The mathematical and computer modeling of the worm tool shaping

K L Panchuk[1], **A A Lyashkov**[1], **T V Ayusheev**[2]

[1]Omsk State Technical University, Mira ave., 11, Omsk, 644050, Russia
[2]East Siberian State University of Technology and Management, Klyuchevskaya ave., house 40, building 1, Ulan-Ude, 670013, Russia

E-mail: Panchuk_kl@mail.ru

Abstract. Traditionally mathematical profiling of the worm tool is carried out on the first T. Olivier method, known in the theory of gear gearings, with receiving an intermediate surface of the making lath. It complicates process of profiling and its realization by means of computer 3D-modeling. The purpose of the work is the improvement of mathematical model of profiling and its realization based on the methods of 3D-modeling. Research problems are: receiving of the mathematical model of profiling which excludes the presence of the making lath in it; realization of the received model by means of frame and superficial modeling; development and approbation of technology of solid-state modeling for the solution of the problem of profiling. As the basic, the kinematic method of research of the mutually envelope surfaces is accepted. Computer research is executed by means of CAD based on the methods of 3D-modeling. We have developed mathematical model of profiling of the worm tool; frame, superficial and solid-state models of shaping of the mutually enveloping surfaces of the detail and the tool are received. The offered mathematical models and the technologies of 3D-modeling of shaping represent tools for theoretical and experimental profiling of the worm tool. The results of researches can be used at design of metal-cutting tools.

1. Introduction

The means and the methods of three-dimensional modeling in modern CAD represent the powerful tool for designing metal-cutting tools, in particular at the stage of their profiling. The determination of geometrical form of the surface of the tool at the set form of the detail and parameters of the positioning of the detail concerning the tool is understood, first of all, under profiling.

Now in the field of profiling the set of theoretical approaches are created. The kinematic method considered in the known works of the founder of the theory of gear gearings F. L. Litvin [1, 2] and method of envelope surfaces which has been reviewed in scientific works, for example [3, 4, 5] prevail among them.

Among the set of tasks of profiling of the cutting tools the incomplete one in respect of theoretical study and implementation by means of CAD is the task of determination of the surface of the worm tool for handling of the screw surface of the detail. In the existing mathematical models of profiling of this tool the first T. Olivier method of formation of the mutually envelope surfaces is used, according to which there is an intermediate surface at models (the surface of the making lath) [1]. It complicates the process of mathematical profiling and its implementation by means of CAD. Therefore, receiving of mathematical model with simpler algorithm of profiling is an actual task. Another actual task accompanying the first is the accomplishment of virtual computer modeling of technological process of shaping of the worm tool. In the case of such modeling, the important technological subtleties and features of profiling, hidden in case of mathematical modeling, are found.

2. Formulation of the Problem

The purpose of the given work is the improvement of mathematical model of profiling of the worm tool and its approbation with the use of tools of frame, superficial and solid-state modeling. Research problems are: receiving mathematical model of profiling with simpler, in comparison with known, algorithm of definition of the form-building surface of the tool; realization of this model based on methods of frame and superficial modeling; development of technology of solid-state modeling of shaping of the mutually envelope surfaces of the detail and the tool.

3. Theory and Methods

The basis for the mathematical model containing simple algorithm of profiling of the worm tools have served the following theoretical position [1, 2]:
1. Total normal at the point of contact of mutually envelope surfaces belongs to the complex of straight lines of the kinematic screw of the instantaneous relative motion of bodies with these surfaces.
2. The mutually envelope surfaces of the detail and the tool are formed on the first T. Olivier method. It means that the intermediate surface, which is provided in the method, forms two surfaces by two independent movements, one of these surfaces belongs to the detail, and another - to the worm tool. The contact of the mutually envelope surfaces in each from several couple of products: the detail – the making lath and the making lath – the worm tool, occurs on the line. The contact of the surfaces in the main couple the detail – the worm tool occurs in the point.
On the basis of the given theoretical positions the geometrical scheme of profiling of the worm tool is developed and its mathematical description in the form of algorithmic sequence of calculation formulas presented in list form is executed.

