

Adomian decomposition method used to solve the one-dimensional acoustic equations

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Abstract. In this paper we propose the use of Adomian decomposition method to solve one-dimensional acoustic equations. This recursive method can be calculated easily and the result is an approximation of the exact solution. We use the Maple software to compute the series in the Adomian decomposition. We obtain that the Adomian decomposition method is able to solve the acoustic equations with the physically correct behavior.

1. Introduction

Physical phenomena are interesting study for the last decades. Many methods have been developed successfully to solve partial differential equations that help to analyze real-world problems. One interesting real-world problem is wave propagation. This paper considers the one-dimensional wave equations, in particular the one-dimensional acoustic equations. Acoustic equations can be derived from the nonlinear elastic equations, as described by LeVeque [1].

In this paper, the acoustic equations are solved using the Adomian decomposition method. Series in the Adomian decomposition method converges to the exact solution. The method attacks the problem in a straightforward way without any transformation formulas [2]. The method implements an integral operator to solve differential equations. When the exact solution cannot be expressed explicitly, the Adomian decomposition method results in approximations of the exact solution [3–6]. More terms involved in the Adomian decomposition solution series result in a more accurate approximation. To our knowledge, our work is the first in applying the Adomian decomposition method to solve the acoustic equations.

The rest of the paper is organized as follows. First, we write the problem description. Then we present the Adomian decomposition method. After that, we provide some computational results and discussion. Finally, concluding remarks are drawn.

2. Problem description

In this section, we describe the problem (the mathematical model) that we want to solve. Starting from the general model, we simplify the model into the simplest form of acoustic equations.

The general form of the acoustic equations are [7,8]

$$p_t + K(x) u_x = 0, \quad (1)$$



$$\rho(x) u_t + p_x = 0. \quad (2)$$

Here $p(x, t)$ denotes the pressure, $u(x, t)$ represents the velocity, x is the one-dimensional space variable, and t is the time variable. In addition, $K(x)$ is the bulk modulus of compressibility and $\rho(x)$ is the density. We use derivative operators $p_t = \partial p / \partial t$, $p_x = \partial p / \partial x$, $u_t = \partial u / \partial t$ and $u_x = \partial u / \partial x$.

Taking $K(x) = 1$ and $\rho(x) = 1$, we obtain the acoustic equations in the simplest form

$$p_t + u_x = 0, \quad (3)$$

$$u_t + p_x = 0. \quad (4)$$

Our goal in this paper is to solve (3) and (4) using the Adomian decomposition method.

3. Adomian decomposition method

In this section, we present the Adomian decomposition method to solve the acoustic equations.

To see how the Adomian decomposition method works, let us start by noting derivative operators $L_t = \partial / \partial t$ and $L_x = \partial / \partial x$, so (3) and (4) become

$$L_t p + L_x u = 0, \quad (5)$$

$$L_t u + L_x p = 0. \quad (6)$$

The inverse for derivative operators for L_t and L_x are $L_t^{-1} = \int_0^t (\cdot) dt$ and $L_x^{-1} = \int_0^x (\cdot) dx$. In this paper, we only take the inverse with respect to the t variable. Applying the L_t^{-1} to the both sides of (5) and (6), we obtain

$$L_t^{-1} L_t p + L_t^{-1} L_x u = 0, \quad (7)$$

$$L_t^{-1} L_t u + L_t^{-1} L_x p = 0, \quad (8)$$

or

$$p(x, t) = p(x, 0) - L_t^{-1} L_x u, \quad (9)$$

$$u(x, t) = u(x, 0) - L_t^{-1} L_x p. \quad (10)$$

The variables $p(x, t)$ and $u(x, t)$ have to be written in series of Adomian polynomials

$$p(x, t) = \sum_{n=0}^{\infty} p_n(x, t), \quad (11)$$

$$u(x, t) = \sum_{n=0}^{\infty} u_n(x, t). \quad (12)$$

Applying the Adomian polynomials to the both sides, where $p_0 = p(x, 0)$ and $u_0 = u(x, 0)$, we obtain

$$\sum_{n=0}^{\infty} p_n = p(x, 0) - L_t^{-1} L_x \sum_{n=0}^{\infty} u_n, \quad (13)$$

$$\sum_{n=0}^{\infty} u_n = u(x, 0) - L_t^{-1} L_x \sum_{n=0}^{\infty} p_n, \quad (14)$$

or

$$p_0 + p_1 + p_2 + p_3 + \cdots = p(x, 0) - L_t^{-1} (L_x (u_0 + u_1 + u_2 + u_3 + \cdots)), \quad (15)$$

$$u_0 + u_1 + u_2 + u_3 + \cdots = u(x, 0) - L_t^{-1}(L_x(p_0 + p_1 + p_2 + p_3 + \cdots)). \quad (16)$$

