

Dynamics of vibration isolation system with rubber-cord-pneumatic spring with damping throttle

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Abstract. The study refers to the important area of applied mechanics; it is the theory of vibration isolation of vibroactive facilities. The design and the issues of mathematical modeling of pneumatic spring perspective design made on the basis of rubber-cord shell with additional volume connected with its primary volume by means of throttle passageway are considered in the text. Damping at the overflow of air through the hole limits the amplitude of oscillation at resonance. But in contrast to conventional systems with viscous damping it does not increase transmission ratio at high frequencies. The mathematical model of suspension allowing selecting options to reduce the power transmission ratio on the foundation, especially in the high frequency range is obtained

1. Introduction

The actual problem for vibration insulation of various objects is reducing the power transmission ratio on the foundation in a wide range of frequencies. Pneumatic springs based on rubber-cord shells [1–4] with a throttling passageway between main 1 and supplementary 2 cavities are prospective for this purpose (Figure 1). Damping at the overflow of air through this passageway limits the amplitude of oscillation at resonance. But in contrast to conventional systems with viscous damping, it does not increase the transmission factor at high frequencies. It is due to air flowing through the passageway between the cavities of pneumatic spring is carried out to a certain flow fluctuations rate, and if the resistance in the damper is high enough to make a gate which disables further cavity. Thus, the efficiency of such a system is much higher at high frequencies.

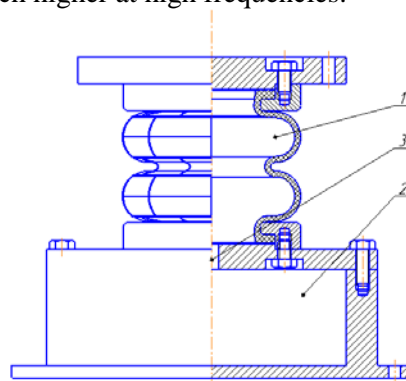


Figure 1. Pneumatic springs based on rubber-cord shell with throttling passageway between main and supplementary cavities



2. Setting a problem

To quantify this effect compared to a conventional viscous damping it is necessary to create a mathematical model of such pneumatic suspension and carry out the necessary calculations.

3. Theory

Thermodynamic processes in the pneumatic springs will be regarded as quasistationary and proceeding at steady expiration modes [1]. The process of gas compression is considered adiabatic and it is usually true for frequencies greater than 3 Htz.

Gas enthalpy i changes due to changes in internal energy u and gas expansion or compression work A , i.e.

$$\frac{di}{dt} = \frac{du}{dt} + \frac{dA}{dt} = Gc_p T_0, \quad (1)$$

where G is mass flow of gas;

c_p, c_v are specific heat of gas at constant pressure and volume;

T_0 is absolute gas temperature.

Besides $\frac{dA}{dt} = p \frac{dV}{dt}$ end $\frac{du}{dt} = Gc_v T_0$ if we assume the gas to be ideal, then using the gas to be

ideal, then using the gas equation and having differentiated it with respect to time, $GT_0 = \frac{1}{R} \frac{d}{dt}(pV)$

we will obtain $\frac{du}{dt} = \frac{c_v}{R} \frac{d}{dt}(pV)$.

Thus, the rate of change of enthalpy in the main chamber 1 is

$$Gc_p T_0 = \frac{c_v}{R} (p_1 V_1) + p_1 \frac{dV_1}{dt}, \quad (2)$$

where p_1, V_1 are pressure and volume in chamber 1;

R is gas constant.

Given that $c_p - c_v = R$, $\frac{c_p}{c_v} = k$, where $k = 1.4$ is adiabatic index from (1) after some transformations we will get

$$G = \frac{1}{RT_0} \left(\frac{V_1}{k} \cdot \frac{dp_1}{dt} + p_1 \frac{dV_1}{dt} \right). \quad (3)$$

Since the volume of the supplementary chamber 2 (Figure 1) does not change, and G is the same, then from (3) one can derive

$$G = \frac{V_2}{kRT_0} \frac{dp_2}{dt}, \quad (4)$$

where p_2, V_2 are pressure and volume of supplementary chamber.

