

The Preventive Control of a Dengue Disease Using Pontryagin Minimum Principal

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Abstract. Behaviour analysis for host-vector model without control of dengue disease is based on the value of basic reproduction number obtained using next generation matrices. Furthermore, the model is further developed involving a preventive control to minimize the contact between host and vector. The purpose is to obtain an optimal preventive strategy with minimal cost. The Pontryagin Minimum Principal is used to find the optimal control analytically. The derived optimality model is then solved numerically to investigate control effort to reduce infected class.

1. Introduction

In the host-vector epidemic model, the population is divided into the host, in this case humans, and the vector, the *Aedes aegypti* mosquito as the carrier of the dengue virus. Research on the dynamic behaviour of the spread of dengue fever using a host-vector model by assuming that there is immunity after recovery has been performed by Esteva [1]. But in reality, dengue fever is caused by the dengue virus that has four serotypes, and a person can be infected by dengue fever again with a different serotype. The mathematical model that addresses this has been done by Soewono and Apriatna [2]. Their model has assumed of becoming susceptible again after recovering from a dengue fever, but has not considered the possibility of infected hosts that cannot spread the dengue fever.

Therefore, this paper discusses the extension of a dengue fever model where after recovery a host can be susceptible again. In addition, it is also assumed that there are some infected hosts but cannot spread the dengue fever. The proposed model (i.e. the first proposed model) and then will be analysed on its behaviour based on the value of the basic reproduction number ([3], [4], [5]).

Furthermore, the first model will be expanded by adding a control variable. Research on the optimal control using control insecticide has been carried out by Rodrigues [6]. The results show that the control must be done repeatedly for the best result. But this leads to the increasing of the cost control. Dumont's research result [7] shows that using a combination treatment with chemicals, larvae control and adult mosquitoes can decrease the epidemic. Of course it is not easy to be done as it requires more data.

On the other hand, WHO [8] brings out some strategies to resolve the epidemic by prevention and control. Therefore, this paper also discusses the behaviour of our first proposed model if we give a preventive control variable to the model (i.e. second proposed model). This second model then will be



analysed analytically by Pontryagin Minimum Principle. The behaviour of the first and second model (before and after being given controls) will be done numerically. At the end, we obtain the most effective control recommendations to minimize the dengue epidemic.

2. Mathematical Model of Dengue Disease without Control

The host population is divided into susceptible class, infectious class and recoverable (immune) class. Vector population are divided into susceptible class and infectious class. There is no recoverable (immune) class in the vector population as mosquitoes will always carry a dengue virus in their body for all their life. The spread of the dengue fever starts from the infected host is bitten by the vulnerable mosquito, resulting in the transmission of dengue virus to the mosquito, so the mosquito becomes infected. If the infected mosquito then bites the vulnerable host, then there will be a transmission which result on the host becomes infected. And so on.

Assume $S_H(t)$ represents the host population on the vulnerable class at t , $I_H(t)$ represents the host population for the infectious class at t , $R_H(t)$ represents the host population for the recoverable class at t , $S_V(t)$ denotes the vector population for the vulnerable class at t , and $I_V(t)$ denotes the vector population for infectious class at t . The population total for the host and the vector is denoted by N_H and N_V , respectively, and assumed they have a constant value.

The birth and death rate in the host population are assumed to be equal, denoted by μ_H . If b denotes the number of mosquito bites per day, then for N_V mosquitoes there will be bN_V mosquito bites per day. This means that the host will receive as much as $\frac{bN_V}{N_H} \frac{I_V}{N_V}$ infected mosquito bite per day. If β_H denotes the transmission rate from the mosquito to the host, then the infection rate of the susceptible hosts is defined as $\beta_H \frac{bN_V}{N_H} \frac{I_V}{N_V}$ or $\beta_H \frac{bI_V}{N_H}$. We assumed that the recoverable host can be susceptible again to the dengue fever, and α denotes this state. The recovery rate is denoted by γ_H .