The results of p and u decompositions are

$$p_0(x, t) = p(x, 0), \quad (17)$$

$$p_1(x, t) = -L_t^{-1}(L_x(u_0(x, t))), \quad (18)$$

$$p_2(x, t) = -L_t^{-1}(L_x(u_1(x, t))), \quad (19)$$

$$p_3(x, t) = -L_t^{-1}(L_x(u_2(x, t))), \quad (20)$$

$$\vdots$$

$$u_0(x, t) = u(x, 0), \quad (21)$$

$$u_1(x, t) = -L_t^{-1}(L_x(p_0(x, t))), \quad (22)$$

$$u_2(x, t) = -L_t^{-1}(L_x(p_1(x, t))), \quad (23)$$

$$u_3(x, t) = -L_t^{-1}(L_x(p_2(x, t))), \quad (24)$$

$$\vdots$$

For computational experiments in the next section, we choose initial conditions

$$p(x, 0) = 0.1 \operatorname{sech}^2(0.2x), \quad (25)$$

$$u(x, 0) = 0, \quad (26)$$

for (3) and (4). We choose hyperbolic secant function because the function is smooth, so it has continuous derivatives. The amplitude phase and constant are taken to be 0.1 and 0.2, respectively. In all calculations in this paper, all quantities are assumed to have SI units with the MKS system.

4. Computational results

Adomian decomposition method needs some recursive iterations to get the approximation to the exact solution. We note that more iterations lead to more accurate solution with this method if the series is not yet convergent to the exact solution.

Using initial conditions (25) and (26), Adomian decomposition method leads to the following series formulas

$$p_{k+1}(x, t) = -L_t^{-1} L_x \sum_{k=0}^{\infty} u_k, \quad k \geq 0, \quad (27)$$

$$u_{k+1}(x, t) = -L_t^{-1} L_x \sum_{k=0}^{\infty} p_k, \quad k \geq 0, \quad (28)$$

where the exact solution is given by

$$\lim_{n \rightarrow \infty} P_n = p(x, t), \quad (29)$$

$$\lim_{n \rightarrow \infty} U_n = u(x, t). \quad (30)$$

The n -term approximations of the density p and the velocity u are

$$P_n[p] = \sum_{k=0}^{n-1} p_k(x, t), \quad n \geq 0, \quad (31)$$

$$U_n[u] = \sum_{k=0}^{n-1} u_k(x, t), \quad n \geq 0. \quad (32)$$

Using the Maple software, we obtain that the results of the iterations (up to p_4) for the pressure solution to our problem are expressed as follows:

$$p_0 = \frac{1}{10} \operatorname{sech}\left(\frac{1}{5}x\right)^2, \quad (33)$$

$$p_1 = 0, \quad (34)$$

$$p_2 = \frac{1}{125} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^2 t^2 - \frac{1}{50} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right) t^2, \quad (35)$$

$$p_3 = 0, \quad (36)$$

$$\begin{aligned} p_4 = & \frac{1}{9375} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^4 t^4 - \frac{11}{3750} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^2 \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right) t^4 \\ & + \frac{1}{375} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right)^2 t^4. \end{aligned} \quad (37)$$

Furthermore, the results of the iterations (up to u_4) for the velocity solution to our problem are expressed as follows:

$$u_0 = 0, \quad (38)$$

$$u_1 = \frac{1}{25} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right) t, \quad (39)$$

$$u_2 = 0, \quad (40)$$

$$\begin{aligned} u_3 = & \frac{2}{1875} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^3 t^3 \\ & - \frac{4}{375} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right) \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right) t^3, \end{aligned} \quad (41)$$

