# Conceptual or procedural mathematics for engineering students at University of Samudra

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**Abstract**. This study we investigate whether the emphasis in mathematics courses for engineering students would benefit from being more conceptually oriented than more procedurally oriented way of teaching. In this paper, we report in some detail from twenty-five engineering students comes from three departements ; mechanical engineering, civil engineering and industrial engineering. The aim was to explore different kinds of arguments regarding the role of mathematics in engineering courses, as well as some common across contexts. The result of interview showed that most of engineering students feel that conceptual mathematics is more important than procedural mathematics for their job the future.

**Keywords:** mathematics courses; engineering students; conceptual and procedural knowledge.

## Introduction

Several universities in Indonesian, especially faculty engineering in University of Samudra, the courses of mathematics is a subject of fundamental for engineering students in the future. Mathematics would became the basis for engineering students due to studied the others subjects in the curriculum. In the last two decades, several studies on mathematics at the workplace have been conducted. Most of these studies dealt with rather "lightweight" use of school mathematics. Just a few studies captured presumably heavy users of mathematics like engineers and scientists. Many engineering educators have proposed changes to the way that mathematics is taught to engineers: by having engineering professors teach mathematics rather than mathematics professors, by better integrating mathematics and engineering or by making large changes to the engineering curriculum. For example, uniting mathematics and engineering courses, or having mathematics and engineering educators collaborate in providing mathematics instruction to engineers. Her recommendations are based in a phenomenographic study of first year students. experiences learning mathematics (Cardella, 2008). Many of engineering students in the university are enrolled in the basic first-year courses mathematics, and to accommodate such large numbers, course enrollments are split into multiple large lectures and supplemented with smaller recitation sections.

According to Kent and Noss, mathematics ‘is andwill remain crucial’ for engineering education. It is not clear, however, how the mathematical education of engineering students should be organized. For example, the types of mathematical knowledge engineering students need, changes of practice and education suggested by the advance of computer technology, or how and when mathematics should be taught, are still discussed in the literature (Kent and Noss, 2003). This paper explores this issue by analysing interviews with twenty five of engineering students where the focus was on their views on the relevance of procedural and conceptual aspects of mathematics in the education. The aim of this study was to explore views and arguments concerning the role of conceptual versus procedural aspects of mathematics in engineering education.

**Blackground**

Correspondingly, the mathematical education of engineers has two major goals: It should enable students to understand, set up and use the mathematical concepts, models and procedures that are used in the application subjects like engineering mechanics, machine dynamics or control theory. In order to clarify this first goal, a lecturer has to investigate the use of mathematics in application textbooks or scripts of colleagues. Although this is also demanding and time-consuming for a non-engineer, the information is quite readily available. The second major goal of mathematics education is to provide students with a sound mathematical basis for their future professional life. For clarifying this rather nebulous goal, one needs information on the mathematical expertise or qualifications an engineer needs in his or her daily life (Alpers, 2010).

In his review of studies on the use of mathematics in engineering workplaces, Alpers identified several categories that ‘capture the way mathematical thinking or activities occur during the work on practical problems’ (Gainsburg, 2007). One category concerns the contextual embedding of mathematical models, concepts and procedures; another involves a recognition that the operation of technological tools still requires understanding of mathematical notations and graphics; while another category points at an appreciation that mathematics is important and necessary but not sufficient for engineering design, where also ‘engineering judgement’ and ‘sceptical reverence’ are critical.

Current experiences demonstrate that this belief is unfounded as engineering faculty members question the meaning of *mathematical competence*. Educators note, both through personal experience and in the research literature (Hiebert and Lefevre, 1986) that engineering students are not adapting the skills they learn in their math and physics classes to their engineering studies under traditional pedagogies. Often, the students just *don't see* how problem-solving strategies and skills learned in one setting apply to exercises and support thinking in another setting. Moreover, the difference of knowing about math, knowing about physics, and knowing about engineering concepts is both emphasized and encouraged by the curriculum in place in each of these disciplines. Indeed, transfer to new contexts of higher-level cognitive abilities from one domain to another is not automatic and may require special attention (Hiebert and Carpenter, 1992).

