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Students' mathematical representations on secondary school in solving trigonometric problems

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Abstract. This research aimed to analyse students' mathematical representations on secondary school in solving trigonometric problems. This research used qualitative method. The participants were 4 students who had high competence of knowledge taken from 20 students of 12th natural-science grade SMAN-1 Kota Besi, Central Kalimantan. Data validation was carried out using time triangulation. Data analysis used Huberman and Miles stages. The results showed that their answers were not only based on the given figure, but also used the definition of trigonometric ratio on verbal representations. On the other hand, they were able to determine the object positions to be observed. However, they failed to determine the position of the angle of depression at the sketches made on visual representations. Failure in determining the position of the angle of depression to cause an error in using the mathematical equation. Finally, they were unsuccessful to use the mathematical equation properly on symbolic representations. From this research, we could recommend the importance of translations between mathematical problems and mathematical representations as well as translations among mathematical representaions (verbal, visual, and symbolic) in learning mathematics in the classroom.

1. Introduction

Representation is one of the process standard should enable students to know and do from kindergarten to K-12. The representation standard should enable students to create and use representations to organize, record, and communicate mathematical ideas. Representations can be expressed in the form of visual, verbal, and symbolic. Visual representations include illustrate, show, or work with mathematical ideas using diagrams, pictures, number lines, graphs, and other math drawings. Verbal representations include using language (words and phrases) to interpret, discuss, define or describe mathematical ideas, bridging informal and formal mathematical language. Symbolic representations include recording or working with mathematical ideas using numerals, variables, tables, and other symbols [9, 11].

Other representation standard should enable students to select, apply, and translate among mathematical representations to solve problems. For instance, translations between mathematical problems and mathematical representations as well as translations among mathematical

representations. Thus, students must be able to move flexibly in between form of representations. [4, 9, 11]. It suggests using representations could help to facilitate when solving problem more efficient. For instance, visual representation could facilitate in solving algebraic problems than using symbolic representation, despite previous students were more likely to prefer symbolic representations over visual representations [10].

Translations between mathematical problems and mathematical representations as well as translations among mathematical intended to get the description of students' matematical representations abilities by themselves without provided (helped) by their teachers [18]. Illustration translations between mathematical problems and mathematical representations could be shown in Figure 1 as well as illustration translations among mathematical representations could be shown in Figure 2 as follows:

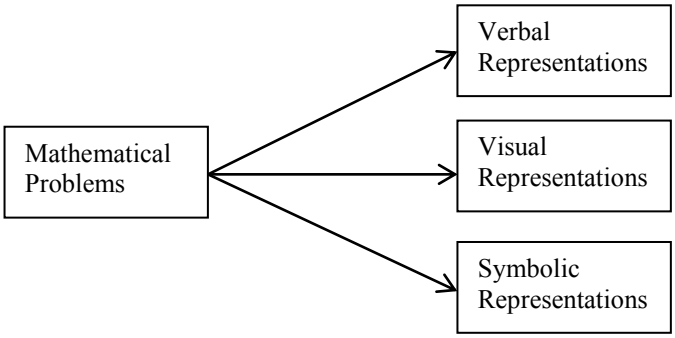

Figure 1. Illustration translations between mathematical problems and mathematical representations.

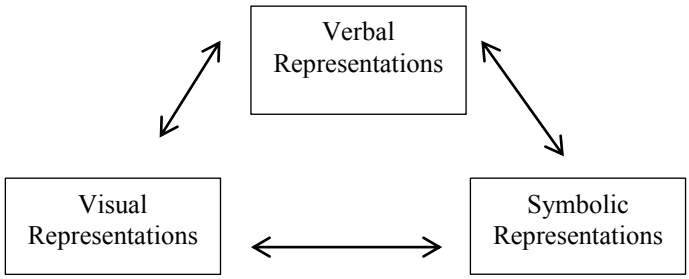

Figure 2. Illustration translations among mathematical representations.

Some of the benefits of representations could encourage students' mathematical ideas, especially in problem-solving ability (e.g., [11], [15], [20]). Besides in solving problem, representations useful in understanding the abstract concepts of mathematics. For instance, in transition between arithmetic and algebra by geometric representations as well as in teaching factoring second-degree polynomials (e.g, [3], [6], [12], [13]). Representations are not only to goal curriculum standard, but an important aspect to improving students' educational value as follows: (1) to help students consolidate their knowledge and improve skills, (2) to help teachers and students enrich their concept of mathematics and mathematical teaching, (3) to help students overcome the psychological barriers, (4) to help teachers assess students' learning result, and (5) to help teachers improve their own literacy [23].

