

Numerical Calculation of Total Radial Forces and Rotary Moments From the Cylinders Surface

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Abstract. The determining of the acting moments of force when the cylinder is characterized by roughness after the deformation of contacting surfaces of the rotating cylinders and the substrate while interacting with incompressible viscous liquid is still one of the main problems in printing. The numerical calculation of the radial forces and rotary moments with integral formulas for deforming surfaces was carried out with regard to the discrete grid at the grid points of the upper and lower cylinders surfaces and their deformation speeds. The paper considers the results of the numerical modeling for the quantitative assessment of rotary moments of radial forces from the cylinders surfaces, occurring during the transfer of incompressible viscous liquid on the substrate between two rotating cylinders. The definition of dynamic forces (radial and tangential force, moment of force) in the discrete form on the example of one computational cell is presented. It is noted, that the consideration of boundary deformation results in the decrease of the calculated dynamic values by 3-5 times as well as to the reducing of the ink bleeding near the boundaries of the printing surface.

1. Introduction

The competitiveness trends in the modern printing equipment market dictate the terms of their continuous improvement. The main purpose of offset printing machines is the obtaining of multiple identical printouts by applying viscous incompressible liquid (ink) on the coated substrate by contact under pressure.

On the other hand, their purpose is also the implementation of the printing process with its inherent variety of deformations caused by contact phenomena in offset printing machines.

The presence of different deformations has a significant impact on the production quality and operation of various mechanical systems.

The deformations in the real printing process and the impossibility of their total elimination constitute an important scientific problem, i.e. their account and control, the solution of which requires comprehensive analysis of the phenomena, because the deformations are fundamental in assessing the quality of printed products and the equipment performance.

A prerequisite for achieving the desired quality of the final printed product [1] during the operation of offset printing machines is the consideration of the effect of the process of transferring viscous



incompressible liquid (ink) to the substrate on the cylinder dynamic characteristics while simulating and the following design of the printing units in offset printing machines.

The search for new approaches to solving the above problem, taking into account that the theoretical developments, in particular the development of mathematical models simulating the work of the printing machine functional units, are among the priorities and has not lost its relevance today.

2. Statement of the problem

In [2], there is a stress on the active control of the viscous incompressible liquid flow with respect to the rotating cylinder only, but not with account for the liquid transfer to the substrate, contacting with the rotating cylinder surface. Computer simulation of the non-monotonic distribution of the ink film during its transfer to the substrate in the printing machine is given in [3], where the issue of vibration compensation is considered.

The attention to this problem is also paid in the following papers [4, 5, 6, 7] where there is only information given on the questions of the liquid transfer between two parallel plates at the strain, rapture and liquid output stages. The influence of the surface tension and the contact angles on the liquid transfer is estimated.

In [8, 9] the results of numerical modeling of calculating the indices of viscous incompressible liquid transfer on a substrate, taking into account the deformation, are presented. However, the effect of the rotational torques is not considered by the authors

The definition of the acting moments of force occurring as a result of the cylinder raggedness when the cylinder surface deformation occurs while interacting with viscous liquid and the other cylinder is still in the focus of researchers.

In its turn, the need to select the projection of the tangential and the normal stress to the appropriate direction (the radius vector to the axis of the cylinder, or to the normal radius of the cylinder in the rotation plane) during the integration of these values has also both scientific and practical significance for the printing machines design and operation.

3. Methods

The numerical calculation for radial forces and moments with these integral expressions for the deformable surfaces was carried out based on the discrete grid at the points of the upper and lower cylinders surfaces and their deformation rates, which is essential in determining the shear stresses.

The simulation is performed using an algorithm developed by the authors of the numerical solution of the Navier-Stokes equations for viscous incompressible liquid on a two-dimensional grid with the help of finite-difference approximations of differential operators on a compact template.

The laminar type of viscous incompressible fluid flow at the point of surfaces contact and a constant cylinders rotation speed is considered.

The numerical calculation on defining the dynamic characteristics was performed on the grid 80×80 for the initial liquid size with thickness $\delta_H = 2.0 \cdot 10^{-6}$ m and width $\delta_S = 8.0 \cdot 10^{-6}$ m, for kinematic viscosity $\nu = 0.012$ m²/s, density $\rho = 10^3$ kg/m³, liquid surface tension coefficient $C_n = 0.03$ N/m.