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**Conceptual and procedural approaches in mathematics work**

Although conceptual and procedural knowledge cannot always be separated, it is useful to distinguish between the two types of knowledge to better understand knowledge development. After interviewing a number of mathematics education researchers, Baroody, *et al* (2007) and colleagues suggested that conceptual knowledge should be defined as 'knowledge about facts, generalizations, and principles', without requiring that the knowledge be richly connected (Rittle and Alibali, 1999). As with conceptual knowledge, the definition of procedural knowledge has sometimes included additional constraints. Within mathematics education, Star (2005) noted that sometimes: 'the term procedural knowledge indicates not only what is known (knowledge of procedures) but also one way that procedures (algorithms) can be known (e.g. superficially and without rich connections)' (p. 408).

At many universities, mathematics studies in engineering education begin with calculus and linear algebra courses, often taught at mathematics departments that treat their subject as valuable knowledge in its own right, rather than as a service subject for practical use outside mathematics. In this context, there is a strong tradition regarding the content of these courses as well as teaching approaches used. However, even within this 'traditional' approach to mathematics teaching, there is a difference between taking a mainly conceptual or procedural approach. Teaching with a focus on conceptual understanding may start by providing a contextual basis for the new mathematical knowledge requiring students to connect to their prior knowledge. Taking a more procedural approach, the lecturer may present definitions, notations and procedures without first providing meaningful contexts to concepts and methods involved (Hiebert and Lefevre, 1986).

While the distinction between these two aspects of doing, teaching and learning mathematics has prevailed, it has also been criticized. In mathematics education, this distinction between conceptual and procedural knowledge was much discussed in the 1980s, mainly within a psychological framework with reference to Piaget and mental schemes. Hiebert and Lefevre defined conceptual knowledge in mathematics as a connected network of knowledge in which 'the linking relationships are as prominent as the discrete pieces of information (Hiebert and Carpenter, 1992). As an example of conceptual knowledge, they note the construction of a relationship between the algorithm for multi-digit subtraction and knowledge of the positional place values of digits (Hiebert and Carpenter, 1992). Procedural knowledge was defined by two parts, constituted by step-by-step procedures for solving mathematical tasks, on one hand, and knowledge of the symbolic representations used in such procedures, on the other hand. To be competent in mathematics, then involves not only of knowledge of concepts and knowledge of procedures but also of relations between these two types of knowledge (Skemp, 1978).

Research suggests that there is a complex interplay between conceptual and procedural knowledge (Peled and Zaslavski, 2008; Rittle and Alibali, 1999); conceptual knowledge may influence or even become procedural with repeated exposure,while procedural knowledgemay support the development of conceptual knowledge (Artique, 2007; Baker and Czarnocha, 2002) Some authors see these two constructs as independent, while others have emphasized dynamic and evolutionary aspects between them. Gray and Tall (1994) coined the term 'procept' for an integrated knowledge of processes and concepts that is characteristic for successful learners of mathematics. Kieran (2013) claims, however, that the distinction is a false dichotomy, with reference to the nature of algebraic symbols. Also, in the context of more advanced mathematics, Wu argues from a number of examples that 'in mathematics, skills and understanding are completely intertwined (Engelbrecht, *et al*., 2012).

In the context of mathematics in engineering education, the relation between conceptual and procedural approaches to solving mathematics tasks has been investigated within the larger project for the present study. That this relation is highly complex was illustrated by Engelbrecht, Bergsten and K agesten, who found that first-year engineering students tended to 'proceduralize' tasks designed to have a conceptual focus, sometimes requiring "quite sophisticated mathematical work". The study by Engelbrecht, Bergsten and K agesten on junior engineering students' achievement and views on conceptually versus procedurally focused mathematics tasks showed that the students considered procedural questions as more common in their mathematics curriculum, while conceptual (mathematical) questions were seen as more common in engineering subjects; they also expressed a higher confidence in their performance on the procedural tasks. However, both types of tasks were seen as relevant for their engineering studies. In (Engelbrecht, *et al*., 2009), strong correlations between performance and confidence for the procedural items for junior and senior students were found, while for the conceptual items, these correlations were much stronger for the senior than for the junior students. This points to an increased familiarity with the conceptual aspects of mathematics through its use in applied subjects throughout the education.