The growth rate for the population vector is denoted by A and the mortality rate for the vector is denoted by μ_V . As one mosquito will bite $\frac{b}{N_H}$ per day per person, then one mosquito will get the virus by $\frac{b}{N_H} I_H$ per day. If β_V denotes the transmission rate from the host to the mosquito, the infection rate per susceptible mosquito is $\beta_V \frac{b}{N_H} I_H$. Furthermore, in this first proposed model, we also calculate the percentage of the infected hosts that cannot spread the dengue fever (ϵ). It means there will be $(1-\epsilon)$ infected hosts that can spread the dengue fever. Thus, we obtain the infection rate per susceptible mosquito as $\beta_V \frac{b}{N_H} (1-\epsilon) I_H$.

The following Fig.1 is the transmission diagram for the spread of the dengue fever by taking all the above assumptions.

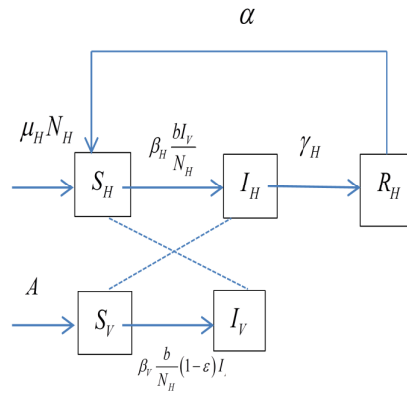


Figure 1. Dengue Fever Transmission Diagram

The mathematics model for the dengue fever transmission is given as follow.

$$\frac{dS_H(t)}{dt} = \mu_H N_H - \frac{\beta_H b S_H I_V}{N_H} - \mu_H S_H + \alpha R_H \quad (1a)$$

$$\frac{dI_H(t)}{dt} = \frac{\beta_H b S_H I_V}{N_H} - \mu_H I_H - \gamma_H I_H \quad (1b)$$

$$\frac{dR_H(t)}{dt} = \gamma_H I_H - \mu_H R_H - \alpha R_H \quad (1c)$$

$$\frac{dS_V(t)}{dt} = A - \frac{\beta_V b S_V (1 - \epsilon) I_H}{N_H} - \mu_V S_V \quad (1d)$$

$$\frac{dI_V(t)}{dt} = \frac{\beta_V b S_V (1 - \epsilon) I_H}{N_H} - \mu_V I_V \quad (1e)$$

where $S_H + I_H + R_H = N_H$ and $S_V + I_V = N_V$. As μ_V denotes the mortality transmission for the vector, thus the total of mortality for vector population is denotes by $\mu_V N_V$. In the other hand, the rate of change of the vector population is denoted by $\frac{dN_V}{dt}$, which is the recruitment rate subtracted by the total of mortality, means that $\frac{dN_V}{dt} = A - \mu_V N_V$. Hence, for $t \rightarrow \infty$, the solution of N_V towards $\frac{A}{\mu_V}$.

We obtain $S_V + I_V = \frac{A}{\mu_V}$.

Let the proportion for each class is denoted by

$$x_1 = \frac{S_H}{N_H}, x_2 = \frac{I_H}{N_H}, x_3 = \frac{R_H}{N_H}, x_4 = \frac{S_V}{A/\mu_V}, \text{ and } x_5 = \frac{I_V}{A/\mu_V}$$

and given that $x_3 = 1 - (x_1 + x_2)$, $x_4 = 1 - x_5$. Equation (1a) can be written as

$$\begin{aligned} \frac{1}{N_H} \left(\frac{dS_H(t)}{dt} \right) &= \frac{1}{N_H} \left(\mu_H N_H - \frac{\beta_H b S_H I_V}{N_H} - \mu_H S_H + \alpha R_H \right) \\ \Leftrightarrow \frac{dx_1}{dt} &= \mu_H - \frac{\beta_H b A / \mu_V}{N_H} x_1 x_5 - \mu_H x_1 + \alpha (1 - (x_1 + x_2)) \end{aligned} \quad (2a)$$

while for (1b), we obtain

$$\begin{aligned} \frac{1}{N_H} \left(\frac{dI_H(t)}{dt} \right) &= \frac{1}{N_H} \left(\frac{\beta_H b S_H I_V}{N_H} - \mu_H I_H - \gamma_H I_H \right) \\ \Leftrightarrow \frac{dx_2}{dt} &= \frac{\beta_H b A / \mu_V}{N_H} x_1 x_5 - \mu_H x_2 - \gamma_H x_2 \end{aligned} \quad (2b)$$