3.1. Computational formulas

1. Equations of a face profile of a screw surface of the detail

$$x_{d1} = a - r\sin\sigma, y_{d1} = r\cos\sigma, \sigma_0 \le \sigma \le \sigma_n \tag{1}$$

2. Module profile point radius vector

$$A = \sqrt{x_{d1}^2 + y_{d1}^2} \tag{2}$$

3. Angle between radius vector and a tangent to the profile in their common point

$$\sigma_f = \arccos\frac{a\cos\sigma}{A} \tag{3}$$

4. The equations of the contact line l_1 of surfaces of the detail and the making lath, h_1 – the screw parameter of the surface of the detail

$$\sqrt{R_1^2 - (A\cos\sigma_f)^2} = C,$$

$$x_{p1} = \frac{A}{R_1}\left[A\cos^2\sigma_f + C\sin\sigma_f\right],$$

$$y_{p1} = \frac{A}{R_1}\left[C - A\sin\sigma_f\right]\cos\sigma_f, \tag{4}$$

$$z_p = h_1\left[arccos(\frac{A}{R_1}cos\sigma_f) - arctg\frac{y_{d1}}{x_{d1}} - \sigma_f\right]$$

5. The size of the current shift of the contact line l_1 along the axis of the detail, β – an angle between axes of the detail and the tool

$$s_i = -(z_p + \frac{y_{p1}}{tg\beta}) \tag{5}$$

6. The equations 4–6 describe the profiling line l_P at mutually envelope of surfaces of detail and tool

$$z_{pi} = -\frac{y_{p1}}{tg\beta} = z_p + s_i \tag{6}$$

7. The size of initial shift of the contact line l_1 along the detail axis

$$s_0 = -(\frac{y_{p1}(\sigma_0)}{tg\beta} + z_p(\sigma_0)) \tag{7}$$

8. Angles of rotation of the detail and the tool in the course of mutually enveloping

$$\varphi_1 = \frac{1}{h_1}|s_0 - s_i| \tag{8}$$

9. Angles of rotation of the detail and the tool in the course of mutually enveloping, u – the transfer relation

$$\varphi_2 = \frac{1}{u}\varphi_1 \tag{9}$$

10. Formulas of transformation of systems of coordinates of the detail and the tool, B – distance between axes of the detail and the tool

$$\begin{aligned} x_2 &= (x_{p1} - B)cos\varphi_2 + (y_{p1}cos\delta + z_{p1}sin\delta)sin\varphi_2, \\ y_2 &= (y_{p1}cos\delta + z_{p1}sin\delta)cos\varphi_2 + (B - x_{p1})sin\varphi_2, \\ z_2 &= -y_{p1}sin\delta + z_{p1}cos\delta \end{aligned} \tag{10}$$

11. The equations of an axial profile of the required surface of the tool, h_2 – the screw parameter of the surface of the tool

$$\begin{aligned} x_{t2} &= x_2 cos(\frac{z_2}{h_2}) + y_2 sin(\frac{z_2}{h_2}), \\ y_{t2} &= y_2 cos(\frac{z_2}{h_2}) - x_2 sin(\frac{z_2}{h_2}) \end{aligned} \tag{11}$$

12. Equations of the face profile of the required surface of the tool

$$\begin{aligned} x_{02} &= \sqrt{x_2^2 + y_2^2}, \\ z_{02} &= z_2 - h_2 arctg(\frac{y_2}{x_2}) \end{aligned} \tag{12}$$

The mathematical model presented by calculation formulas (1) - (12) allows to make computer visualization (figure 1) of the contact line l_1 of surfaces of the screw detail and the making lath and the profiling line l_P (the line of gearing on terminology in the theory of gear gearings [1, 2]). From the lines of l_1 and l_P, on the basis of kinematics of the mutually envelope movements of a detail and the tool, transition to the surface of the worm tool is carried out. In figure 2 the frame models of an initial surface of the detail and the surface of the tool which are in the position of the contact are presented. Superficial models of these contacting products are given in figure 3.

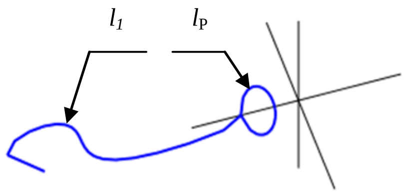

Figure 1. Line of the contact and line of profiling (closed).

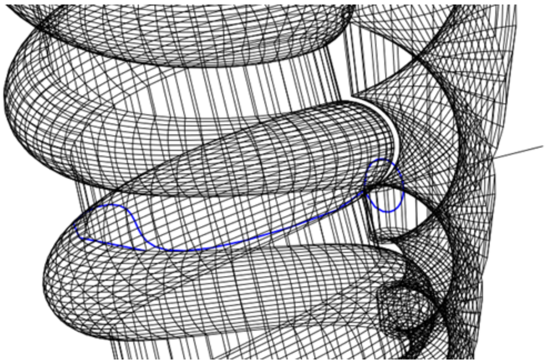

Figure 2. Frame model of the contacting surfaces.