**Methodology**

The sample of the research were twenty-five engineering students comes from three departements ; mechanical engineering, civil engineering and industrial engineering. We address various populations engineering students views on the role of mathematics in engineering education with a particular focus on conceptual/procedural approaches. We selected engineering students who had the highest and the lowest IPK of populations, using a combination of convenience and snowball sampling as sample of research (Geertz, 1973). In earlier parts of the project, we have problematized the distinction between conceptual and procedural approaches, and surveyed junior (second year) and senior (fourth year) engineering students from both departement.

In the overall project as well in this study, we use the following working definitions regarding the approach to the mathematical education of engineers: A *conceptual approach* includes translations between verbal, visual (graphical), numerical and formal/algebraic mathematical expressions (representations); linking relationships; and interpretations and applications of concepts (for example, by way of diagrams) to mathematical situations. A *procedural approach* includes (symbolic and numerical) calculations, employing (given) rules, algorithms, formulae and symbols.

The research question, *How do engineering students view the relevance and role of conceptual and procedural mathematical skills in the education ?*, was used as a guide to structure the interviews and the analysis of the interview protocols. In this paper, we report, as a case study, in some detail from three of these interviews for the full results from different engineering students, selected to represent three different 'poles' of engineering students, one being more on research and product development oriented and one more on site practice oriented (Bryman, 2004). By looking at these different cases, which may represent both, what Bryman calls a *unique case* and an *exemplifying case*, we hoped to be able to find different kinds of arguments regarding the role of mathematics in engineering work, as well as some that were common across contexts (Geertz, 1973).

The interviews were audio-recorded and transcribed. In the semi-structured interviews, (Geertz, 1973) the concepts of conceptual and procedural approaches were initially explained to the interviewees, using our working definitions and a few examples of items from the research instrument used with the students. To investigate the research question above, the interviews were organized by the following guiding questions:

1. What is your opinion about the subjects of mathematics in your departement?
2. Do the mathematics courses you could be relevant to the subject futher in your departement ? Why ?
3. If the subjects is relevant to your futher, which method is more useful for your project, conceptual or procedural mathematics ?

For the analysis of the two interview protocols, a matrix was set up where the answers from each interviewer were organized in columns sorted by questions. By identifying content categories of responses in a column across the two interviewees, similarities and differences between their views were possible to describe for each question. A picture of the overall view of each interviewee on the procedural/conceptual issue was obtained by a two-step coding procedure, with specific codes sorted within overarching themes.

**Interview 1 – The Civil Engineering Students**

Civil engineering students learn mathematics in the first semester, the second semester and the third semester. In studying subjects they need more advanced mathematics than the other sciences such as physics, chemistry, and biology. Mathematics plays a central part in the formation of civil engineering students, both in the form of an entry requirment to undergraduate courses, and as a core underpinning element of those course.

To the opening interview question, *What is your opinion about the subjects of mathematics in your departement*? Based on interviews, it can be concluded that mathematics is a subject which is very difficult to understanding. Mathematics as both a language and an art, emphasizing that he has not been much involved in computational activities. Firstly, it’s a language to be learned, mathematical language, and then it it’s also a little art, I mean you must have some, you have to have imagination to solve these problems sometimes, and that one can learn mechanically but some have the aptitude for this, and some may work hard on it and so.

While question, *Do the mathematics courses you could be relevant to the subject futher in your departement ? Why ?*. Many of engineering students answered that the mathematics courses could be relevant to the subjects futher in my department. In the practice of civil engineering, ‘computational mathematics’ is perceived as a tremendous opportunity, pushing forward the boundaries of civil engineering design. For example, in the structural design of buildings, there are high-profile, high-budget projects, pioneering the development of techniques that gradually become used across the whole range of buliding projects (Kent and Noss, 2003).