4. Method implementation

Let us consider on the example of a cylinder the defining of the discrete values for stresses at the i -th point in the cylinder transversal direction with computational region $h_x(r)$ to L_z in size (Figure 1), where L_z is the length of the calculated element along the cylinder.

In Figure 1, on the example of the boundary point «04», let us consider the definition in a discrete form the dynamic values from one computational cell.

As the system of polar coordinates (r, φ) is considered with its transformation into the similar system $(x, y) = (R_\varphi, R - r)$, then the step size to the grid h_x will correspond to the dimension at radius $r = R$, u $h_x(r) = h_{x0} = \Delta_\varphi \cdot R$ where Δ_φ is the grid step to the angle φ .

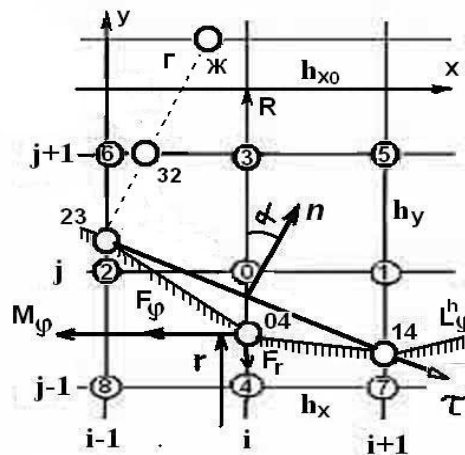


Figure 1. The determination in a discrete form the dynamic values (radial and tangential force, moment of force) from one computational cell.

To determine the active forces moment M_φ on the cylinders axis from the surfaces of contacting with ink, let us calculate the integral values of the following form with regard to the main liquid stress and the conditions of incompressibility and adhesion of viscous liquid on solid boundaries (at constant cylinder radius $r = R$):

$$F_r = \iint_S \left[-(P - P_0) - 2\nu\rho \left(\frac{\partial U_r}{\partial r} \right) \right] R d\varphi dl \quad (1)$$

where r, φ, l are the surface cylindrical coordinates; S, P are the pressure on the cylinder and the liquid surfaces; P_0 is the external atmospheric pressure.

For determining the acting moment of forces M_φ relatively to the cylinders axes it is necessary to integrate the tangent stresses $\nu\rho \left(\frac{\partial U_\varphi}{\partial r} r \right)$ over the surface of the liquid contact with the cylinder while considering the positive moments of forces anticlockwise:

$$M_\varphi = \iint_S \left[-\nu\rho \left(\frac{\partial U_\varphi}{\partial r} \right) \right] R^2 d\varphi dl \quad (2)$$

To find out the total radial force and the moment of force for the given let us do the summation over the calculated cells of the deformed surfaces, which generates force stress, interacting with the contacting liquid, with which a power generated voltage.

Thus, if the computational point «0», nearest by index i to the boundary point «04» of the cylinder, belongs to the liquid region, then in this boundary point we calculate discrete values $F_{ri}, F_{\varphi i}, M_{\varphi i}$ at radius value $r_{i,j(04)} = R + y_{i,j(04)}$ and angle $\varphi_i = x_i / R$.

Normal and tangent directions n, τ with angle α to the radius-vector r can be determined here by the neighbor points coordinates $(x_{i-1}, y_{i-1(23)}), (x_{i+1}, y_{i+1(14)})$ and by angle β , being a discrete step along φ in a polar coordinate system and defined on a uniform grid in the form of $\beta = h_{x0} / R$.

Thus, let us define this angle α in the following form:

$$\alpha = \arctg \left(\frac{(r_{i,j(14)} - r_{i,j(23)}) \cdot \cos \beta}{(r_{i,j(14)} + r_{i,j(23)}) \cdot \sin \beta} \right) \quad (3)$$