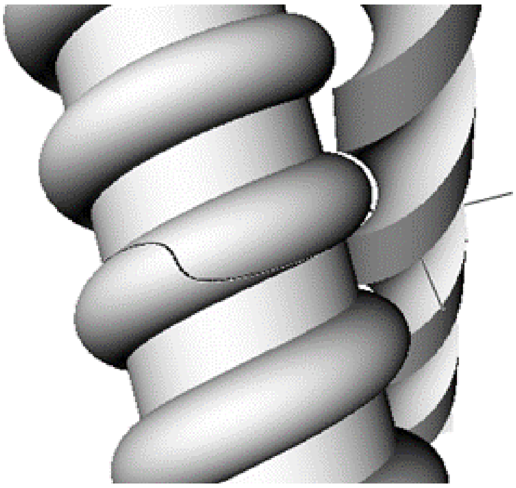

Figure 3. Superficial model of the contacting surfaces.

Among modern profiling techniques the method based on solid modeling is becoming increasingly common. It is regarded, for example, in scientific works of Dimitriou Vasilis, Vidakis Nectarios and Antoniadis Aristomenis [6], A.A. Lyashkov and K.L. Panchuk [7]. The following factors serve as motivation to it:

IOP Conf. Series: Journal of Physics: Conf. Series **858** (2017) 012024 doi:10.1088/1742-6596/858/1/012024

1. The need in the mathematical research and the solution of the transcendental equations or their systems connecting geometrical and kinematic parameters of the process of shaping on the basis of mutually envelope of surfaces of the detail and the tool disappears at solid-state modeling.
2. Solid-state modeling allows solving two interconnected shaping problems at the virtual imitating level. One of them is the task of geometrical synthesis including solutions of the direct and return task of profiling. In case of the direct task the determination of the surface of the tool is carried out, and in case of the return – a detail surface on the received tool surface for the quality evaluation of the solution of the direct task also is carried out. Solutions of both tasks of profiling are based on the virtual computer reproduction of relative movements of the detail and the tool provided on the practice by means of tuning of a kinematic chain of the machine equipment. The other task of the shaping is the task of technological synthesis, namely – consecutive removal by the cutting tool of amount of material of billet of the detail before receiving its necessary geometrical form.

Virtual imitating modeling allows receiving the quality characteristics of technological process of the shaping and numerical parameter values of the cut-off layers with the subsequent introduction of necessary geometrical and technological amendments to the scheme of shaping.

Solution of both tasks of synthesis is carried out in the mode of virtual experiment in which, if necessary, modification or adjustments of basic data for receiving of required results of profiling without accomplishment of material and finance costs is possible.

The algebraic logic of the theory of sets forms theoretical base of virtual technological experiment. The model of billet of the detail and model of the tool are considered as three-dimensional sets of geometrical elements for which transaction of the difference of sets of $M_1 = M_2 \backslash M_3$ is carried out where M_1, M_2 and M_3 – respectively a resultant set, a set of elements of billet of the detail and the tool. In the process of a cyclic shaping the set of M_1 after each iterative step accomplishment of transaction of the difference is updated and the cycle of the shaping is repeated for already updated set of M_1. As a result we receive discrete virtual modeling of the process of shaping.

4. Results of Experiments

In this work, proceeding from essence of solid-state modeling, the technological sequence of the solution of the considered profiling task is developed and virtually realized. The initial stage of solid-state modeling of shaping is the creation of initial models of the detail and billet. For the considered case the detail with the screw flute is the initial one. In the beginning model surfaces on its face section (figure 4) are created, and then they are combined with the model of billet (figure 5). These surfaces are used for cutting off its parts from the billet, the result of which is the detail model with a screw flute (figure 6).

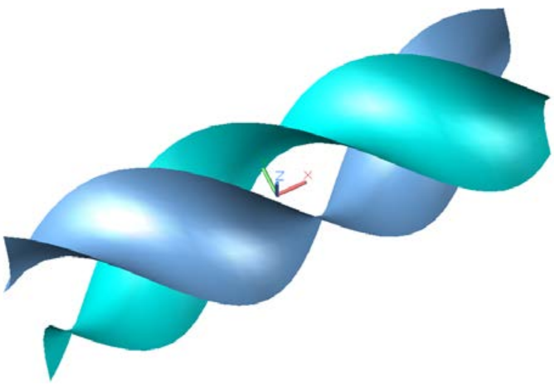

Figure 4. Initial screw surfaces of the detail.