When they asked about the emphasis on procedural or conceptual aspects of mathematics for engineering education, they says the he believes a little on both parts should be studied. They want to learn mathematics procedurally as to compute than the concepts or theories. While they having some difficulties to explain why the procedural parts of his mathematics experience is needed, but they still have not realized the importance of the issue the sustainability of other subjects.

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According to them, the courses of mathematics very difficult to learn because of less understanding of mathematics concepts and mathematical formulas that are difficult to remember. In addition, a simple counting techniques is also required by students in solving problems in mathematics. In education mathematics, engineering students are thus advised to encourage independence and confidence, and be able to collaborate with others on a challenging problem, do not sit with 'thumbin the answer key and looking'.

**Interview 2 – The mechanical engineering students**

Mechanical engineering students learn mathematics in the first semester and the second semester. To the opening interview question, *What is your opinion about the subjects of mathematics in your departement*? Based on interviews, it can be concluded that mathematics is a subject which is very difficult to understanding. Most of students come from vocational school, so many courses in they school not relevant to mathematics courses.

While question, *Do the mathematics courses you could be relevant to the subject futher in your departement ? Why ?*.Most of them answered that mathematics is relevant to the subject futher in our departement, such as technical mathematics, calculus, fluida mechanical, and etc. But, they have been difficulties to studied it. It happens because of the level of student knowledge that lower and mathematical formulas that are difficult to remember so the understanding of mathematical concepts and calculation in solving problems mathematical became less.

When they asked about the emphasis on the procedural aspects or conceptual mathematics for engineering education. They asked that conceptual mathematics is more important than procedural aspects for work futher. A *conceptual approach* includes translations between verbal, visual (graphical), numerical and formal/algebraic mathematical expressions (representations); linking relationships; and interpretations and applications of concepts (for example, by way of diagrams) to mathematical situations. A *procedural approach* includes (symbolic and numerical) calculations, employing (given) rules, algorithms, formulae and symbols. According to Baker (2002), in particular, engineering students must be able to express ideas that arise in engineering in the form of mathematical terms and then use their problem-sloving skills to understand the consequences. Based on my experineces in teaching ordinary differential equations to engineering students, students see their maathematical education as simply a vast collection of specific procedures. The question raised here is whether better coordination of the content in first-year math through conceptual and procedural approach could be improve student ability to use mathematics in subsequent engineering courses.

**Interview 3 – The Industrial Engineering Students**

To the opening interview question, *What is your opinion about the subjects of mathematics in your departement*? Based on interviews, it can be concluded that mathematics is a subject which is very difficult to understanding. Mathematics as both a language and an art, emphasizing that he has not been much involved in computational activities. Firstly, it's a language to be learned, mathematical language, and then it it's also a little art, I mean you must have some, you have to have imagination to solve these problems sometimes, and that one can learn mechanically but some have the aptitude for this, and some may work hard on it and so.

While question, *Do the mathematics courses you could be relevant to the subject futher in your departement ? Why ?*. Many of engineering students asked that the mathematic courses could be relevant to the subjects futher in my departement because some subjects require understanding of mathematical concepts and procedures, such as calculus, thermodinamic, industrial management, and the others.

They like to learn mathematics procedurally as to compute than the concepts or theories. While having some difficulties to pinpoint exactly why the procedural part of his mathematical experience is needed, They still hangs on to its importance even if he does not explicitly use it in his own work futher. Regarding the relevance of procedural versus conceptual mathematics for his present job, They continues to talk in rather general terms, acknowledging that he does not deal much with mathematics directly.

It’s probably more about, creativity, and see the possibilities, the job they have today maybe a little more towards, instead of this calculating so maybe a little more to the formulation and problem-solving way, they would say. But perhaps it is far from that as well. Despite the uncertainty expressed here, he again emphasizes that ‘one must be accurate and can’t be sloppy’ and that ‘both are important’, but that the key issue is problem solving, ‘in order to formulate the problems and be able to see something and structure and sort out’. It is clear that a conceptual approach, expressed through seeing structure and order, is more relevant for his own work than managing the calculations.