Research related to mathematical representations previously showed that relationship between visual representations and problem-solving as well as in solving routin and non-routine problems (e.g., [14], [17]). Other research related to translation among mathematical representations showed students

were successful than the pre-service teachers in understanding of functions, likewise in representing the fractions on number lines through other representations (e.g., [1], [2]). From the research, representations were used in analyzing the understanding of prime numbers, making generalizations on algebra material, as well as in representing the law of cosines without using the Pythagorean theorem and trigonometry (e.g., [16], [21], [22]).

This research focused on trigonometry. Trigonometry is one of content standard that students should enable in school mathematics [11], either in national curriculum 2006 (KTSP) or national curriculum 2013 (K-2013). On the other hand, trigonometry is one of difficult materials in the classroom according to the local mathematics teacher experiences. The research related to trigonometry showed that there were difficulties in understanding trigonometric function by standard instruction [19] also in solving routine and no routine problems [5].

Based on the explanations above, this research was conducted to answer the question: How do students' mathematical representations abilities on secondary school in solving trigonometric problems?

2. Method

This research used qualitative method. The participants were 4 students, named Anne, Bob, Clara, and Tina who had high competence of knowledge taken from 20 students of 12th natural-science grade in SMAN-1 Kota Besi, Central Kalimantan. They were given 2 problems to determine their abilities of translation between mathematical problems and mathematical representations as well as translation among mathematical representations (verbal, visual and symbolic). The form of the problems are shown as follows:

Problem 1
Use the figure and right-triangle definition of sine to explain why, $\sin 20° < \sin 40° < \sin 60°$ and $\cos 20° > \cos 40° > \cos 60°$.

Problem 2
The pilot of aircraft is flying at an altitude of 1,300 m above the sea level saw a cruise ship sailing in the position of the angle of depression is $40°$.

- Make a sketch regard to the angle of depression, both pilot position and cruise ship position as well.
- How far the distance of cruise ship to the point at sea level which precise under the aircraft.

Meanwhile, data validation was carried out using time triangulation with data collection 2 times (test and interview). Data analysis used Miles and Huberman model which consists of three stages: (1) data reduction, (2) data display, and (3) conclusion drawing/verification.

3. Results

This research aimed to analyse the students' mathematical representations abilities, including verbal, visual and symbolic representations. Verbal representations analysis used Problem 1, as well as visual and symbolic representations used Problem 2. In this paper, their responses (i.e., test results and interviews) are not exposed completely. However, some responses in representing their abilities on mathematical representations. The results with respect to each their abilities on mathematical representations are shown the following.

3.1. Students' verbal representations abilities

Student's verbal representations ability who named Anne on the first test, she answered only based on the given figure. However, on the interview she was able to answer based on the definition of trigonometric ratio. On second test, she was able to answer based on the figure as well as used the definition of trigonometric ratio.

Student's verbal representations ability who named Bob on the first test, he answered only based on the value of the sine without used the definition trigonometric ratio. However, on the interview he was able to answer based on the definition of trigonometric ratio. On the second test, he was able to answer based on the given figure as well as used the definition of trigonometric ratio.

Students's verbal representations abilities who named Clara and Tina were almost similar on the first test. They didn't use the definition trigonometric ratio, but only based on the given figure. However, on the interview they were able to answer based on the definition of trigonometric ratio. On the second test, they were able to answer based on the given figure as well as used th definition of trigonometric ratio.

The excerpt of interview with one of them to explain orally on verbal representations as follows:

Interviewer:	Could you explain how to answer this question?
Student:	Since the concept of sine is invers by the concept of cosine Sir.
Interviewer:	Now, if we use the definition of cosine, How?
Student:	Based on the definition of cosine is the ratio of adjacent and hypotenuse.
Interviewer:	Thus, could you explain it completely?
Student:	Since the adjacent length of the angle 20° is longer than the adjacent length of the angle 40°as well as the adjacent length of the angle $40^\circ$is longer than the adjacent length of the angle 60°. Thus, $\cos 20^\circ > \cos 40^\circ > \cos 60^\circ$.