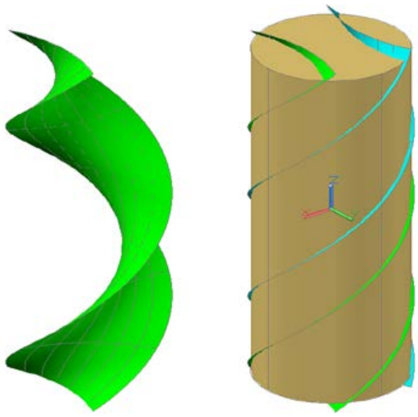

Figure 5. Cutting off the amounts of material from model of billet of the detail.

At the following stage the billet model (a cylindrical body) for the tool lath (figure 7) is added to the model of the detail.

The model of an instrumental lath is formed on the basis of the model of the detail which is given forward movement along forming a cylindrical body (figure 8).

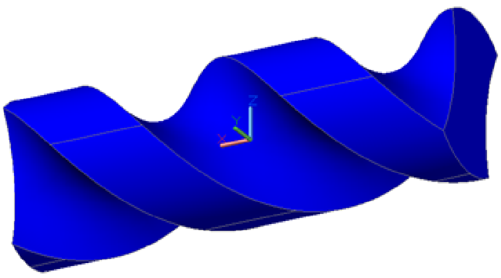

Figure 6. Result of cutting off.

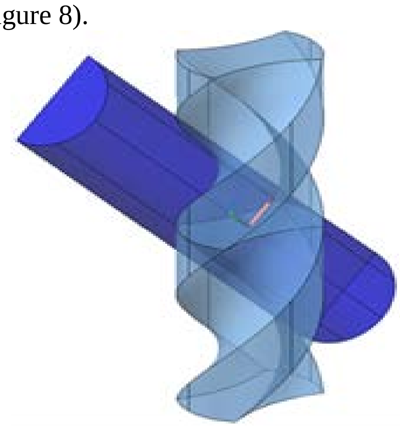

Figure 7. Addition of model of cylindrical billet.

The process of shaping is carried out in the automated mode on the base of the developed algorithms and programs.

The accuracy of modeling is defined by the size of an increment of the parameter of forward movement of the product. This parameter is set in the dialogue mode before modeling. The result of modeling and face section of an instrumental lath are given in figure 9. This section is used for receiving model of the cylindrical surface interfaced to surfaces of the detail and the instrument (figure 10). Then the cylindrical surface is used for creation of solid-state model of other instrumental lath used for shaping the instrument (figure 11).

IOP Conf. Series: Journal of Physics: Conf. Series **858** (2017) 012024 doi:10.1088/1742-6596/858/1/012024

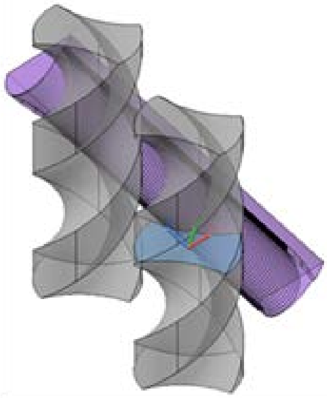

Figure 8. Shaping of the surface of the tool lath.

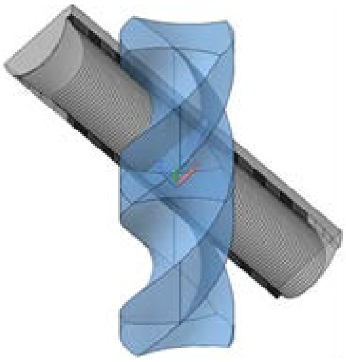

Figure 9. Result of modeling of the surface of an instrumental lath.

In figure 12 the models of the second lath and of the cylindrical billet for the tool are shown. The model of the tool is formed on the basis of the received model of the second instrumental lath making the screw movement with the set parameter (figure 13). The process of shaping is also carried out in the automated mode. Accuracy of modeling is determined by the size of the increment of parameter of screw movement of the product. This parameter is also set in the dialogue mode before accomplishment of the modeling.