## Discussion

The development of conceptual and procedural knowledge in mathematics was simultaneously. The conceptual knowledge is knowledge that learners have about something. The conceptual knowledge in mathematics is knowledge of concepts, facts, definitions, and formulas but the conceptual knowledge refers to knowledge about facts and concepts of mathematics of students or the factors that influence the thinking and interesting it learning (Anderson, 2006).

The conceptual knowledge is relatively static but a procedural knowledge is more dynamic. When the procedural knowledge was activated, the result is not as a retrieval of information, but a transformation of information. For example, the results work on the problems of 999/3 is 333. The input information (999/3) has been converted into an output (333) that is different from the input.

Another difference between conceptual knowledge and procedural knowledge is on the speed of activating. When procedural knowledge has been studied once, this knowledge will be working quickly and automatically. For example, a skilled reader review a passage (articles, etc.) quickly. Reviewers process is a procedural knowledge, because the reading material is converted into a meaning. Activation of conceptual knowledge is slower and is more conscious. Gagne (in Winkel, 2006), said that the difference between conceptual knowledge and procedural knowledge is follows. Someone had learned verbal information, if he can tell you about the information. The information is called as verbal, because we know it in the form of sentence. Someone had learned an intellectual skill, if he knows how to do something as opposed to know what that something was.

Mathematicals problems is a useful tool to study the conceptual and procedural knowledge, because problem solving in mathematics specifically requires the application of both types of knowledges. It was was due to problem solving requires an understanding of the problem situation to produce a settlement. Although it is generally argued that the problem solving involves the understanding, and they often succes using the procedure without passing through the process of understanding.

For several years, researchers and educational practitioners were shocked by the systematic procedural errors committed by students. Observations on procedural errors that occur due to flaws in the conceptual knowledge. This shows that the procedural knowledge is based on the conceptual knowledge. A person who is given only the conceptual knowledge must construct the procedural knowledge. Two type of knowledges are often, and almost always that could not be separated each others (Shapiro, 2000).

The twenty five of engineering students interviewed for this study have very different department and are working in different areas and types of engineering. Already these examples thus highlight the potential complexity in designing what could be called ‘engineering education’, and in particular its mathematical component. The result of interview showed that most of engineering students feel that conceptual mathematics is more important than procedural mathematics for their job the future. But, if the students only understanding about the conceptual knowledge in mathematic education is not complete. It must construct the procedural knowledge. Based on the conceptual and procedural knowledge in mathematics education for engineering students especially student in university of Samudra is very important and could not be separated each others.

This message also echoes well the voices from the other twenty of five engineering students who were interviewed in the overall study described above (Bryman, 2002) Still, while the selection of the interviewees aimed to cover a wide spectrum of types of engineering work, the diversity and development of this professional field can never be ‘covered’ in any reasonable sense by one sample. By providing a ‘thick description’ of the arguments put forward in relaxed conversation settings by experienced engineers from different professional contexts, we hope to have pointed to some key concerns about the issue discussed relevant to their work. However, the intricacy of the relation between the procedural and conceptual aspects of mathematical work, as it has been described in these interviews, will need a continued empirical and epistemological attention in research to better understand the role of procedural knowledge in mathematics for engineering curriculum design.

There are complexities in the different uses of mathematics in engineering practice: not only the direct usefulness of mathematicals techniques and ideas to practice (for example, to perform a load case analysis of a certain kind of structure), but uses indirect usefulness of mathematics to practice. That is, it’s formative role in development of an engineering students-the ways in which mathematics contributed to the development of technical expertise and judgement. For example, matrix alegbra role in developing an understanding of many engineering principles, but in the form of an explicit matrix may be of little practical ‘use’ for the working engineering. It is easy to see that the direct use of mathematics in practice has changed, but the changes indirectly uses are less apparent.

Futhermore, various kinds of mathematics may be relavant at different times in engineering careers. Take the case of calculus; it is important for formal technical engineering education to respect the principles of basic engineering principles, while in practice it may rarely have explicit for many engineering students but yet it still present, ‘half remembered’as part of the appreciation of the analysis engineering education.