Using the same way, we derive (1e) as follow.

$$\begin{aligned} \frac{1}{A / \mu_V} \left(\frac{dI_V(t)}{dt} \right) &= \frac{1}{A / \mu_V} \left(\frac{\beta_V b S_V (1 - \varepsilon) I_H}{N_H} - \mu_V I_V \right) \\ \Leftrightarrow \frac{dx_5}{dt} &= \beta_V b (1 - \varepsilon) (1 - x_5) x_2 - \mu_V x_5. \end{aligned} \quad (2c)$$

The equilibrium point of System (2) is achieved when $\frac{dx_1}{dt} = 0, \frac{dx_2}{dt} = 0, \frac{dx_5}{dt} = 0$. Using this condition, we have two equilibrium points that are $(1, 0, 0)$, which is called as disease free, and (x_1^*, x_2^*, x_5^*) , with

$$\begin{aligned} x_1^* &= \frac{N_H (\mu_H + \gamma_H) \mu_V ((\mu_H + \gamma_H) + \alpha) \mu_V - (\mu_H + \alpha) (\varepsilon \beta_V b - \beta_V b)}{(\beta_V b - \varepsilon \beta_V b) (\beta_H b A (\mu_H + \gamma_H) + \alpha \beta_H b A) - (\varepsilon \beta_V b - \beta_V b) (\mu_H + \alpha) (N_H (\mu_H + \gamma_H) \mu_V)} \\ x_2^* &= \frac{(\mu_H + \alpha) \beta_H b A (\beta_V b - \varepsilon \beta_V b) - (\mu_H + \alpha) (N_H (\mu_H + \gamma_H) \mu_V^2)}{(\beta_V b - \varepsilon \beta_V b) ((\beta_H b A (\mu_H + \gamma_H) + \alpha \beta_H b A)) - (\varepsilon \beta_V b - \beta_V b) (\mu_H + \alpha) (N_H (\mu_H + \gamma_H) \mu_V)} \\ x_5^* &= \frac{(\mu_H + \alpha) \beta_H b A (\beta_V b - \varepsilon \beta_V b) - (\mu_H + \alpha) (N_H (\mu_H + \gamma_H) \mu_V^2)}{((\beta_H b A (\mu_H + \gamma_H) + \alpha \beta_H b A) \mu_V - (\mu_H + \alpha) \beta_H b A (\varepsilon \beta_V b - \beta_V b))}, \end{aligned}$$

which is called as the endemic equilibrium point.

Then, we will discuss about the value of secondary infections as a result of the first infection. This value is called the basic reproduction number. To determine this value, we use the next generation matrix method [9].

Defined matrix F as a matrix whose entries are the first derivative of all terms that "enters" the infectious class at $(1, 0, 0)$.

$$F|_{(1,0,0)} = \begin{bmatrix} \frac{\partial \left(\frac{\beta_H b A}{N_H \mu_V} \right)_{x_1 x_5}}{\partial x_2} & \frac{\partial (\beta_V b (1 - \varepsilon))_{x_2}}{\partial x_2} \\ \frac{\partial \left(\frac{\beta_H b A}{N_H \mu_V} \right)_{x_1 x_5}}{\partial x_5} & \frac{\partial (\beta_V b (1 - \varepsilon))_{x_2}}{\partial x_5} \end{bmatrix}_{(1,0,0)} = \begin{bmatrix} 0 & \beta_V b (1 - \varepsilon) \\ \frac{\beta_H b A}{N_H \mu_V} & 0 \end{bmatrix}$$