Assuming that the connecting passageway between the chambers is a capillary (the length is much greater than the diameter) the flow through it can be determined from Poiseuille formula [5]

$$G = \frac{\pi d^4 \rho (p_1 - p_2)}{128 \mu \ell}, \quad (5)$$

where d , ℓ are diameter and length of capillary;

ρ is air density;

μ is dynamic viscosity of air.

If we express the density from the gas equation, $\rho = \frac{p}{RT_0}$ and take $p = \frac{p_1 + p_2}{2}$ as average pressure, from expression (5) one can write

$$G = \frac{B}{2RT_0} (p_1^2 - p_2^2), \quad B = \frac{\pi d^4}{128\mu\ell}. \quad (6)$$

There can be carried out linearization of expressions (3), (4) and (6) in the close to parameters corresponding to their steady-state values in the close to the equilibrium position of the pneumatic spring $G = G_0$, $V_1 = V_{10}$, $p_1 = p_2 = p_0$. At the same time one can exclude from the obtained expressions ΔG and Δp_2 and combine them into a single expression

$$\left[V_{10} \left(\frac{sV_2}{kBp_0} + 1 \right) + V_2 \right] \Delta p_1 = kp_0 \left(\frac{sV_2}{kBp_0} + 1 \right) \Delta V_1, \quad (7)$$

where s is operator of differentiation.

In case we divide this expression by Δx , where x is pneumatic spring absolute movement of masses, and consider that $\frac{dV_1}{dx} = F_0$ is effective area of pneumatic spring and $\frac{\Delta p_1}{\Delta x} = \frac{dp}{dx}$ then expression (7) can be written as

$$\frac{dp_1}{dt} = \frac{kp_0 F_0}{V_{10} + \frac{V_2}{1 + \frac{V_2}{kBp_0} \cdot s}}. \quad (8)$$

Stiffness c of pneumatic spring can be expressed as follows:

$$c = \frac{d(p_1 F_0)}{dx} = F_0 \frac{dp_1}{dx} + p_1 \frac{dF_0}{dx}. \quad (9)$$

If we take $F_0 = \text{const}$, from (8) и (9) we will obtain

$$c = \frac{kp_0 F_0^2}{V_{10} \left(1 + \frac{\frac{V_2}{V_{10}}}{1 + \frac{V_2}{kBp_0} \cdot S} \right)} = \frac{c_{10}}{1 + \frac{N_1}{1 + B_1 S}}, \quad (10)$$

where $c_{10} = \frac{kp_0 F_0^2}{V_{10}}$ is static stiffness of the spring without considering the supplementary cavity

$$B_1 = \frac{V_2}{kBp_0}, \quad N_1 = \frac{V_2}{V_{10}}.$$

If one makes the substitution $s = \omega j$ in (10) then at low frequency vibrations (ω approaches 0) it follows from (10) that

$$c_0 = \frac{c_{10}}{1 + N_1} = \frac{kp_0 F_0^2}{V_{10} + V_2},$$

i.e. with overflow of air through the capillary the pressure values in both chambers have time to equalize and stiffness of the spring is determined with their total volume. At high frequencies (ω approaches infinity) equation (10) implies $c_\infty = c_{10}$, which means that the pressure of the main chamber has no time to be transferred to the additional stiffness and is determined only with the volume of the main cavity. At intermediate frequencies stiffness takes intermediate values. Furthermore one can observe oscillation damping effect at low frequencies due to overflow of the gas

Equation of spring mass motion m can be written as $(ms^2 + c)x = P$, where P is harmonic forcing process acting in the x direction. Then the force transmitted to the foundation $P_f = cx$, and the transfer function characterizing the efficiency of vibration protection, change to

$$W(s) = \frac{P_f}{P} = \frac{c}{ms + c}, \quad (11)$$

where c is dynamic stiffness determined from the expression (10).