$$u_4 = 0. \quad (42)$$

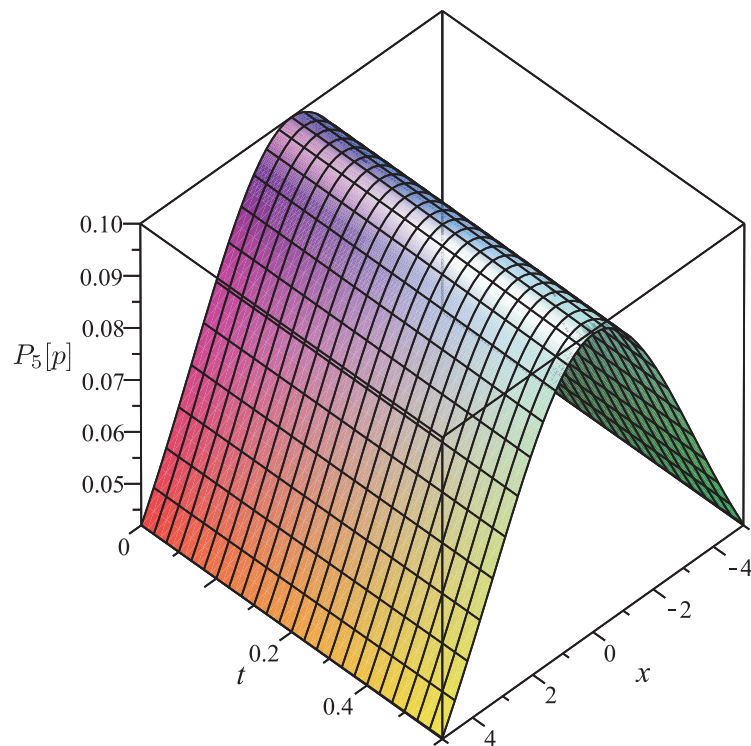


Figure 1. The approximate pressure $P_5[p]$ of the Adomian decomposition.

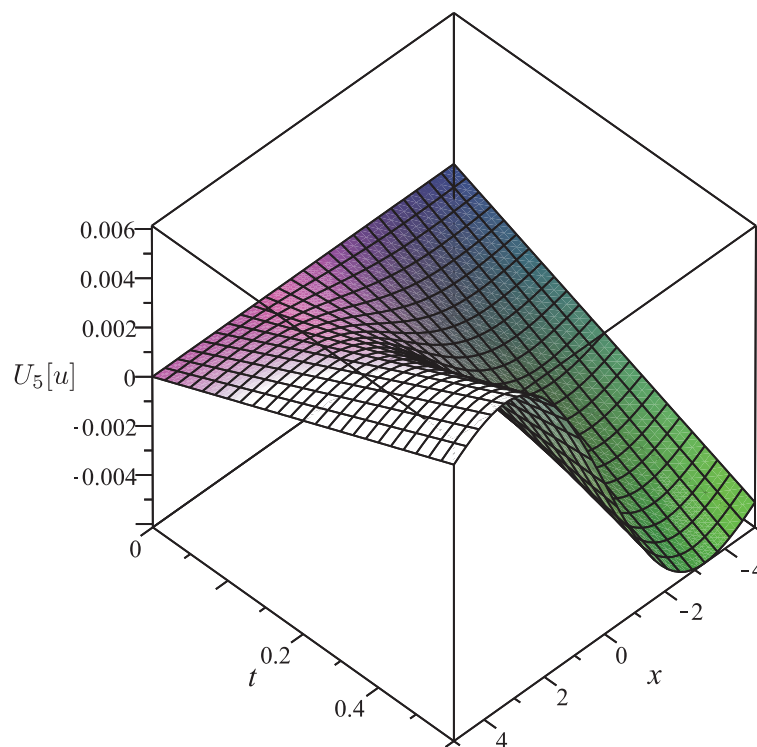


Figure 2. The approximate velocity $U_5[u]$ of the Adomian decomposition.

Now, we compute $P_5[p] = p_0 + p_1 + p_2 + p_3 + p_4$ and $U_5[u] = u_0 + u_1 + u_2 + u_3 + u_4$ using the above results. Therefore, we obtain that the approximate pressure and velocity are

$$\begin{aligned}
 P_5 = & \frac{1}{10} \operatorname{sech}\left(\frac{1}{5}x\right)^2 + \frac{1}{125} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^2 t^2 \\
 & - \frac{1}{50} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right) t^2 \\
 & + \frac{1}{9375} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^4 t^4 \\
 & - \frac{11}{3750} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^2 \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right) t^4 \\
 & + \frac{1}{375} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right)^2 t^4,
 \end{aligned} \tag{43}$$

and

$$\begin{aligned}
 U_5 = & \frac{1}{25} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right) t + \frac{2}{1875} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right)^3 t^3 \\
 & - \frac{4}{375} \operatorname{sech}\left(\frac{1}{5}x\right)^2 \tanh\left(\frac{1}{5}x\right) \left(\frac{1}{5} - \frac{1}{5} \tanh\left(\frac{1}{5}x\right)^2\right) t^3,
 \end{aligned} \tag{44}$$

respectively.

Results of the pressure P_5 the velocity U_5 are plotted in figure 1 and figure 2, respectively. From these figures, the pressure from the origin point propagates to the left and to the right directions as the time evolves. The velocity mimics the pressure wave propagation, because the velocity is negative when the pressure wave propagates to the left, and the velocity is positive when the pressure wave propagates to the right. This is the correct behavior that we expect. In figure 2, the velocity tends to zero for large x and t .

5. Conclusion

We have solved the acoustic equations using the Adomian decomposition method. The method produces solutions any time and space point. The Adomian decomposition method can be implemented in a computer software with an inexpensive computation. The method is projected to be successful for solving multidimensional problems of acoustics.

Acknowledgments

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