We also define matrix V whose entries are the first derivatives from the terms that are “out” from the infectious class at $(1,0,0)$.

$$V|_{(1,0,0)} = \begin{bmatrix} \frac{\partial(\mu_H + \gamma_H)x_2}{\partial x_2} & \frac{\partial(\beta_V b x_2 x_5 + \mu_V x_5)}{\partial x_2} \\ \frac{\partial(\mu_H + \gamma_H)x_2}{\partial x_5} & \frac{\partial(\beta_V b x_2 x_5 + \mu_V x_5)}{\partial x_5} \end{bmatrix}_{(1,0,0)} = \begin{bmatrix} \mu_H + \gamma_H & \beta_V b x_5 \\ 0 & \beta_V b x_2 + \mu_V \end{bmatrix}_{(1,0,0)} = \begin{bmatrix} \mu_H + \gamma_H & 0 \\ 0 & \mu_V \end{bmatrix}$$

Let G is a product of matrix F and V^{-1} , then

$$G = \begin{bmatrix} 0 & \frac{\beta_V b(1-\varepsilon)}{\mu_V} \\ \frac{\beta_H b A}{N_H \mu_V (\mu_H + \gamma_H)} & 0 \end{bmatrix}$$

The value of the basic reproduction number is obtained from the biggest eigen value from matrix G ,

which is $\bar{R}_0 = \sqrt{\frac{\beta_V b(1-\varepsilon)\beta_H b A}{N_H \mu_V^2 (\mu_H + \gamma_H)}}$. If

$$R_0 = \frac{\beta_V b(1-\varepsilon)\beta_H b A}{N_H \mu_V^2 (\mu_H + \gamma_H)}, \quad (3)$$

then $\bar{R}_0 = \sqrt{R_0}$.

Based on the value of the basic reproduction number (3), it is easy to show if $R_0 < 1$, then equilibrium point $(1,0,0)$ is asymptotically stable, and if $R_0 > 1$, the equilibrium point $(1,0,0)$ is not stable. Furthermore, the endemic equilibrium point (x_1^*, x_2^*, x_5^*) is stable if $R_0 > 1$.

3. Optimal Control

This section discusses about the extension of the transmission model of dengue fever (1) controlled by the prevention control variable so that the contact between the host and the mosquito can be reduced, and denoted by u_1 , where $0 \leq u_1 \leq 1$. We obtain the following model

$$\frac{dS_H(t)}{dt} = \mu_H N_H - (1-u_1) \frac{\beta_H b S_H I_V}{N_H} - \mu_H S_H + \alpha R_H \quad (4a)$$

$$\frac{dI_H(t)}{dt} = (1-u_1) \frac{\beta_H b S_H I_V}{N_H} - \mu_H I_H - \gamma_H I_H \quad (4b)$$

$$\frac{dR_H(t)}{dt} = \gamma_H I_H - \mu_H R_H - \alpha R_H \quad (4c)$$

$$\frac{dS_V(t)}{dt} = A - (1-u_1) \frac{\beta_V b S_V (1-\varepsilon) I_H}{N_H} - \mu_V S_V \quad (4d)$$

$$\frac{dI_V(t)}{dt} = (1-u_1) \frac{\beta_V b S_V (1-\varepsilon) I_H}{N_H} - \mu_V I_V \quad (4e)$$

It can be seen that the infection from the susceptible host occurs because there is a contact with the infected mosquito, thus the infection rate is $\beta_H \frac{bS_H I_V}{N_H}$. In the other hand, assume that the prevention control in order to reduce the contact between the host and the mosquito is denoted by u_1 , thus $(1-u_1)$ is the possibility of control failure. Hence, the infection rate will become $(1-u_1) \frac{\beta_H bS_H I_V}{N_H}$ if the prevention control is given. Meanwhile, the infection of the susceptible mosquito happens because there is a contact with the host, thus the infection rate per susceptible mosquito is $\beta_V \frac{b}{N_H} S_V (1-\varepsilon) I_H$. If the prevention control is given, then the infection rate will become $(1-u_1) \beta_V \frac{b}{N_H} S_V (1-\varepsilon) I_H$. The prevention efforts in this case could be a counselling about the dengue fever, the provision of the abate powder or the fogging readiness if necessary.

