# Gas dynamic model of electrothermal thrusters of small spacecraft and possibility of applying microwave heating of a working

**V N Blinov, VV Shalay, I S Vavilov, V V Kositsin, V I Ruban, A I Lykyanchik, P S Yachmenev, A S Vlasov**

Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia

E-mail: Petrov@mail.ru

**Abstract**. This paper is devoted to development and approbation of the gas dynamic model of ammonia thruster with low power consumption and ultra small thrust for picosatellite weighing up to 5 kg and possibility of applying microwave heating of a working fluid. It is shown, that simplest electrothermal thruster consisting of propellant tank, solenoid valve, expension cavity and heating chamber can provide ultra small trust due to gas dynamic processes and small heat supply. The results of the study set tasks for further design of small spacecrafts microwave generators.

## 1. Introduction

Twenty-first century with its microminiaturization, electronics, nanotechnology and the aspiration for cheaper without quality loss begets rigid boundary value problems, including intensive commercial, military and scientific exploration of space. In recent years, in Russia has increased significantly the interest of States to the space industry. This is due to the natural processes of restoration science intensive high-tech industries, the launch of the first missile from a new cosmodrome Vostochny, the need to create your navigation system, etc. These processes are especially noticeable in the field of designing and manufacturing small spacecraft.

Space ceases to be the fate of elected and universities and laboratories all over the world are intensively develop their satellite destinations. An example of such adaptation to the new realities of technical progress is creating such small spacecrafts like a CubeSat. At the relative cheapness of manufacture of such equipment and their components can be displayed in near-Earth space group and breeding method of orbits of functioning. Jet micro-engines with different thrust creation are used for exploring and correction of putting into orbit errors.

Ammonia trust systems with ohmic heating element have been designed in PA "Polet" in Omsk and were supplied to customer worldwide. Developments were transferred to OmSTU futher. This systems showed its reliability and posibility to perfom the task, but its have been designed for heavy satellites. Time set the task to minimize dimensions, mass, energy consumption of satellite while maintaining reliability. Respectively, should be developed and go on the market engines with less consumption of working fluid, less weight, less power, less prices and high reliability.

Also should also consider other ways of transmission of energy to working body, have as less as possible intermediaries in this way, which ultimately increases the efficiency of propulsion. One such way is microwave heating. It is distinguished by three-dimensional heat transfer and lack of contact with the working fluid.

## 2. Formulation of the problem

Objective in this research is gas-dynamic calculation of working fluid flow tract of small power elementary electrothermal thruster, which used on ultra small spacecrafts like CubeSat for creating micro-thrust and the use microwave enegergy for working fluid heating.

## 3. Theory

The scheme of the model electrothermal truster is shown in Figure 1.

