on the RCE surface with their radiuses, the following transformation can be applied:

$$R_r(a) = R_{SB}\frac{\sin\alpha}{\sin\alpha_m}, \tag{33}$$

Where $R_r(\alpha)$ gives the radius values on the RCE surface, arranged in ascending order and corresponding to the fiber inclination values on the assembly drum.

Matched angle β is determined by the expression

$$\beta = \alpha\sin(\chi(\alpha_н)\mathrm{R}_r(\alpha)). \tag{34}$$

6. The RCE effective radius

For further consideration of the RCE geometric parameters, three specific radiuses determining the RCE shape as shown in fig. 2 can be pointed out.

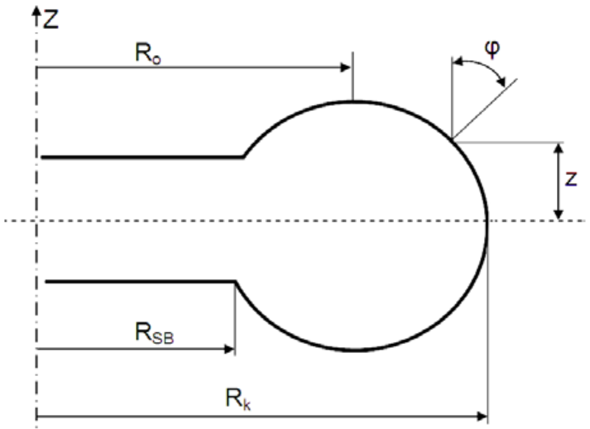

Figure 2. The RCE specific radiuses:
R_o – effective radius;
R_{sb} – assembly drum radius;
R_k – RCE equator radius.

By transforming equation (2) with respect to the effective radius R_o, under $R(\alpha_d) = R_d$,we obtain that

$$R_o^2 = \frac{R_d^2\cos\beta_K + R_{SB}^2\cos\alpha}{\cos\beta_K + \cos\alpha}. \tag{35}$$

The next designations will be useful:

$$C_1 = \frac{\cos\alpha}{\cos\alpha + \cos\beta_K}; \tag{36}$$

$$C_2 = \frac{\cos\beta_K}{\cos\beta_K + \cos\alpha}. \tag{37}$$

It should be noted that

$$C_1 + C_2 \equiv 1. \tag{38}$$

By using (36)-(38), the equation (35) can be formed as

$$R_o^2 = (R_{SB}^2 - R_d^2)C_1 + R_d^2, \tag{39}$$

or for C_2:

$$R_o^2 = (R_d^2 - R_{SB}^2)C_2 + R_{SB}^2. \tag{40}$$

The obtained formulas are equivalent and allow estimation of the effective radius R_o changes caused by changing of angle α from zero to α_d.
Consideration of the α changing interval starting point obtains: $\alpha = 0$, $\beta = 0$ while $C_1 = C_2 = 0,5$.
For both formulas (33) - (34) the following expression is actual:

$$\mathrm{R_o} = \sqrt{\frac{\mathrm{R_{SB}^2 + R_d^2}}{2}} = \frac{\mathrm{R_{SB}}}{\sqrt{2}}\left(1 + \frac{1}{\sin^2 \alpha_\mathrm{d}}\right) \tag{41}$$

Consideration of the α changing interval end point obtains: $\alpha = \alpha_d$, $\beta = \pi/2$ while $R_r = R_H$, $C_1 = 1$, $C_2 = 0$ and $R_o = R_{sb}$.
Thus, when the angle α changes from zero to α_d, the effective radius decreases with a coefficient

$$K_{R_o} = 0{,}7071\left(1 + \frac{1}{\sin^2 \alpha_d}\right). \tag{42}$$

7. Conclusion

The obtained equations can be used during the RCE design process for the selection of the optimal fiber initial angle under given angles in manufactured RCE. Analysis of the given expressions applicability boundaries shows the character of RCE carcass working conditions changing depending on the initial angles laid during the design. The angles exit beyond the permissible values leads to loss of the RCE structural stability under the influence of external loads [8].

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