To investigate the optimality of those efforts, given the objective function J , namely to minimize the infected host class and the cost to implement the u_1 control,

$$J(u_1) = \int_0^T (BI_H(t) + Cu_1^2(t)) dt \quad (5)$$

where B is the weights for the infected hosts class, while C is the weights for prevention efforts. It means that we will find the solution from (5), which minimizes the number of the infected hosts with a minimal $u_1(t)$ control cost. In other words, will be sought so an u_1^* optimal control such that

$$J(u_1^*) = \min \{J(u_1) | u_1 \in \Gamma\} \quad (6)$$

with system (4) as a constraint and $\Gamma = \{u_1 | u_1 \text{ measurable, with } 0 \leq u_1 \leq 1\}$ is a control set.

Further, we define Hamiltonian as follow.

$$\begin{aligned} H = & BI_H(t) + Cu_1^2(t) \\ & + \delta_1 \left(\mu_H N_H - (1-u_1) \frac{\beta_H bS_H I_V}{N_H} - \mu_H S_H + \alpha R_H \right) + \delta_2 \left((1-u_1) \frac{\beta_H bS_H I_V}{N_H} - \mu_H I_H - \gamma_H I_H \right) \\ & + \delta_3 (\gamma_H I_H - \mu_H R_H - \alpha R_H) + \delta_4 \left(A - (1-u_1) \frac{\beta_V bS_V (1-\varepsilon) I_H}{N_H} - \mu_V S_V \right) \\ & + \delta_5 \left((1-u_1) \frac{\beta_V bS_V (1-\varepsilon) I_H}{N_H} - \mu_V I_V \right) \end{aligned}$$

where $\delta_1, \delta_2, \dots, \delta_5$ are costate variable. Furthermore, we use Minimum Pontryagin Principal to give an idea about the existence of optimal control through the following Lemma.

Lemma 1

Given u_1^* as the optimal control that minimizes $J(u_1)$ for Γ with System (4) as the constraint then there are costate variables $\delta_1, \delta_2, \dots, \delta_5$ such that

$$\begin{aligned}
\frac{d\delta_1}{dt} &= \delta_1 \left((1-u_1) \frac{\beta_H b I_V}{N_H} + \mu_H \right) - \delta_2 (1-u_1) \frac{\beta_H b I_V}{N_H} \\
\frac{d\delta_2}{dt} &= -B + \delta_2 (\mu_H + \gamma_H) - \delta_3 \gamma_H + \delta_4 (1-u_1) \frac{\beta_V b S_V (1-\varepsilon)}{N_H} - \delta_5 (1-u_1) \frac{\beta_V b S_V (1-\varepsilon)}{N_H} \\
\frac{d\delta_3}{dt} &= -\alpha \delta_1 + \delta_3 (\mu_H + \alpha) \\
\frac{d\delta_4}{dt} &= \delta_4 (1-u_1) \frac{\beta_V b I_H (1-\varepsilon)}{N_H} - \delta_5 (1-u_1) \frac{\beta_V b I_H (1-\varepsilon)}{N_H} \\
\frac{d\delta_5}{dt} &= \delta_1 (1-u_1) \frac{\beta_H b S_H}{N_H} - \delta_2 (1-u_1) \frac{\beta_H b S_H}{N_H} + \delta_5 \mu_V
\end{aligned}$$

with transversality conditions $\delta_i(T) = 0, i = 1, \dots, 5$, and the u_1^* satisfies the optimality condition

$$u_1^* = \min \left\{ \max \left\{ 0, \frac{(\delta_2 - \delta_1) \frac{\beta_H b S_H^* I_V^*}{N_H} + (\delta_5 - \delta_4) \frac{\beta_V b S_V^* (1-\varepsilon) I_H^*}{N_H}}{2C} \right\}, 1 \right\}.$$