Having substituted (11) into (10) and expressed the module of the transfer function (power transfer coefficient on the foundation - K_t), we obtain

$$K_t = \frac{\sqrt{(B_1 \omega)^2 + 1}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 - (B_1 \omega)^2 \left[1 - N_0 \left(\frac{\omega}{\omega_0}\right)^2\right]}}, \quad (12)$$

where $\omega_0^2 = \frac{c_0}{m}$; $c_0 = \frac{c_{10}}{1 + N_1} = \frac{kp_0 F_0^2}{V_{10} + V_2}$; $N_0 = \frac{V_{10}}{V_0} = 1 + N_1$.

We are going to consider the weight fluctuations on the same pneumatic spring without additional volume V_2 but with a conventional viscous damper creating a resistance force $P_{\text{res}} = B_2 \dot{x}$. Then we can write $(ms^2 + B_2 s + c_{10})x = P$, and for the force on the foundation one can get $P_f = (c_{10} + B_2 s)x$. In this case, the force transmission ratio on the foundation is determined this way

$$K'_t = \frac{\sqrt{\left(\frac{B_2}{c_{10}} \omega\right)^2 + 1}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_{10}}\right)^2\right]^2 + \left(\frac{B_2}{c_{10}} \omega\right)^2}}; \quad \omega_{10}^2 = \frac{c_{10}}{m}; \quad c_{10} = \frac{kp_0 F_0^2}{V_{10}}. \quad (13)$$

4. The results of the numerical experiment and discussion

To compare the effect of vibration isolation of these two schemes with expressions (12) and (13) the calculations were carried out. The following inputs for a pneumatic spring with rubber-cord shell I-08 [7]: $m = 35\text{kg}$, $V_{10} = 0.7 \cdot 10^{-3} \text{ m}^3$, $V_2 = 1.4 \cdot 10^{-3} \text{ m}^3$, $F_0 = 72 \cdot 10^{-4} \text{ m}^2$, $p_0 = 1.54 \cdot 10^5 \text{ Pa}$; for a throttle

passageway $d = 1 \cdot 10^{-3}$ m, $\ell = 2 \cdot 10^{-2}$ m, $\mu = 18.2 \cdot 10^{-6}$ Pa·s are provided as an example. B_2 resistance coefficient for the second scheme was chosen so that the value of K_t at resonance for both schemes was similar and could be compared on vibration isolation efficiency at the super resonance zone of frequency changes.

The results of the calculations (see Figure 2 and particularly Figure 3) demonstrate that the scheme with a pneumatic spring and the throttle passageway at high frequencies allows reducing the K_t value in about 4 times compared to the scheme with a conventional viscous damper (curves 1 and 2). Furthermore both schemes indicate the same efficiency when passing through the resonance zone at low frequencies. Furthermore there is an optimum diameter of the throttle passageway, ensuring the most damping while passing through resonance zone (curve 1 in Figure 2). By increasing the diameter of the passageway (from optimum) resistance value is reduced, which causes an increase in vibration amplitude at resonance (curve 3 in Figure 2). With further increase in the diameter the damping passageway tends to zero, and turns into pneumatic spring with the static volume $V_{10} + V_2$. By decreasing the diameter (from optimum) damping is also reduced as less amount of air has time to flow from chamber 1 to chamber 2 and back. In addition to that there is a growth of the oscillation amplitude at resonance (curve 4 in Figure 2). Following further decrease in the diameter the damping goes to zero and the support turns into a pneumatic spring with static volume V_{10} .

At high frequencies, the small deviations from the optimum diameter of the throttling passageway have very little effect on the values of K_t (Figure 3, curves 1, 3, 4). This is due to the fact that at high frequencies of the passageway turns into a gate which disables supplementary cavity. Damping does not occur, and that is necessary to improve the vibration isolation in this frequency range.