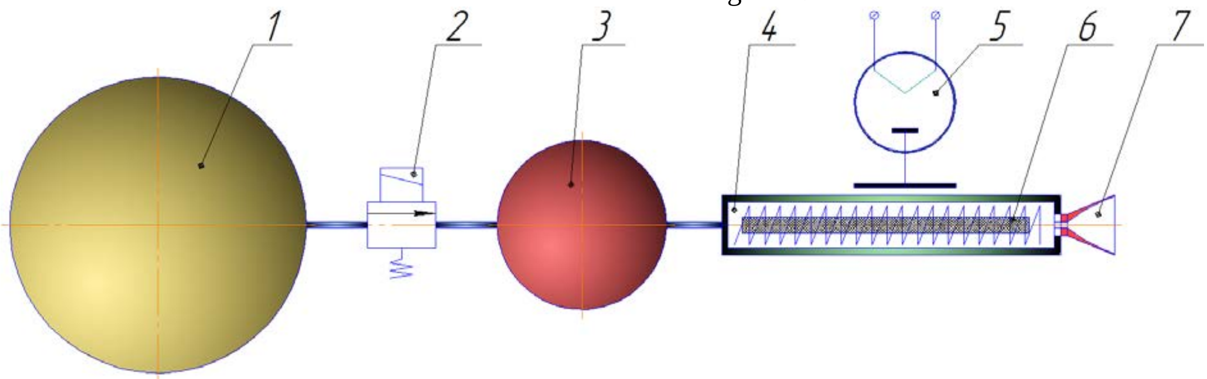


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**Figure 1.**Scheme elementary electrothermal micromotor:
1– high-pressure tank with working fluid; 2–valve solenoid; 3 – expansion cavity; 4– the working fluid heating chamber ; 5 –magnetron; 6 –radar absorbing material; 7 – Nozzle.

Ammonia has been chosen as working fluid. The choice of ammonia as a working body is caused, primarily, a low boiling point and high pressure on the saturation line. Ammonia in critical condition in tank 1(fig. 1), this condition ensures a constant pressure at the outlet of the tank at a constant temperature inside the tank, regardless of the volume of the working fluid. Using ammonia as the working fluid to simplify truster construction without using expulsion system.
Thermodynamic parameters of ammonia in the tank 1 further denoted by index "1". Output Parameters of expension cavity 2 denoted by index "2".
Consider the expiration of ammonia from the high preassure tank to expension cavity. Low temperature gas obeys the equation of state Van-Der-Waals with high accuracy under high pressure. Expiration process is accompanied by the Joule-Thomson effect, which leads to a drop in the temperature of the gas at adiabatic throttling.
In General, a change in temperature when adiabatic throttling can be expressed as the ratio [1]:

$$\frac{\Delta T}{\Delta P}=\frac{\frac{b\cdot R\cdot T}{(V-b)^2}-\frac{2\cdot a}{V^2}}{c_p\cdot\left(\frac{\partial P}{\partial V}\right)_T},$$

where $a,b$ - Van-der-Waals constants; $c_p$ -isobaric heat capacity; $R$ -Universal gas constant; $P$, $T$, $V$ - the pressure, temperature and volume, respectively.

Calculating the derivative $\left(\frac{\partial P}{\partial V}\right)_T$ from Clapeyron equation we get:

$$\frac{\Delta T}{\Delta P}=\frac{\frac{2\cdot a}{R\cdot T}-b}{c_p} \tag{1}$$

Ammonia Van-der-Waals Constants:

$$a_{NH_3}=\frac{27}{64}\cdot\frac{R^2\cdot T_{cr}^2}{P_{cr}},\ b_{NH_3}=\frac{R\cdot T_{cr}}{8\cdot P_{cr}},$$

where $P_{cr}=113.2\cdot 10^5$ Pa; $T_{cr}=405.55$ K - ammonia critical point parameters [2].
Thermodynamic parameters of ammonia in the tank $P_1,\ T_1,\ c_{p1}$.
Thermodynamic parameters of ammonia in the expansion cavity $P_2,\ T_2,\ c_{p2}$.
Then from equation (1) we have:

$$T_2=\left(\frac{\frac{2\cdot a_{NH_3}}{R\cdot T_1}-b_{NH_3}}{c_{p1}}\right)\cdot(P_2-P_1)+T_1 \tag{2}$$

Pressure and heat capacity of ammonia in the tank depend on temperature of the tank, which, in turn, can take different values depending on the light, the work of the on-board equipment. Temperature range of engine construction is in the range -10° C ... + 400°C in space ( e.g., telemetry processing small spacecraft QL series showed the construction temperature range 0°C ... 100° C).
According to [2], it is possible plot the dependence of ammonia pressure in the tank on its temperature at saturation line. Relation is shown in Fig. 2. Gaseous ammonia area is highlighted.