Proof. The differential equations developing the costate variables are obtained by differentiation of the Hamiltonian function that evaluated at the optimal control. Then the costate system can be written as follow

$$\begin{aligned}
\frac{d\delta_1}{dt} &= -\frac{\partial H}{\partial S_H} = \delta_1 \left((1-u_1) \frac{\beta_H b I_V}{N_H} + \mu_H \right) - \delta_2 (1-u_1) \frac{\beta_H b I_V}{N_H} \\
\frac{d\delta_2}{dt} &= -\frac{\partial H}{\partial I_H} = -B + \delta_2 (\mu_H + \gamma_H) - \delta_3 \gamma_H + \delta_4 (1-u_1) \frac{\beta_V b S_V (1-\varepsilon)}{N_H} - \delta_5 (1-u_1) \frac{\beta_V b S_V (1-\varepsilon)}{N_H} \\
\frac{d\delta_3}{dt} &= -\frac{\partial H}{\partial R_H} = -\alpha \delta_1 + \delta_3 (\mu_H + \alpha) \\
\frac{d\delta_4}{dt} &= -\frac{\partial H}{\partial S_V} = \delta_4 (1-u_1) \frac{\beta_V b I_H (1-\varepsilon)}{N_H} - \delta_5 (1-u_1) \frac{\beta_V b I_H (1-\varepsilon)}{N_H} \\
\frac{d\delta_5}{dt} &= -\frac{\partial H}{\partial I_V} = \delta_1 (1-u_1) \frac{\beta_H b S_H}{N_H} - \delta_2 (1-u_1) \frac{\beta_H b S_H}{N_H} + \delta_5 \mu_V
\end{aligned}$$

with transversality conditions $\delta_i(T) = 0, i = 1, \dots, 5$. On the interior of the control set, $0 \leq u_1 \leq 1$, the optimal control at the solutions $S_H^*, I_H^*, R_H^*, S_V^*, I_V^*$ of the corresponding state system (4) is derived by

$$\begin{aligned}
\frac{\partial H}{\partial u_1} &= 0 \\
\Leftrightarrow 2Cu_1^* + \delta_1 \frac{\beta_H b S_H^* I_V^*}{N_H} - \delta_2 \frac{\beta_H b S_H^* I_V^*}{N_H} + \delta_4 \frac{\beta_V b S_V^* (1-\varepsilon) I_H^*}{N_H} - \delta_5 \frac{\beta_V b S_V^* (1-\varepsilon) I_H^*}{N_H} &= 0 \\
\Leftrightarrow u_1^* &= \frac{(\delta_2 - \delta_1) \frac{\beta_H b S_H^* I_V^*}{N_H} + (\delta_5 - \delta_4) \frac{\beta_V b S_V^* (1-\varepsilon) I_H^*}{N_H}}{2C}
\end{aligned}$$

Hence, we obtain

$$u_1^* = \min \left\{ \max \left(0, \frac{(\delta_2 - \delta_1) \frac{\beta_H b S_H^* I_V^*}{N_H} + (\delta_5 - \delta_4) \frac{\beta_V b S_V^* (1 - \varepsilon) I_H^*}{N_H}}{2C} \right), 1 \right\}.$$

4. Numerical Simulation

In this section, we present numerical simulation to describe the optimal solution of the host-vector model. The parameter values are given as below.

Table 1. Values of Parameters used in Host-Vector Model

Parameter	Value	Reference
α	0,9619	[10]
μ_V	0,071428	[11]
b	0,5	[1]
β_V	1	[1]
β_H	0,75	[1]
μ_H	0,9619	[12]
γ_H	0,1428	[1]

A susceptible host who is bitten by infected mosquito cannot transmit the virus directly, he/she needs 3-14 days in this condition [13], so the value of ε is between $\frac{1}{3}$ and $\frac{1}{14}$. Using $A = 400$, $N_H = 10000$ and parameter in Table 1, we have $R_0 = 7 > 1$. It means without control, any solution will tend to the endemic equilibrium point. The initial values used were $S_H(0) = 0,6$; $I_H(0) = 0,3$; $I_V(0) = 0,1$. As shown in Figure 2, for susceptible hosts' class, the population rose initially, but dropped later on. This is because the population will go to the infectious class. While with the presence of the control, the population of the susceptible class increased over time.