For approximate definition of the values $(P - P_0)$ when calculating F_r , M_φ in place of P let us use $P_{i,j(04)}$, defined either by the pressure value in the nearest computational point $P_{i,j(0)}$ with the precision $O(h_y)$, or $O(h_y^2)$ by recomputation from the expression, obtained on the basis of the equation of motion by component y at $h_x = \beta \cdot R$, $h'_x = x_{i(0),j(0)} - x_{i(02),j(02)}$, $h_y = r_{i,j(3)} - r_{i,j(0)}$, $h'_y = r_{i(0),j(0)} - r_{i(04)}$ in the following form:

$$P_{i,j(04)} = P_{i,j(0)} \frac{h_y^2 - h_y'^2}{h_y^2} - P_{i,j(3)} \frac{h_y'^2}{h_y^2} + \rho \frac{h'_y(h_y + h'_y)}{h_y} \times$$

$$\times \left[- \left(\frac{\partial U_r}{\partial t} + U_r \frac{\partial U_r}{\partial r} + \frac{U_\varphi R}{r} \frac{\partial U_r}{\partial \varphi R} - \frac{(U_\varphi + \omega r)^2}{r} \right)_{i,j} + \right. \quad (4)$$

$$\left. + v \left(\nabla^2 U_r - \frac{U_r}{r^2} - \frac{2R}{r^2} \frac{\partial U_\varphi}{\partial \varphi R} \right)_{i,j} \right] + O(h_y^2)$$

where $\nabla^2 U_r = \frac{1}{r} \frac{\partial U_r}{\partial r} + \frac{\partial^2 U_r}{\partial r^2} + \frac{R^2}{r^2} \frac{\partial^2 U_r}{(\partial \varphi R)^2}$, ω – angular rotation speed.

For calculating $P_{i,j(04)}$ it is necessary to replace the first and the second partial derivatives of U_r , U_φ to finite-difference.

Here, it is necessary to use all additional points at the boundary intersection with the nodal lines (Figure 2).

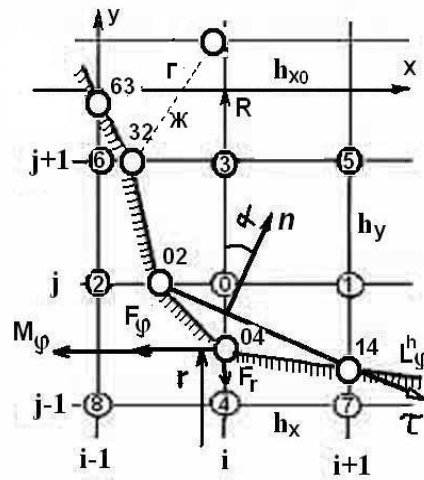


Figure 2. The definition of the dynamic forces (radial and transversal forces, moment of force) in the discrete form from one computational cell at additional grid points on the grid lines of various directions under very curved boundaries.

Besides, for approximate definition of the derivatives in the expression $\frac{\partial U_\varphi}{\partial r} \cos(\alpha) + \frac{\partial U_r}{r \partial \varphi} \sin(\alpha)$, used in the stress projections, let us use finite-difference approximations $\Lambda_r^h U_\varphi$ and $\Lambda_\varphi^h U_r$ for $\frac{\partial U_\varphi}{\partial r}$

and $\frac{\partial U_r}{\partial \varphi R}$ (on the non-uniformity grid points with steps h and h' let us use notations $\Lambda_r^h U_\varphi$ and $\Lambda_\varphi^{h'} U_r$ for them):

$$\frac{\partial U_\varphi}{\partial r} = \frac{1}{(h_y + h_y')} \left(\frac{h_y}{h_y} U_{\varphi i, j(04)} - \frac{(h_y^2 - h_y'^2)}{h_y h_y'} U_{\varphi i, j(0)} - \frac{h_y'}{h_y} U_{\varphi i, j(3)} \right) + O(h_y^2) = \Lambda_r^h U_\varphi + O(h_y^2) \quad (5)$$

$$\frac{\partial U_r}{\partial \varphi R} = \frac{1}{(h_x + h_x')} \left(\frac{h_x'}{h_x} U_{r i, j(1)} + \frac{(h_x^2 - h_x'^2)}{h_x h_x'} U_{r i, j(0)} - \frac{h_x}{h_x'} U_{r i, j(02)} \right) + O(h_x^2) = \Lambda_\varphi^{h'} U_r + O(h_x^2) \quad (6)$$