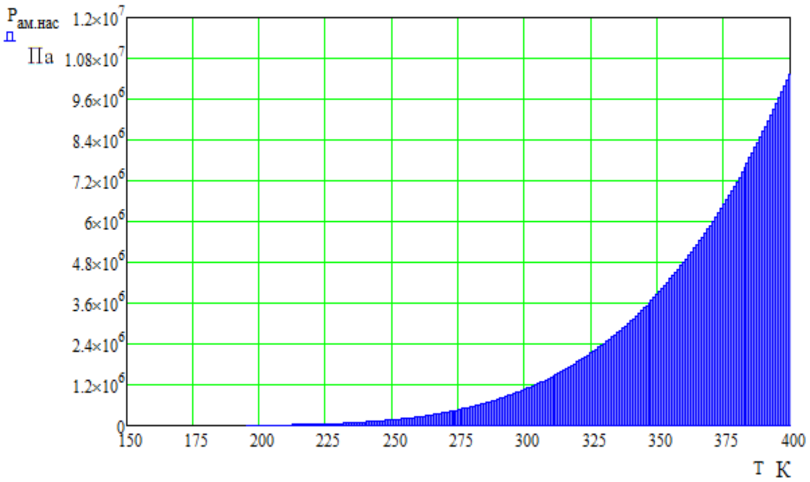


**Figure 2.**Dependence of ammonia pressure on temperature at saturation line.

Expansion cavity pressure is value assigned and is chosen according to the required microengine trust.
In this way dependence (2) allows to get ammonia temperature value in expansion cavity. It is possible to judge in which state of aggregation is working fluid based on obtained parameters and ratio is shown in Fig. 2.
Working fluid falls into heating chamber 4 after cavity 3 (Fig. 1), where gas spead increased by supplying heat. The heating value is calculated from the condition guaranteed obtaining gas fraction of ammonia in the Chamber.
The temperature in the Chamber can be determined from the ratio:

$$\begin{aligned} &Q = m \cdot c_{p1} \cdot \left(T_3 - T_2\right) \\ &Q = m \cdot r_3 \end{aligned} \tag{3}$$

where $Q$ - heat is spent on evaporation; $m$ - ammonia mass; $r_3$ - specific heat of of vaporization of ammonia at the thermodynamic parameters of the gas in the expansion chamber.

Expressing the temperature $T_3$ of the system (3), we get: $T_3 = \frac{r_3}{c'_{p2}} + T_2$. Here is $c'_{p2}$ the heat capacity of liquid ammonia, if the parameters of the working body in expansion cavity presuppose the existence of a liquid phase. If the ammonia in the cavity gaseous can be taken $c'_{p2} = c_{p1}$.

Further, movement of gas is considered as a one-dimensional flowing with heating. Heat supply process makes thermal resistance, full pressure falls [3].

Gas velocity is determined from the Bernoulli equation at existing pressure drop and heating value. In case of the expiration from expansion cavity are:

$$V_2 = \sqrt{\frac{2 \cdot P_2}{\rho_2 \cdot \left(2 \cdot \frac{T_3}{T_2} - 1\right)}} \tag{4}$$

where $\rho_2$ is the density of the gas at the inlet in heating Chamber (density expansion cavity). This value can be determined from equation of State of Van der Waals:

$$\left(P_2 + \frac{a_{NH_3} \cdot \rho_2^2}{M^2}\right) \cdot \left(\frac{M}{\rho_2} - b_{NH_3}\right) = T_2 \cdot R$$

where is the $M$ molecular mass.

Ammonia isentrope index in expension cavity can be determined from the ratio:

$$k_2 = \frac{c''_{p2}}{c''_{p2} - R_{NH_3}} \tag{5}$$

where $c''_{p2}$ is the heat capacity of the gas phase; $R_{NH_3} = 488.162\ \text{kJ/(kg} \cdot \text{K)}$ -ammonia gas constant.

Thus, determining the temperature by the expression (3) and isentrope index by the expression (5), it is can be calculated the critical sound velocity at the outlet from expansion cavity:

$$a_{cr2} = \sqrt{\frac{2 \cdot k_2 \cdot R_{NH_3} \cdot T_2}{k_2 + 1}}.$$

Define relative velocity $\lambda_2 = \frac{V_2}{a_{cr2}}$.