3.2. Students' visual representations abilities

Student's visual representations ability who named Anne on the first test, she was not able to determine the object positions (pilot and cruise ship) in the sketch made, as well as in determining the position of the angle of depression. On the other hand, students who named Bob, Clara, and Tina were successful to determine the object positions (pilot and cruise ship), although they failed to determine the position the angle of depression. On the second test, they (i.e., Anne, Bob, Clara, and Tina) were successful to determine the object positions (pilot and cruise ship). However, they also failed to determine the position of the angle of depression.

The following one of students' solution of problem of visual representations is shown in Figure 3 below:

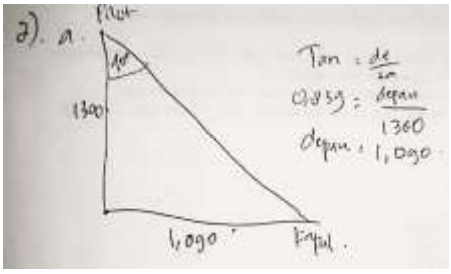

Figure 3. Students' solution of problem on visual representations.

3.3. Students' symbolic representations abilities

Student's symbolic representations ability who named Anne on the first test, she was able to determine the mathematical equation based on the sketches made as well as on the second test. However, the result was error because failure in determining the position of the angle of depression on the previous sketch made. On the other hand, students who named Bob, Clara and Tina on the first test, they didn't

answer or write the solution of problem. However, they were able to determine the mathematical equation. Meanwhile, on the second test, they were able to determine the mathematical equation. However, they failed to use the mathematical equation properly.

The following one of students' solution of problem of symbolic representations could be shown in Figure 4 below:

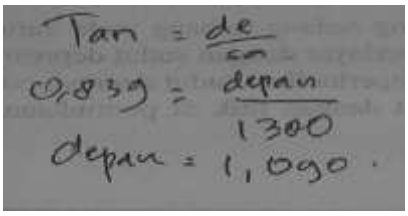

Figure 4. Students' solution of problem on symbolic representations.

The summary data analysis of students' mathematical representations abilities could be shown in Table 1, Table 2, Table 3, and Table 4 as follows:

Table 1. Summary of Anne's mathematical representations abilities.

Student	Mathematical Representations	The First Data Collection	The Second Data Collection	Conclusion
Anne	Verbal	On the first test, Anne only answer based on the given figure. However, on the interview Anne was able to answer based on the definiton of trigonometric ratio.	On the second test, Anne answer based on the given figure as well as based on the definiton of trigonometric ratio.	Anne was able to answer based on the given figure as well as based on the definition of trigonometric ratio.
	Visual	On the first test, Anne failure in determining the object positions and the position of the angle of depression. On the interview, Anne successful in determining the object positions. However, Anne failure in determining position of the angle of depression.	On the second test, Anne successful in determining the object positions. However, Anne failure in determining position of the angle of depression.	Anne successful in determining the object positions. However, Anne failed to determine the position of the angle of depression.
	Symbolic	On the first test, Anne Successful in determining the mathematical equation. However, she failed to use the mathematical equation properly.	On the second test, Anne Successful in determining the mathematical equation. However, she failed to use the mathematical equation properly.	Anne successful in determining mathematical equation. However, Anne failed to use mathematical equation properly.

Table 2. Summary of Bob's mathematical representations abilities.

Student	Mathematical Representations	The First Data Collection	The Second Data Collection	Conclusion
Bob	Verbal	On the first test, Bob only answer based on the value of trigonometric ratio. However, on the interview Bob was able to answer based on the definiton of trigonometric ratio.	On the second test, Bob answer based on the given figure as well as based on the definiton of trigonometric ratio.	Bob was able to answer based on the given figure as well as based on the definition of trigonometric ratio.
	Visual	On the first test, Bob successful in determining the object positions. However, she failure in determining the position of depression angle.	On the second test, Bob successful in determining the object positions. However, Bob failure in determining the position of depression angle.	Bob successful in determining the object positions. However, Bob failed to determine the position of the angle of depression.
	Symbolic	On the first test, Bob failure in determining the mathematical equation. However, on the interview Bob was able to determine the mathematical equation.	On the second test, Bob failure in determining the mathematical equation. However, on the interview Bob was able to determine the mathematical equation.	Bob successful in determining the mathematical equation. However, Bob failed to use the mathematical equation properly.