Based on the expressions of types (5) – (6) for calculating the pressure on the computational region boundary $P_{i, j(04)}$ instead of (4) we obtain the following approximate formula:

$$P_{i, j(04)} = P_{i, j(0)} \frac{h_y^2 - h_y'^2}{h_y^2} - P_{i, j(3)} \frac{h_y'^2}{h_y^2} + \rho \frac{h_y' (h_y + h_y')}{h_y} \times \left[- \left(\Lambda_r^h U_r + U_r \Lambda_r^h U_r + \frac{U_\varphi R}{r} \Lambda_\varphi^{h'} U_r - \frac{(U_\varphi + \omega r)^2}{r} \right)_{i, j} + \right. \\ \left. \times \left[\frac{1}{r} \Lambda_r^h U_r + \Lambda_{rr}^{h'} U_r + \frac{R^2}{r^2} \Lambda_{\varphi\varphi}^{h'} U_r - \frac{U_r}{r^2} - \frac{2R}{r^2} \Lambda_\varphi^{h'} U_\varphi \right]_{i, j} \right] \quad (7)$$

Besides, for an approximate definition of the derivatives in the expression $\frac{\partial U_\varphi}{\partial r} \cos(\alpha) + \frac{\partial U_r}{r \partial \varphi} \sin(\alpha)$,

used in (5)-(6), let us take finite-difference approximations $\Lambda_r^h U_\varphi$ and $\Lambda_\varphi^{h'} U_r$ for $\frac{\partial U_\varphi}{\partial r}$ and $\frac{\partial U_r}{\partial \varphi R}$ (at the non-uniformly grid points with steps h and h' let us denote them $\Lambda_r^h U_\varphi$ and $\Lambda_\varphi^{h'} U_r$).

From here, values F_{ri} , $F_{\varphi i}$, $M_{\varphi i}$ near the boundary of the considered cylinder with computational node (i, j) could be obtained from the following expressions (if there is no the cylinder boundary contact with the fluid, these values for the i -th radial direction are set to zero):

$$F_{ri} = L_z (h_x r_{i, j(04)} / R) / \cos(\alpha) [- (P_{i, j(04)} - P_0) \cdot \cos(\alpha) - \nu \rho (\Lambda_r^h U_\varphi \cos(\alpha) + (R / r_{i, j(0)}) \Lambda_\varphi^{h'} U_r \sin(\alpha)) \cdot \sin(\alpha)] \quad (8)$$

$$F_{\varphi i} = L_z (h_x r_{i, j(04)} / R) / \cos(\alpha) [- (P_{i, j(04)} - P_0) \cdot \sin(\alpha) - \nu \rho (\Lambda_r^h U_{\varphi i, j(0)} \cos(\alpha) + (R / r_{i, j(0)}) \Lambda_\varphi^{h'} U_{r i, j(0)} \sin(\alpha)) \cdot \cos(\alpha)] \quad (9)$$

$$M_{\varphi i} = r_{i, j(04)} L_z (h_x r_{i, j(04)} / R) / \cos(\alpha) [- (P_{i, j(04)} - P_0) \cdot \sin(\alpha) - \nu \rho (\Lambda_r^h U_{\varphi i, j(0)} \cos(\alpha) + (R / r_{i, j(0)}) \Lambda_\varphi^{h'} U_{r i, j(0)} \sin(\alpha)) \cdot \cos(\alpha)] \quad (10)$$

Total values F_r , M_φ over the all cylinder surface boundary points, contacting with the liquid, can be obtained in the following form: (8) и (10)

$$F_r = \sum_{i=1}^{N_1} F_{ri} e_{ri}, \quad M_\varphi = \sum_{i=1}^{N_1} M_{\varphi i} \quad (11)$$

where N_1 is the number of calculated points over coordinate x (or φ); e_{ri} is the unit vector for the i -th direction of the vector radius near the boundary (i, j) - th point of the computational grid.

5. Results and discussion

The computed values of radial forces on the axis and of the moments of forces for cylinders during the interaction with the flowing fluid including the moment of the fluid division between the cylinders surfaces are shown. Figure 3 presents the change in radial forces for the case with no cylinders surface deformation; while Figure 4 shows the detailed calculation for the case of the interaction with the deforming cylinder surfaces and the substrate.

It can also be noted, that when there is no boundaries deformation, in the middle of the period of interaction between the cylinders and the liquid, the peak of power loads occurs when the liquid is compressed to the maximum.

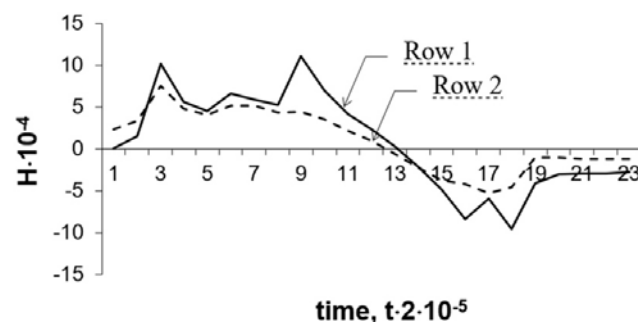


Figure 3. Radial forces of the pressure on cylinders 1 and 2 without deformation during filtration on cylinder 2 (row 2).