We will use the gas-dynamic function

$$q(\lambda) = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \cdot \lambda \cdot \left(1 - \frac{k-1}{k+1} \cdot \lambda^2\right)^{\frac{1}{k-1}} \tag{6}$$

For expiration from expansion cavity according to (6) we have:

$$q(\lambda_2) = \left(\frac{k_2+1}{2}\right)^{\frac{1}{k_2-1}} \cdot \lambda_2 \cdot \left(1 - \frac{k_2-1}{k_2+1} \cdot \lambda_2^2\right)^{\frac{1}{k_2-1}}$$

Gas-dynamic function

$$z(\lambda_2) = \lambda_2 + \frac{1}{\lambda_2} \quad (7)$$

Gas flows into the heating Chamber after expension cavity, which generally represents a vortex tube. For the convenience of research consider the heating chamber as two consecutive elements: local resistance and cylindrical heating pipe (fig. 3).

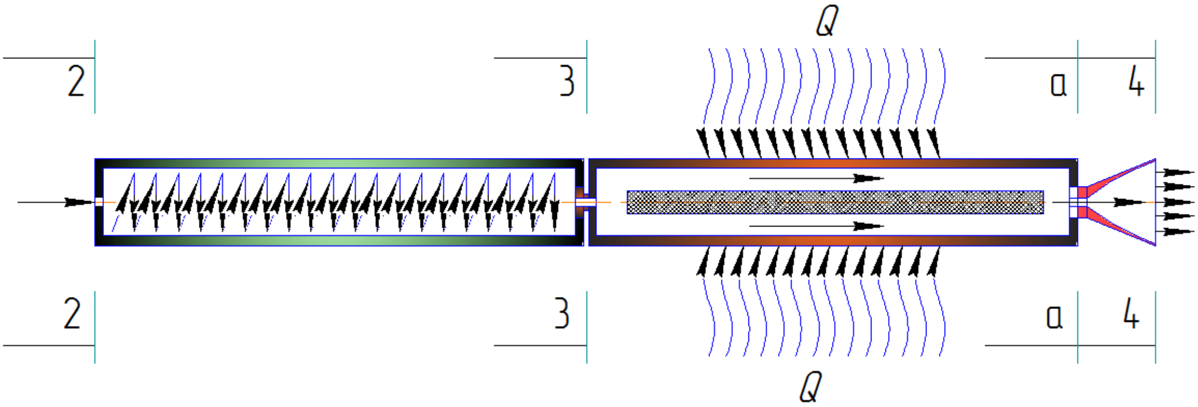


**Figure 3.** Heating chamber physical model: section 2-2 – input gas from expansion cavity; section 3-3 – input gas into heating pipe; a-a – nozzle critical section; section 4-4 – nozzle cross-section; Q – working fluid heating

Relative velocity in critical cross-section a-a at the thermal crisis (achieving sound speed) $\lambda_a = 1$. Then according to expression (7) we have

$$z(\lambda_a) = \lambda_a + \frac{1}{\lambda_a} = 1 + \frac{1}{1} = 2 .$$

Brake temperature in nozzle (on a plot of 3-3-a-a) is equal $T_3 = T_2$.
Preasure loss in engine tract (plot 2-2-3-3 Fig. 3) can be determined by Darcy-Weisbach formule [4] for smooth-wall tube:

$$\Delta P_{TR} = 32 \cdot \nu_2 \cdot \frac{l_{TR}}{d_{TR}^2} \cdot \rho_2 \cdot V_2 ,$$

where $\nu_2$ is the kinematic viscosity of the gas at the inlet into heating Chamber; $l_{TP}$, $d_{TP}$ -length and diameter pipeline respectively.
Then the heating chamber pressure:

$$P_3 = P_2 - \Delta P_{TR} .$$

According to the obtained values of the thermodynamic parameters of the gas in the heating chamber $P_3$ and $T_3$ the reference data [2] define isobaric heat capacity of ammonia $c''_{p3}$. Similarly, the definition of ammonia isentrope index in the expansion cavity heating chamber isentrope index is defined by the expression (5):