One provides on Fig.1 an example of a pneumatic spring with rubber-coated balloon-type shell of small size. But this scheme with the throttle pasageway can also be implemented for other types of shells [1, 7]. It is particularly suitable for a pneumatic spring with a diaphragm shell, which involves the construction of two volumes (Figure 4). Calculations carried out for this spring with $m = 480$ kg, $V_{10} = 6 \cdot 10^{-3}$ m³, $V_2 = 9 \cdot 10^{-3}$ m³, $F_0 = 6 \cdot 10^{-2}$ m², $p_0 = 1.8 \cdot 10^5$ m² showed that optimal values for the throttle passageway is $d = 1.8 \cdot 10^{-3}$ m, when $\ell = 3 \cdot 10^{-2}$ m.

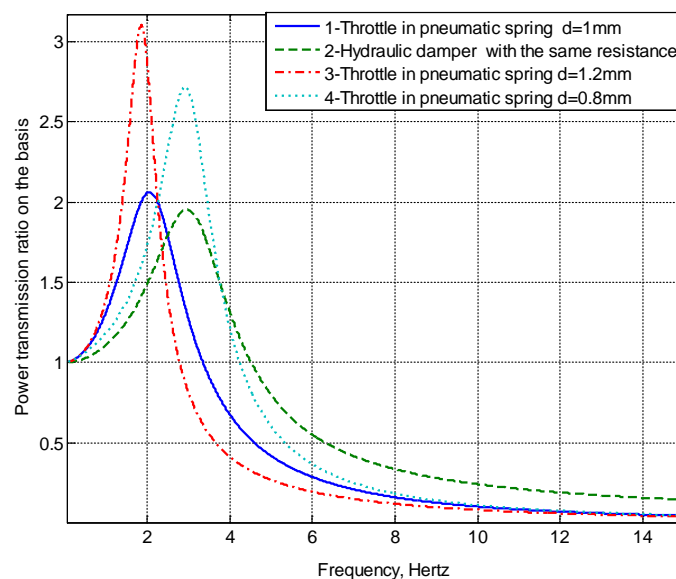


Figure 2. Dependence of the power transmission ratio on the basis of frequency for throttling passageway with pneumatic springs between the primary and secondary volume of comparison with the same spring but without additional volume of a hydraulic damper with the same resistance (in the frequency range up to 15 Hz)

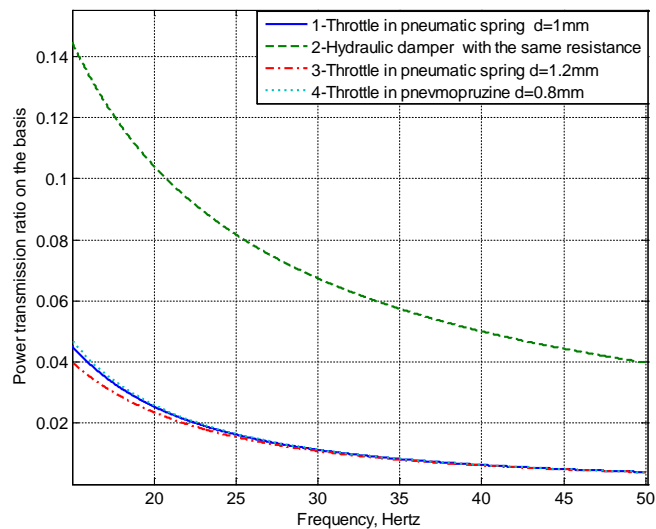


Figure 3. The same for the frequency range above 15 Hz.

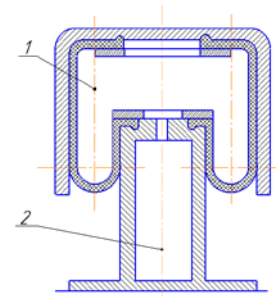


Figure 4. Pneumatic springs based on rubber-cord shell of diaphragm type with a throttling passageway between the main and supplementary chambers

5. Summary and conclusions

Thus the proposed analytical relationships allow making a preliminary selection of the parameters of the throttle passageway considering other given parameters of suspension on pneumatic spring to provide vibration isolation efficiency, both in the low-frequency and high-frequency oscillations.

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