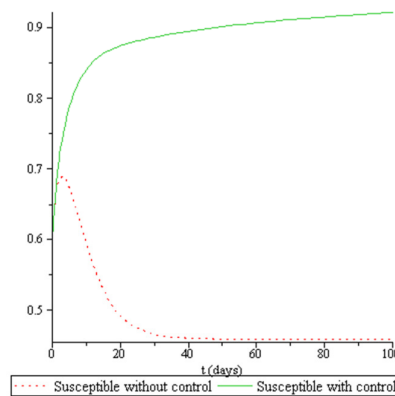


Figure 2. Susceptible human with and without control

Figure 3 explains that without control, the population of the infected hosts' increases. Conversely when the control is given, the population decreases sharply. In Figure 4, it can be seen that the vector population increases, both with and without controls. But, the population with control rose faster and higher than without control.

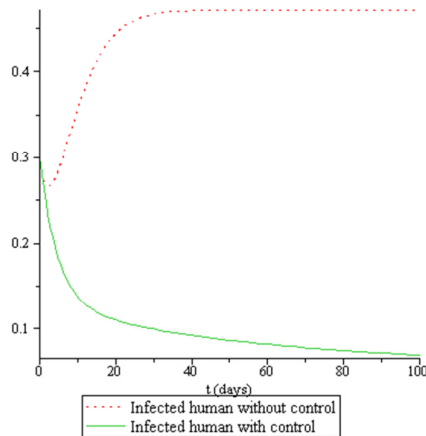


Figure 3. Infected human with and without control

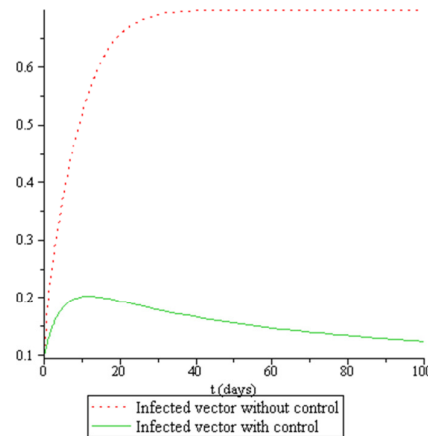


Figure 4. Infected vector with and without control

Figure 5 shows the optimal control function u_1 when the value in (5) are chosen $B = 1$ and $C = 50$. It can be seen that with preventive control can reduce the number of infectious host significantly.

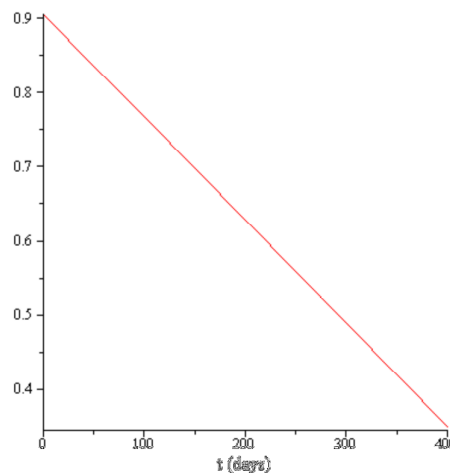


Figure 5. Optimal control function

5. Conclusion

This paper has discussed the development of a mathematical model for the dengue fever spread in the form of host-vector. The model then is extended by giving a prevention control for the spread. We described analytically that using Pontryagin Minimum Principle there is an optimal control. We also proved numerically that with a control strategy, we can decrease the population of infectious class both for the host and the vector.

Acknowledgments

The authors would like to thank to DIPA Faculty of Mathematics and Natural Sciences, Yogyakarta State University for the support and the research grant for this research.

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