$$k_3 = \frac{c''_{p3}}{c''_{p3} - R_{NH_3}} .$$

The critical velocity in the heating Chamber is equal to:

$$a_{cr3} = \sqrt{\frac{2 \cdot k_3 \cdot R_{NH_3} \cdot T_3}{k_3 + 1}}.$$

Total pressure losses in the half-thermal nozzle at the thermal crisis ( $\lambda_a = 1$) can be determined by [3]:

$$P_4 = P_3 \cdot \left(\frac{1+\lambda_2}{2}\right)^2 \cdot \left[\frac{1 - \frac{k_2 - 1}{k_2 + 1}\cdot \lambda_2^2}{\frac{2}{k_2+1}}\right]^{\frac{1}{k_2-1}}$$

or (8)

$$\sigma_{cr} = \frac{P_4}{P_3} = \left(\frac{1+\lambda_2}{2}\right)^2 \cdot \left[\frac{1 - \frac{k_2 - 1}{k_2 + 1}\cdot \lambda_2^2}{\frac{2}{k_2+1}}\right]^{\frac{1}{k_2-1}}$$

Gas consumption through supersonic nozzle is defined by the expression [3]:

$$G_a = \frac{P_3 \cdot F_a}{\sqrt{T_3}} \cdot \left[\frac{2}{k_3 + 1}\right]^{\frac{k_3+1}{2\cdot(k_3-1)}} \cdot \sqrt{\frac{k_3}{R_{NH_3}}} \tag{9}$$

Here $F_a$ - is the critical cross-section square of nozzles (specified by the Designer).
To determine the cross-section of the supply channel of heating chamber use the condition of equality of ammonia rates in the studied cross sections of the engine tract. We have:

$$F_к = \frac{G_a}{q(\lambda_2) \cdot B_{1G} \cdot \frac{P_2}{\sqrt{T_2}}} \tag{10}$$

where $B_{1G} = \sqrt{k_2 \cdot \left(\frac{2}{k_2+1}\right)^{\frac{k_2+1}{k_2-1}}} \cdot \frac{1}{\sqrt{R_{NH_3}}}$ is a constant [5].

From the expression (10) is define the geometric parameters of the flow cross section of the supply channel. Section may be round, rectangular or triangular.
The engine trust is determined by the formula:

$$P_C = P_3 \cdot P_к \cdot \left[\sigma_{cr} \cdot \frac{f(\lambda_4)}{q(\lambda_4)} - f(\lambda_2)\right] \tag{11}$$

$$P_C = P_a \cdot F_a \cdot \left(\lambda_a + \frac{1}{\lambda_a}\right) \cdot \left(\frac{2}{k_3+1}\right)^{\frac{1}{k_3-1}} \tag{12}$$

where: $f(\lambda_4) = (\lambda_4^2 + 1) \cdot \left(1 - \frac{k_3 - 1}{k_3 + 1} \cdot \lambda_4^2\right)^{\frac{1}{k_3-1}}$ is the gas-dynamic function;

$q(\lambda_4) = \left(\frac{k_3+1}{2}\right)^{\frac{1}{k_3-1}} \cdot \lambda_4 \cdot \left(1 - \frac{k_3 - 1}{k_3 + 1} \cdot \lambda_4^2\right)^{\frac{1}{k_3-1}}$ - gas-dynamic function.