Table 3. Summary of Clara's mathematical representations abilities.

Student	Mathematical Representations	The First Data Collection	The Second Data Collection	Conclusion
Clara	Verbal	On the first test, Clara only answer based on the given figure. However, on the interview, Clara was able to answer based on the definiton of trigonometric ratio.	On the second test, Clara answer based on the given figure as well as based on the definiton of trigonometric ratio.	Clara was able to answer based on the given figure as well as based on the definition of trigonometric ratio.
	Visual	On the first test, Clara failure in determining the object positions and the position of the angle	On the second test, Clara successful in determining the object positions. However,	Clara successful in determining the object positions. However, Clara failed to determine

		of depression.	Anne failure in determining position of the angle of depression.	the position of the angle of depression.
	Symbolic	On the first test, Clara failure in determining the mathematical equation. However, on the interview, Clara was able to determine the mathematical equation.	On the second test, Clara also failure in determining the mathematical equation. However, on the interview, Clara was able to determine the mathematical equation.	Clara successful in determining the mathematical equation. However, Clara failed to use the mathematical equation properly.

Table 4. Summary of Tina's mathematical representations abilities.

Student	Mathematical Representations	The First Data Collection	The Second Data Collection	Conclusion
Tina	Verbal	On the first test, Tina only answer based on the given figure. However, on the interview, Tina was able to answer based on the definiton of trigonometric ratio.	On the second test, Tina answer based on the given figure as well as based on the definiton of trigonometric ratio.	Tina was able to answer based on on the given figure as well as based on the definition of trigonometric ratio
	Visual	On the first test, Tina failure in determining the object positions and the position of the angle of depression.	On the second test, Tina successful in determining the object positions. However, Tina failure in determining position of the angle of depression.	Tina successful in determining the object positions. However, Tina failed to determine the position of the angle of depression.
	Symbolic	On the first test, Tina failure in determining the mathematical equation. However, on the interview, she was able to determine the mathematical equation.	On the second test, Tina failure in determining the mathematical equation. However, on the interview, she was able to determine the mathematical equation.	Tina successful in determining the mathematical equation. However, she failed to use the mathematical equation properly.

4. Discussion

Based on the results above showed students were able to answer based on the given figure, as well as based on the definition of trigonometric ratio. Fathermore, students were able to determine the object positions to be observed. However, they failed to determine the position of the angle of depression. Finally, students were able to determine the mathematical equation. However, they unsuccessful to use the mathematical equation properly. According to Bruner's theory that representation consists of three modes, there are enactive, iconic and symbolic. The first mode of representation is enactive where student should have direct experience. The second mode representation is iconic where student should

have been able to memorize of their experiences. The third mode representation is symbolic where student should be able to understand the concept in learning trigonometry. For instance, students successful to prove the problem based on the definition of trigonometric ratio, to determine the object position, as well as to determine the mathematical equation. On the other hand, students unsuccessful to determine the position of the angle of depression as well as to use the mathematical equation properly because they didn't have direct experience, memory and understand concept to solve the problem they may encounter.

According to Goldin's theory, representation is divided into external representations and internal representations. External representations comprise students' representations, such as verbal, visual and symbolic. Meanwhile, internal representations comprise visual imagination, problem-solving strategies, as well as natural language students [7]. Based on Goldin's theory, interaction between the internal representations and the external representations is considered in learning mathematics. For instance, students successful to prove the problem based on the definition of trigonometric ratio, to determine the object position, as well as to determine the mathematical equation. Meanwhile, students unsuccessful to determine the position of the angle of depression. Finally, they failed to use the mathematical equation properly because interaction between the internal representations and the external representations is underdeveloped.

5. Conclusions

This research could be concluded students' mathematical representations abilities on secondary school in solving trigonometric problems as follows: (1) students were able to answer based on the given figure, as well as based on the definition of trigonometric ratio, either written or orally on verbal representations, (2) students were able to determine the object positions to be observed. However, they failed to determine the position of the angle of depression on visual representations, and (3) students were able to determine the mathematical equation. However, they unsuccessful to use the mathematical equation properly on symbolic representations. Based on these conclusions, we recommend the importance of translations between mathematical problems and mathematical representations as well as translations among mathematical representations (verbal, visual, and symbolic) in learning mathematics in the classroom. For the future research, we suggest to analyse the students' mathematical representations among secondary students, either on trigonometry or other materials.