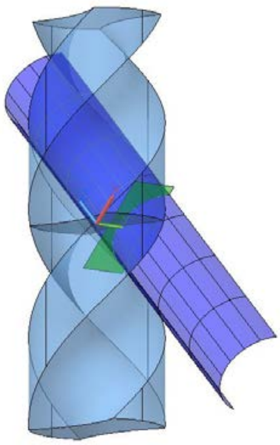

Figure 10. Surface of the tool lath.

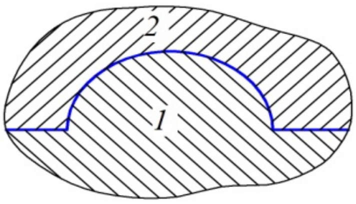

Figure 11. Two bodies with one surface

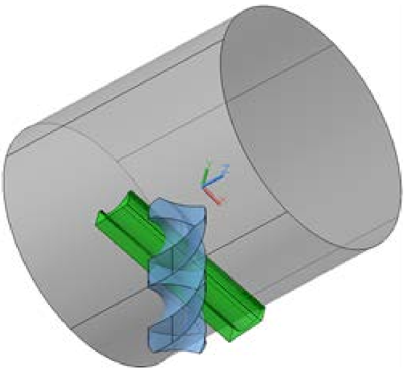

Figure 12. Accession of the surface of the tool lath to model of billet of the tool.

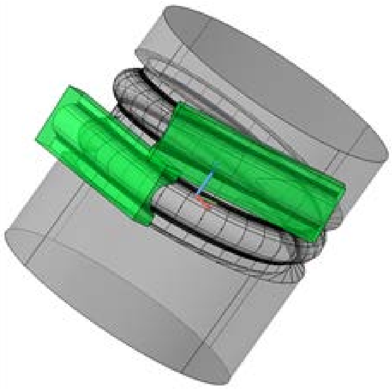

Figure 13. Instrument surface shaping by screw movement of an instrumental lath.

The result of modeling of the part of the instrument together with the detail and cylindrical surface of an instrumental lath is shown in figure 14. The same models are given in figure 15, but the instrument body is defined on its axial section from the previous stage of modeling.

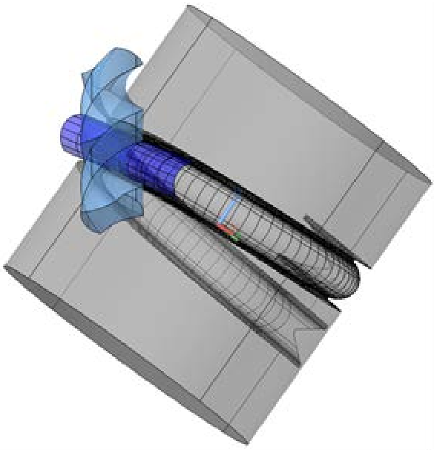

Figure 14. Result of shaping of the surface of the tool.

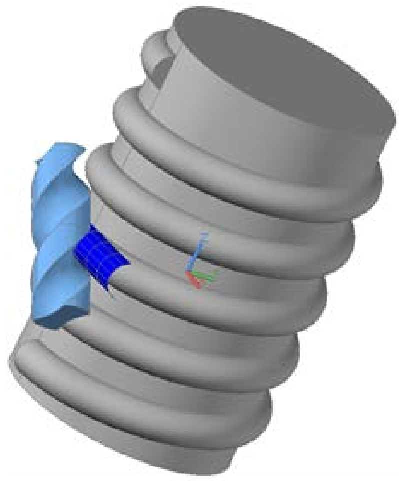

Figure 15. Result of the full shaping of the surface of the tool.

For the assessment of quality of the executed shaping of the worm tool, the received models can be used for comparison of results of solid-state modeling with the results of frame and superficial modeling. For this purpose it is possible to construct, for example, sections by various planes. So in figure 16 the section of models by the plane passing through the tool axis, and in figure 17 – through the detail axis is shown. The comparative analysis of the axial sections of the worm tool received by various ways of modeling shows their coincidence to within the fourth sign after the comma. Besides, from images in figures 16 and 17 it is possible to draw the conclusion that the practical contact of the surface of the detail and the tool occurs on some area though according to mathematical model they have pointed contact.

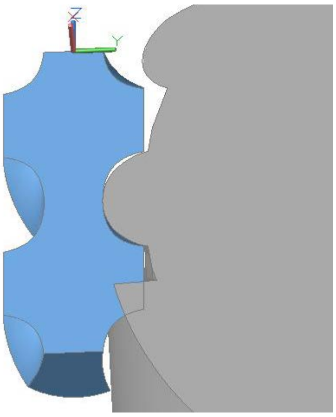

Figure 16. Visualization of the contact of surfaces of the detail and the tool in the axial section of the tool.