**Conclusion**

The twenty five of engineering students have been interviewed for this study have very different departement and are working in different areas and types of engineering. The result of interview showed that most of engineering students feel that conceptual mathematics is more important than procedural mathematics for their job the future.

So, the math educators are very focused on improvements in teaching their discipline. Their purpose is to ensure that students understand the concepts relevant to the field. They also appreciate that most students enrolled in the first year in math classes will be continuing in engineering, and so acknowledge that the topics covered should be appropriate for engineering students. However, in math, the emphasis is on improving skills and simplifying conceptual development without explicit attention to its connections to engineering.

**References**

[1] Alpers B 2010 Studies on the mathematical expertise of mechanical engineers *J. Math Model Appl*. 1(3):2–17
[2] Anderson O W & Krathwohl D R 2001 *A Taxonomy for Learning Teaching, and Assessing* (*A Revision of Blooms Taxonomy of Educational Objectives*). New York: Addision Wesley
[3] Artigue M 2007 Digital technologies: a window on theoretical issues in mathematics education. *Proceedings of CERME6*. Larnaca: University of Cyprus 68–82
[4] Baker W and Czarnocha B 2002 Written meta-cognition and procedural knowledge *Proceedings of the 2nd International Conference on the Teaching of Mathematics*.
[5] Baroody A J, Feil Y and Johnson A R 2007 An alternative reconceptualization of procedural and conceptual knowledge *J. Res Math Educ*. 38(2):115–131.
[6] Bryman A 2004 Social research methods Oxford University Press Oxford
[7] Cardella ME 2008 Which mathematics should we teach engineering students? An empirically grounded case for a broad notion ofmathematical thinking *Teach Math Appl*. 27(3):150– 159
[8] Engelbrecht J, Bergsten C and K°agesten O 2009 Undergraduate students' preference for procedural to conceptual solutions to mathematical problems *Int J.Math Educ Sci Technol*. 40(7):927– 940
[9] Engelbrecht J, Bergsten C and K°agesten O 2012 Conceptual and procedural approaches to mathematics in the engineering curriculum: student conceptions and performance *J. Eng. Educ*. 2012;101(1):138–162
[10] Gainsburg J 2007 The mathematical disposition of structural engineers *J. Res Math Educ*. 38:477–506
[11] Gray E and Tall D Duality 1994 ambiguity and flexibility: a proceptual view of simple arithmetic *J. Res Math Educ* 26(2):115–141
[12] Geertz C 1973 Thick description: toward an interpretive theory of culture New York Basic Books 3–30
[13] Hiebert J and Lefevre P 1986 Procedural and conceptual knowledge in mathematics: an introductory analysis Hillsdale (NJ) Erlbaum 1–27
[14] Hiebert J and Carpenter T 1992 Learning and teaching with understanding New York:Macmillan Publishing Company 65–97
[15] Kent P and Noss R 2003 Mathematics in the university education of engineers Available from: http://www.lkl.ac.uk/research/ REMIT/Kent-Noss-report-Engineering-Maths.pdf
[16] Kieran C 2013 The false dichotomy in mathematics education between conceptual understanding and procedural skills: an example from algebra Springer 153–171
[17] Peled I and Zaslavski O 2008 Beyond local conceptual connections: meta-knowledge about

procedures *Learn Math* 28(3):28–35
[18] Rittle J B and Alibali M W 1999 Conceptual and procedural knowledge of mathematics: does one lead to another? *J. Educ Psychol* 91(1):175–189.
[19] Shapiro Stewart 2000 Thinking about mathematics the philosophy of mathematics, New York. Oxford University Press
[20] Silver E 1986 Using conceptual and procedural knowledge: a focus on relationships. Hillsdale (NJ): Erlbaum. p. 181–198.
[21] Skemp R 1978 Relational and instrumental understanding *Arithmetic Teach* 26:9–15
[22] Star J R 2005 Reconceptualizing procedural knowledge *J. Res Math Educ* 36(5):404–411