Trust value increases with growth $\lambda_4$ at the expiration of the gas in the vacuum ( $P_{vac} = 0$ ). In this case, the gas-dynamic pressure function equals $\pi(\lambda_4) = \frac{P_{vac}}{P_4} = 0$ . It is possible to determine the relative velocity at the nozzle exit:

$$\pi(\lambda_4) = \frac{P_{vac}}{P_4} = \left(1 - \frac{k_3 - 1}{k_3 + 1} \cdot \lambda_4^2\right)^{\frac{k_3}{k_3 - 1}} = 0 .$$

Pressure in a critical section at $\lambda_a = 1$ can be determinem from the equation [3]:

$$\frac{P_3}{P_a} = \left(\frac{k_3 + 1}{2}\right)^{\frac{k_3}{k_3 - 1}} .$$

According to the speed of response of valve (Figure 1) can be judged on the extent of the expansion of the cavity:

$$V_{rp} = \frac{G_a \cdot t}{\rho_2} ,$$

where $t$ is the time of opening and closing the valve section.
The power needed to heat the propellant with rate $G_a$ to the temperature $T_3$ is equal to:

$$P = G_a \cdot c''_{p3} \cdot (T_3 - T_2) \tag{13}$$

At the same time, the specific power of electromagnetic vibrations emitted per unit volume of radioabsorbing material is equal to [6]:

$$P_{sp} = 0.278 \cdot 10^{-12} \cdot f \cdot \varepsilon' \cdot tg\delta \cdot |E|^2 \tag{14}$$

where: $\varepsilon' = \frac{\varepsilon_a'}{\varepsilon_0}$ $\varepsilon_a'$ - is the real part of the absolute dielectric permittivity of medium; $\varepsilon_0 = 8.85418 \cdot 10^{-12}$ $\frac{\Phi}{m}$ -Electric constant; $f$ - microwave frequency ( for microwave heating installation are distinguish the following frequencies: 433 Mhz ± 0.2%, 915 Mhz ± 2.73%; 2450 Mhz ± 2.04%, 5800 Mhz ± 1.29%, 22125 Mhz ± 0.56% [6]); $tg\delta = \frac{\varepsilon''_a}{\varepsilon_a'} + \frac{\sigma}{\omega \cdot \varepsilon_a'}$ -dielectric loss tangent; $\varepsilon''_a$ -minimal part of the absolute dielectric permittivity of medium; $\sigma$ -the specific conductivity of the medium; $\omega = 2 \cdot \pi \cdot f$ -circular frequency; $E$ -intensity of the electric field.

Dielectric loss tangent in the General case $tg\delta \ll 1$, and the amplitude of the electric field in dielectric fades. For metallic conductors (and graphite) $tg\delta \gg 1$, due to the high conductivity [7]. The result is a more intense heat at a low intensity electrical field.

An important feature of the conductors used in microwave technology is the depth of the a screen-layer. It depends on the magnetic permeability of medium. In this case, graphite, like diamagnetik, has an advantage over metal because it possesses more depth screen-layer.

## 4. Research Results

The research of elementary electrothermal micro engine was carried out according to a mathematical model.

Input data for calculation are shown in Table I.

**Table 1.**Input data

| Parameter | Value | U. measurement |
|---|---|---|
| Working fluid | ammonia | - |
| Engine case temperature | 280 | K |
| Critical diameter nozzle cross-section | $0.8 \cdot 10^{-3}$ | m |
| The inner diameter of the working fluid piplines | $2 \cdot 10^{-3}$ | m |
| The total length of pipelines | 0.8 | m |
| Pressure in the expansion cavity | $1 \cdot 10^{4}$ | Pa |
| The time of opening-closing of the valve | 2 | s |
| Frequency of microwave radiation | $2.45 \cdot 10^{9}$ | Hz |

Solving equation (1)...(14) were obtained design values of of thermodynamic parameters of engine and its power characteristics. The main results of the research are presented in Table II.