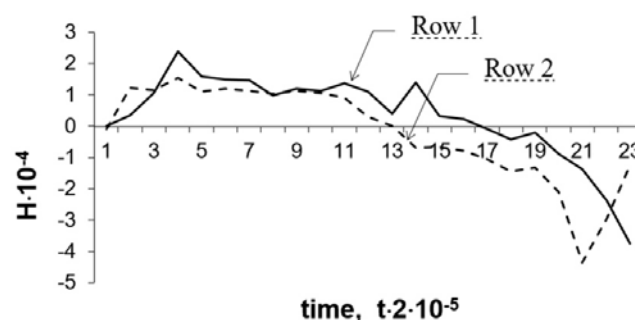


Figure 4. Radial forces of the pressure on cylinders 1 and 2 during deformation and filtration on cylinder 2 (row 2).

It may be noted that in the absence of boundaries deformation power loads peak occurs in the middle of the period of the cylinders interaction with fluid, when its maximum compression occurs. The values of the radial forces are several times greater than for the case of deformation. During the deformation of the cylinders surfaces relative maxima are observed at the beginning of the cylinders interaction during the collision of cylinder 2 with the fluid, and finally during the division of fluid along the two cylinders when compression forces are replaced by the tensile forces that occurs with some delay in contrast to the reaction without deformation. Such values behavior can be also observed for those moments of forces, for which the forces are calculated in transverse directions.

6. Conclusions

The implementation of the approach suggested for solving the task of defining the acting (operating) moments of forces:

- taking account of boundaries deformation reduces the computational dynamic quantities by 3-5 times and the ink spreading around the boundaries of the printed area;

- maximum values occur both at the initial moment of the hydrodynamic shock in the collision of the printing cylinder with liquid and at the end of the separation (gap) of the liquid in the cylinders and its transfer to the substrate;
- when there is no boundary deformation, maximum strain at the cylinders can occur at the moment of the maximum fluid compression. Taking into consideration the deformation, the forces change the direction into the reverse (opposite) one with some delay after the maximum compression of the fluid, when the deformed boundaries recover their original shape.

References

- [1] ISO 12647-2:2004 2012 Graphic technology, Process control for the production of half-tone colour separations, proof and production prints, Part 2: Offset lithographic processes p 19
- [2] Ou Y-R 1992 Decision and Control relative to a rotating cylinder *Decision and Control*, IEEE Conf. Publications vol 4 pp 3399–3404
- [3] Yu J and Wei P 2011 Simulation the axial oil distribution of offset printing *Electrical and Control Engineering (ICECE)*, IEEE Conf. Publications pp 4968–4971
- [4] Huang W-X, Lee S-H, Sung H J , Lee T-M and Kim D-S 2008 Simulation of liquid transfer between separating walls for modeling micro-gravure-offset printing *Int. J. of Heat and Fluid Flow* **29**(5) 1436–1446
- [5] Reis N C Jr, Griffiths R and Santos J M 2004 Numerical simulation of the impact of liquid droplets on porous surfaces *J. Comput. Phys.* **198** 747–770
- [6] Panichkin A V and Varepo L G 2014 The Numerical Calculation of a viscous Incompressible Fluid transfer onto Poros Surface between Rotating Cylinders *Int. Multidisciplinary Microscopy Congress, Springer Proc. in Physics* vol 154 pp 79–83
- [7] Panichkin A V and Varepo L G 2014 The simulation of the viscous fluid splitting at the outlet of the engagement zone between cylinder surfaces and substrate *Dynamics of Systems, Mechanisms and Machines*, IEEE Conf. Publications pp1–4
- [8] Tai C H, Zhao Y and Liew K M 2005 Parallel computation of unsteady incompressible viscous flows around moving rigid bodies using an immersed object method with overlapping grids, *J. of Computational Phys.* **207**(1) 151–172
- [9] Yu J and Lin J 2012 The ink preset algorithm based on the model optimized by chaotic bee colony *5th Int. Congress on Image and Signal Processing* pp 547–551