References

[1] Bannister V R P 2014 Flexible conceptions of perspectives and representations: an examination of pre-service mathematics teachers' knowledge *Int. J. of Edu. in Math., Sci. and Technol.* **2** 223−33

[2] Biber A C 2014 Mathematics teacher candidates' skills of using multiple representations for division of fractions *Academic J.* **9** 237−44

[3] Cabahug J A 2012 The Use of bruner's modes of representations in teaching factoring second-degree polynomials *IAMURE: Int. of Edu.* **1** 85−103

[4] Dindyal J 2009 Mathematical problems for secondary classroom *Mathematical Problem Solving Yearbook 2009* ed B Kaur, Y B Har and M Kapur (Singapore: World Scientific Publishing) chapter 11 pp 208−225

[5] Dundar S and Yaman H 2015 How do prospective teachers solve routine and non-routine trigonometry problems? *Int. Outline and Educational Sci.* **7** 41−57

[6] Flores A 2002 Geometric representations in the transition from arithmetic to algebra *Representations and Mathematics Visualizations* ed F Hitt (Mexico: Ciinvestav-IPN) chapter 1 pp 9−30

[7] Godino J D and Font V 2010 The theory of representations as viewed from onto-semiotic approach to mathematics education *Mediterranean J. for Research in Math. Edu.* **9** 189–210

[8] Hegarty M and Kozhevnikov M 1999 Types of visual-spatial representations and mathematical

problem solving *J. of Educational Psy.* **91** 648−89
[9] Huinker D 2015 Representational competence: a renewed focus for classroom practice in mathematics *Wisconsin Teacher of Mathematics* **2015** 4−8
[10] Mielicki M K and Wiley J 2016 Alternative representations for algebraic problem solving: when are graphs better than equations? *J. of Problem Solving* **9** 3−12
[11] NCTM 2000 *Principles and Standards for School Mathematics* (USA: NCTM inc)
[12] Panasuk R M and Beyranevand M L 2011 Preferred representations of middle school algebra students when solving problems *The Math. Educator* **13** 32−52
[13] Pape S J and Tchoshanov M A 2001 The role of representation(s) in developing mathematical Understanding *Theory into Practice* **40** 118−27
[14] Presmeg N C and Balderas-Canas P E 2002 Graduate students' visualizations in two rate of change problems *Representations and Mathematics Visualizations* ed F Hitt (Mexico: Ciinvestav-IPN) chapter 2 pp 47−62
[15] Sajadi M, Parvaneh A and Rostamy-Malkhalifeh M 2013 The examining mathematical word problems solving ability under efficient representation aspect *ISPAC* **2013** 1−11
[16] Scher D and Goldenberg E P 2001 A multirepresentational journey through the law of cosinus *The Roles of Representation in School Mathematics* ed A A Couco (USA: NCTM inc) chapter 10 pp 117−128
[17] Stylianou D A and Pitta-Pantazi D 2002 Visualization and high achievement in mathematics: a critical look at successful visualization strategies *Representations and Mathematics Visualizations* ed F Hitt (Mexico: Ciinvestav-IPN) chapter 3 pp 31−46
[18] Terwel J, Oers B V, Dijk I V and Eeden P V D 2009 Are representations to be provided or generated in primary mathematics education? effects on transfer *Educational Research and Evaluation* **15** 25–44
[19] Weber K 2005 Students' understanding of trigonometric functions *Math. Edu. Research J.* **17** 91−12
[20] Yee S P and Bostic J D 2014 Developing a contextualization of students' mathematical problem solving *J. of Mathematical Behavior* **36** 1−19
[21] Zazkis R and Liljedahl P 2004 Understanding prime: the role of representation *J. for Research in Math. Edu.* **36** 164−86
[22] Zeljic M and Dabic M 2014 Iconic representation as students' success factor in algebraic generlitations *J. Plus Edu.* **10** 173−84
[23] Zhe L 2012 Survey of primary students' mathematical representation status and study on the teaching model of mathematical representation *J. of Math. Edu.* **5** 63−76