**Table 2.**Research Results

| Parameter | Designation | Value | U. measurement |
|---|---|---|---|
| Van der Waals constants | $a_{NH_3}$ | $4.237 \cdot 10^{5}$ | $N \cdot m^4 / k-mol^2$ |
| | $b_{NH_3}$ | 0.037 | $m^3 / k-mol$ |
| The gas temperature in the expansion cavity | $T_2$ | 213.326 | K |
| The density of the gas in the expansion cavity | $\rho_2$ | 0.0961 | kg/m$^3$ |
| The temperature in the heating Chamber | $T_3$ | 533.239 | K |
| Gas pressure in the heating Chamber | $P_3$ | $9.95 \cdot 10^{3}$ | Pa |
| Critical velocity at the inlet in the heating Chamber | $a_{cr2}$ | 344.049 | m/s |
| Critical velocity at the inlet in the nozzle | $a_{cr3}$ | 540.982 | m/s |

| | | | |
|---|---|---|---|
| Nozzle gas consumption | $G_a$ | $6.385 \cdot 10^{-6}$ | kg/s |
| Engine trust | $P_C$ | $3.496 \cdot 10^{-3}$ | N |
| The power to heat the working fluid | $P$ | 5.477 | W |
| Expansion cavity volume | $V_{RP}$ | $1.328 \cdot 10^{-4}$ | $m^3$ |

As for microwave heating, the resulting gas-dynamic calculation of engine power performance allow to approach the issue of choice of radar absorbing material. According to the formula (14) at the volume of material $V_{RP} = 1 \cdot 10^{-6}$ $m^3$ complex value can be determined:

$$\varepsilon' \cdot tg\delta \cdot |E|^2 = \frac{P}{0.278 \cdot 10^{-12} \cdot f \cdot V_{RP}} = \\ = \frac{5.477}{0.278 \cdot 10^{-12} \cdot 2.45 \cdot 10^9 \cdot 1 \cdot 10^{-6}} = 8.042 \cdot 10^9 \qquad (15)$$

## 5. Results and Discussion

The results of this research showed that the simplest electrothermal thruster for picosatellites has low power consumption and a simple functional dependence of the thermodynamic parameters of the design temperature of the apparatus. Having plotted the engine characteristics for all possible temperatures we can get the field of possible engine thrust and delineate the scope of applicability. Regulation of engine thrust may be controlled by changing of pressure in the expansion chamber, which in turn is achieved by the opening time of the solenoid. Working fluid heating quantity in the chamber is dependent magnitude, but it can be used as a control feedback. Calculation was carried out for case not-constant isentrope index of engine tract, which introduces additional bonds in the construction of mathematical models.

Selecting the method of heat transmission to working body is also relevant. In this research microwave method of power transmission to working fluid is offered. This method differs by fast volumetric heating of the working fluid or surface circumfluous by working fluid.

As in this research as the working fluid ammonia was selected, which is not a conductor and is transparent to microwave radiation, the method considered heat transmission by radar absorbing material (e.g., graphite, copper). It should be noted the possibility of additives (for example, water, which is infinitely soluble in ammonia), which will enhance the conductivity of the working fluid and, accordingly, will disappear in need of radioabsorbing mediators.

## 6. Conclusion

Research leads to the following conclusions:

- ammonia Consumption is 6.385 mg/s;
- Engine trust is 3.496 mN;
- The temperature in the heating chamber 533.329 k;
- Working fluid heating power 5.477 W.

Diamagnetic and ferromagnetic material with high thermal conductivity and a deep screen-layer suitable as radar absorbing inverter microwave energy into thermal energy according to the formula (15).

Findings allow to start designing microwave generators for electrothermal engines with ultra small thrust.

From the calculation follows that the electrothermal thruster can be used to create a corrective micro-thrust for ultrasmall spacecrafts like CubeSat. These apparatus are low-powered, so such propulsion system of this satellites should to consume no more than 10 Watts. Another feature of this class of spacecrafts is that they used in corrective propulsion systems and shall not has high thrust. This is due to the minimization of disturbing moment of the expiration of gas through a jet nozzle.
In addition to the heat in the engine there is only one electricity consumer is a solenoid valve. Currently, production is able to provide developers with low-power small-sized high-speed low flow valves with the consumption up to 2 W.
It can be seen that the total forecasted consumption power of the engine does not exceed 8 W.

## 7. Acknowledgments

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