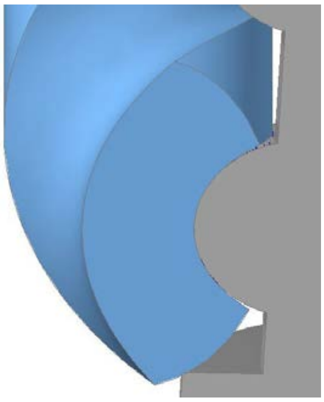

Figure 17. Visualization of the contact of surfaces of the detail and the tool in the axial section of the detail.

5. Discussion of Results

The design of the face profile of the screw detail is more difficult, the geometry of her screw surface is more difficult. The difficult design of the profile can include several natural lines joined on the first or higher order of smoothness. It can consist also of one or several spline lines, the points of the face profile constructed on the discrete set. For screw details of difficult geometry additional researches within the accepted mathematical model is realized by the frame or superficial modeling, for example, identification and research of transitional curves and cutting are required. The solid-state modeling realized at the virtual imitating level allows tracking emergence of transitional curves and cutting. It gives an idea of constructive and technological conditions of their emergence, allows to conduct the corresponding researches and to introduce necessary amendments in the scheme of shaping of the tool. Obviously, frame and superficial modeling of the problem of profiling of the worm tool are expedient for screw details with rather simple geometry, and solid-state modeling – for details of complex geometry. It should be noted also that for the considered problem of profiling the unexplored is the question of modeling of the shaping of the tool without participation of the making lath, i.e. directly, giving of an initial screw surface of the detail of the two-parametrical movement of enveloping.

6. Conclusion

1. The advanced mathematical model of the solution of the problem of profiling of a worm instrument for processing of the screw surface of the detail is offered. The model is based on the first T. Olivier method of shaping of the mutually envelope surfaces and unlike the existing mathematical models doesn't demand the use of an intermediate surface of tool rack. The model is simple concerning computing procedures and allows solving the problem of profiling with visualization of frame and superficial modeling without application of numerical methods.
2. The solution of the problem of profiling of the worm instrument on the basis of solid-state modeling is received. Profiling is carried out at the virtual imitating level in the form of sequence of the technological stages corresponding to the first T. Olivier method.
3. Proposed frame, superficial and solid-state modeling solutions of the problem of profiling of the worm instrument are submitted in total by the necessary set of design-technology instruments of profiling. For the screw details of simple geometry which is used, for example, in some designs of rotors of screw compressor cars for profiling of the worm instrument it is enough to use offered frame and superficial modeling, and for screw details of irregular shape – solid-state modeling. All three types of modeling supplement each other and allow reaching the qualitative solution of the problem of profiling.

References

[1]Litvin F L 1989 Theory of Gearing *NACA Reference Publication 1212 AVSCOM Technical Report 88-C-035* 490
[2]Litvin F L and Fuentes A 2004 Gear Geometry and Applied Theory (Cambridge University Press) 800
[3]Argyris John, Litvin Faydor L, Qiming Lian and Lagutin Sergey A 1999 Determination of envelope to family of planar parametric curves and envelope singularities *Computer Methods in Applied Mechanics and Engineering*. **175(1-2)** pp 175–187. DOI: 10.1016/S0045-7825(98)00367-3
[4]Raikhman G N 2002 Theory of conjugate surfaces formation by generating lines *Mechanism and Machine Theory* **37(9)** pp 999–1005 DOI: 10.1016/S0094-114X(02)00024-1.
[5]Litvin Faydor L, Donno M De, Lian Qiming and Lagutin S A 1998 Alternative approach for determination of singularity of envelope to the family of parametric surfaces *Computer Methods in Applied Mechanics and Engineering* **167(1–2)** pp 153–165 DOI: 10.1016/S0045-7825(98)00117-0.
[6]Vasilis Dimitriou, Nectarios Vidakis and Aristomenis Antoniadis 2006 Advanced Computer Aided Design Simulation of Gear Hobbing by Means of Three-Dimensional Kinematics Modeling *Journal of Manufacturing Science and Engineering* **129(5)** pp 911–918 DOI 10.1115/1.2738947
[7]Lyashkov A A, Panchuk K L 2015 Computer modeling of the pump screw and disc tool cross shaping process *Procedia Engineering* **113** pp 174–180 DOI: 10.1016 / j.proeng.2015.07.314.